Question 1. *Suppose that Q Ethernet stations, all trying to send continuous streams of packets, require $\frac{Q}{2}$ slot times to sort out who transmits next. Assuming the time required to transmit the average packet is 5 slot times, express the efficiency of the network as a function of Q.*

The network described would alternate between sending useful data for 5 times slots and then wasting time due to collisions for $\frac{Q}{2}$ time slots. The total length of each such cycle would be $5 + \frac{Q}{2}$ during which 5 units of time would be spend actually sending data. Accordingly, the efficiency of the network would be

$$\frac{5}{5 + \frac{Q}{2}}$$

or

$$\frac{1}{1 + \frac{Q}{10}}$$

Question 2. *Let A and B be two stations attempting to transmit on an Ethernet. Each has a continuous stream of packets to send. We will call A's packets A_1, A_2, and so on and B's packets B_1, B_2, etc. Let S be the slot time. Suppose that A and B simultaneously attempt to send frame 1, collide and happen to choose backoff times of $0 \times S$ and $1 \times S$ respectively. That is, A decides to try to transmit again immediately and B decides to delay for one slot time so that it will transmit in the second slot. In this case, A transmits successfully and when B's delay completes it detects that the network is now busy with A's transmission and waits. At the end of this transmission, B will immediately attempt to retransmit B_1, while A will also attempt to transmit A_2. These transmissions will collide, but when A and B choose backoff times, they will use different ranges. Because A has only seen one collision since its last successful transmission, it will again choose a delay between $0 \times S$ and $1 \times S$. B, on the other hand, will choose a delay between $0 \times S$, $1 \times S$, $2 \times S$, and $3 \times S$.*

(a) *Give the probability that A wins this second backoff round immediately after the first collision. That is, the probability that A's first choice of backoff times is less than B's.*

There are 8 possible outcomes. Each outcome is described by the pair of delay values the stations choose. Thus, (0, 3) would describe the situation where A chooses no delay while B chooses the maximum delay while (1, 1) describes the situation where they both choose to delay 1 slot leading to a collision. In the outcomes (0,1), (0,2), (0,3), (1,2), and (1,3) A wins. B only wins in the situation (0,1). Another collision occurs in (0,0) and (1,1). All of these outcomes are equally likely, so the probability that A wins outright is 5/8ths.

We can also reason about this using probabilities of independent events rather than counting up the actual cases. The probability that A will succeed is the probability that A transmits with delay 0 and B chooses a delay greater than 0 OR A transmits with delay 1 and B chooses a delay greater than 1. Since A and B decide independently, this works out to:

$$\frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{4} = \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

(b) *Suppose that A does win this second backoff round. A and B will then again compete to transmit A_3 and B_1. As before, their initial transmissions will collide and then they will again choose backoff times. What range of backoff times will A choose from? What range will B choose from?*

A will again choose between 0 and 1. B will choose between 0 and 7.

(c) *What is the probability that A wins this third backoff round immediately.*

This time there will be $2 \times 8 = 16$ possible outcomes. Again, B can only win in (0, 1) or tie in (0,0) and (1,1). In all 13 other cases, A wins immediately. This result occurs with probability 13/16ths. Using probabilities:

$$\frac{1}{2} \times \frac{7}{8} + \frac{1}{2} \times \frac{6}{8} = \frac{7}{16} + \frac{6}{16} = \frac{13}{16}$$

You should observe that, in certain circumstances, Ethernet favors stations that have transmitted recently over stations that have lost collisions recently. This behavior is known as the Ethernet capture effect.

Question 3. *The analysis of Ethernet performance in the paper by Metcalfe and Boggs makes the simplifying assumption that each computer will transmit during a given slot with probability $\frac{1}{N}$ where N is the number of stations trying to transmit. In reality, we know that the probability that a station will transmit depends on the number of collisions it has experienced rather than on N. If a computer has experienced K collisions, it chooses a random delay value from the set $\{0, 1S, 2S, \ldots, (2^K - 1)S\}$. Therefore, the probability it transmits in any given slot is $\frac{1}{2^K}$.*

Metcalfe and Boggs make this simplifying assumption in order to obtain a formula that approximates the expected number of slots that will be wasted either by being left idle or because a collision occurs during the slot. Any formula for the expected number of wasted slots that accurately reflects the actual dynamics of the exponential backoff algorithm would be extremely complicated. To appreciate this, we would like you to determine a formula for the expected number of slots that would be wasted in a single round of a very specific collision resolution scenario.

Suppose that two computers, A and B, attempt to transmit simultaneously on an idle Ethernet. The process of collision resolution in which they will participate can be divided into "rounds." In round 0, both stations transmit immediately and collide. In round 1, they both randomly choose a delay from the set $\{0, S\}$ and transmit after this delay. In this round, the probability of a collision is $\frac{1}{2}$. If they collide again in round 1, then they engage in another round (round 2) in which they choose delays from the set $\{0, S, 2S, 3S\}$. The total number of slots wasted during this process will be the sum of the slots wasted in each round. It is obvious that the maximum number of slots wasted in round K is 2^K. The actual number of slots wasted in a given round, however, depends on the random delays selected.

For example, in round 1, there are four equally likely outcomes. Station A may choose delay 0 while station B chooses S. In this case, A will immediately transmit successfully and no slots will be wasted.. Similarly, if B chooses delay 0 and A chooses S, no slots will be wasted. On the other hand, if both A and B choose 0, they will both transmit immediately and collide. After this collision, they will both move on to round 2 immediately, so round 1 will only waste 1 slot. On the other hand, if the both choose a delay of S, one slot will be wasted by being left idle and another slot will be wasted due to collision. To compute the expected number of slots wasted in round 1, we simply sum the number of slots wasted in each of the four possible outcomes time the probability of each outcome ($\frac{1}{4}$) to obtain:

$$0 \times \frac{1}{4} + 0 \times \frac{1}{4} + 1 \times \frac{1}{4} + 2 \times \frac{1}{4} = \frac{3}{4}$$

That is, the expected number of slots that will be wasted during round 1 will be $\frac{3}{4}$.

Assuming that A and B do collide during round 1 and move on to round 2, determine the expected number of slots that will be wasted during round 2. Justify your answer.

> *To determine how many slots will be wasted in the second round, it is helpful to identify seven possible outcomes. There are three outcomes in which some stations actually transmits successfully. Such a transmission can occur in slot 0, slot 1, or slot 2. There are additionally four outcomes that result in a collisions. Such collisions can occur in slots 0, 1, 2, and 3.*
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> *The probability of a collision in any given slot is $\frac{1}{16}$ because the probability that one station transmits in a given slot is $\frac{1}{4}$ and both stations must transmit in the same slot to have a collision. If a collision occurs in slot i, then a total of $i + 1$ slots will be wasted. Therefore, the expected number of slots wasted in scenarios that end in collisions is*
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> $$(1 + 2 + 3 + 4) \times \frac{1}{16} = \frac{10}{16} = \frac{5}{8}$$
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> *The probability that a station succeeds in slot i is the probability that the station chooses slot i which is just $1/4$ times the probability that the other station chooses a slot later than i. Since there are $3 - i$ slots after slot i the probability of choosing one of these slots is $\frac{3-i}{4}$. That is, the probability a give stations succeeds in slot i is $\frac{3-i}{16}$. Either A or B can succeed with the probability, so the total probability of a success in slot i is double this amount or $\frac{3-i}{8}$*
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> *If a station succeeds in slot i, the only slots wasted are the slots that precede slot i. Since we number slots starting at 0, the number of slots that precede slot i is just i. Therefore, the expected number of slots*

wasted by the outcomes in which some station succeeds is

$$\sum_{i=0}^{3} i \times \frac{3-i}{8}$$

which is just

$$0 \times \frac{3}{8} + 1 \times \frac{2}{8} + 2 \times \frac{1}{8} + 3 \times \frac{0}{8} = \frac{1}{2}$$

Combining the expected number of slots wasted if a collision occurs with those wasted if a station succeeds, the total expected number of slots wasted in round 2 is $1\frac{1}{8}$