

We encode information so that it can be *efficiently transferred* from one place to another — whether it be between the hard drive and main memory of a computer or between people chatting on IM. This homework will explore the tradeoffs among several basic encoding schemes.

Question 1. Show the sequence of binary digits used to encode the four characters in the text **2ft.** in ASCII. A link to the ASCII encoding scheme is available on the course web page.

00110010 01100110 01110100 00101110

Question 2. ASCII is a fixed length code. Each character is a block of eight binary digits or bits. With 8 bit blocks, one can encode a maximum of 256 different characters. But what if we allow ourselves three values (say 0, 1, 2) instead of just two? This question explores the nature of ternary codes.

- (a) Suppose that we were only interested in encoding the 26 upper case letters of the American English alphabet. If we restrict ourselves to fixed length blocks, what is the minimum number of ternary digits per block required for the code?

$$\lceil \log_3(26) \rceil = 3$$

- (b) It's often convenient to write expletives using the characters $\$#:+$. For example, one might write "Holy $\$#:+$, I can't believe Brent and Tom have a question with the word ' $\$#:+$ ' in it." How many more ternary digits do we need to add to our block so these additional four characters can be included?

1 because $26 + 4 = 30$ characters require $\lceil \log_4(30) \rceil = 4$ bits

- (c) In addition to the four characters $\$#:+$, what if we also want to add the three characters $@=!$ to our alphabet. How many more ternary digits do we need now?

0 because 33 characters still requires $\lceil \log_4(33) \rceil = 4$ bits

- (d) In general, if we can encode a maximum of n characters with k ternary digits, how many characters can we encode with $k + 1$ ternary digits? How about $k + 2$? $k + 3$? $k + m$ where m is a non-negative integer?

- $3n = 3^{k+1}$ characters
- $3^2n = 3^{k+2}$ characters
- $3^3n = 3^{k+3}$ characters
- $3^m n = 3^{m+k}$ characters

Question 3. In contrast to fixed length codes, variable length codes may encode different characters with a different number of digits. For example, consider the following variable length binary code over the American English alphabet:

A=1110	B=110000	C=01001	D=11111	E=100	F=00101	G=110011
H=0110	I=1011	J=001001011	K=0010011	L=11110	M=00111	N=1010
O=1101	P=110001	Q=001001001	R=0101	S=0111	T=000	U=01000
V=001000	W=00110	X=001001010	Y=110010	Z=001001000		

Table 1: A variable length binary code for the English Alphabet

Notice that we encode *E* with three bits while *Q* takes nine bits. This is because we expect *E* to appear more frequently in a message than *Q*. In fact, this code is optimal with respect to the frequency of letters in the American English alphabet.

- (a) What is the encoding for the word BORAT using the variable length code above?

110000110101011110000

- (b) Suppose we were interested in only encoding the 26 upper case letters of the American English alphabet using a fixed length binary code. What is the minimum number of binary digits per block required for such a code? How many bits are required to encode the word BORAT with such a code? Is this shorter or longer than the variable-length encoding from part (a)?

5 bits is required to encode the 26 roman upper case characters with a fixed length binary code. Encoding BORAT takes $5 \times 5 = 25$ bits so the variable length code is shorter.

- (c) When decoding a fixed-length code with block size k , one reads the first k digits and decodes the block. However, with a variable length code, no such k exists. Try decoding the phrase 1110111101011110011 using the variable length code above. What does it decode to?

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- ★ (d) When a scheme like the code shown in Table 1 is used to transmit messages, the binary digits are not received all at once. They may instead arrive one by one. For example, if we were receiving the message described in part (b), then at some point, we would have received the bits 111, but not know whether the next bit would be a 0 or a 1. An instant later, we would have received the bits 1110, but again, we would not know what to expect next.

The code shown in Table 1 has an important property. As soon as the last bit in a codeword arrives, we can immediately decode it. The values of the digits that arrive later do not influence the interpretation. We say that such a code is instantaneous. For example, if you receive the bits 111 while processing a message encoded using the scheme in Table 1, you cannot tell whether these bits are part of the encoding of an A, an L or a D. If the next bit is a 0, however, you can instantly tell that the entire sequence received, 1110, encodes an A. Bits received later cannot change this.

By contrast, consider the code shown below in Table 2, which is designed to encode messages containing only letters from the set {A, B, C, D}. If while receiving a message encoded using this scheme we receive the string 011111 but don't know what (if any) additional digits we will receive later, then we cannot determine whether some substring of the bits received thus far should be interpreted as CD, AD, or BD.

A=0 B=01 C=011 D=111

Table 2: A variable length binary code for A, B, C, and D

What feature of the code in Table 1 makes it instantaneous? That is, explain why the code in Table 1 is instantaneous whereas the code in Table 2 is not.

The code in Table 1 is instantaneous because it is prefix-free. In other words, no codeword is the prefix of any other codeword. Because of this, as soon as we encounter a matching codeword, we can instantaneously decode it.

Question 4. This question departs from the encoding theme above in order to help firm up your Java programming vernacular. Below is a sample Java program. It is similar to the one you typed into BlueJ in Laboratory 1. Briefly, identify which lines are described by each of the following programming terms.

- | | |
|---|--|
| (a) Class declaration 4 or 4-20 | (e) Method body 15-17 or 14-18 |
| (b) Instance variable declaration 6,7 | (f) Local variable declaration 15 |
| (c) Constructor definition 9-12 | (g) Assignment statement 16 |
| (d) Method header 14 | (h) Method invocation 10,11,17 |

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1.      import squint.*;
2.      import javax.swing.*;
3.
4.      public class TouchyButton extends GUIManager {
5.
6.          private final int WINDOW_WIDTH  = 150;
7.          private final int WINDOW_HEIGHT = 300;
8.
9.          public TouchyButton() {
10.             this.createWindow( WINDOW_WIDTH, WINDOW_HEIGHT );
11.             contentPane.add( new JButton( "Click Here" ) );
12.          }
13.
14.          public void buttonClicked() {
15.              JLabel j;
16.              j = new JLabel("I'm Touched");
17.              contentPane.add( new JLabel( "I'm Touched" ) );
18.          }
19.
20.      }
```

Figure 1: Some Java code from Laboratory 1