CS 336 Lecture 5– 2/16/98

Announcements

- Exercise 2 is now available on the course web page (due Wednesday).

Fourier Series (cont.)

1. For a periodic signal $s(t)$ of period $P = 1/f$, Fourier's showed that

$$s(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos(2\pi n f t) + b_n \sin(2\pi n f t))$$

where

$$a_n = \frac{2}{P} \int_0^P s(t) \cos(2\pi n f t) dt$$

and

$$b_n = \frac{2}{P} \int_0^P s(t) \sin(2\pi n f t) dt$$

2. Last time, we determined the Fourier coefficients for one component of the encoding of a digital signal using frequency modulation. Namely the $d(t)\sin(2\pi f_1 t)$ term of the frequency modulated signal described by

$$d(t)\sin(2\pi f_1 t) + (1 - d(t))\sin(2\pi f_2 t)$$

In particular, we concluded that

$$d(t)\sin(2\pi f_1 t) = \frac{\sin(2\pi f_1 t)}{2} + \sum_{\text{n = 1 and n odd}}^{\infty} \frac{1}{\pi n} (\cos(2\pi(f_1 - nf)t) + \cos(2\pi(f_1 + nf)t)$$

- This still may not look like a sine wave to you, but recall that a cosine wave is just a sine wave with a phase shift. So, we really have expressed $s_1(t)$ as the sum of sine waves with frequencies of the form $(f_1 \pm nf)$.
- Unfortunately, we again have an infinite set of sine waves.
- Fortunately, the higher frequency waves have smaller coefficients AND the frequencies are centered around the carrier frequency.
- As we expected, the higher the data rate, the wider the spread of frequencies will be.
- Fortunately, in "real" systems, replacing the square wave with squares with rounded corners can further reduce the range of frquencies that are significant.
- If we took the time to account for the other component of the transmitted signal, $(1 - d(t))\sin(2\pi f_2 t)$, we would obtain a similar series of sine waves with frequencies of the form $(f_2 \pm nf)$
- Thus, the spectrum of the actual signal will consist of two sequences of sine waves each centered around one or the frequencies used as a carrier wave.

3. If nothing else, these results should let you understand what is so wonderful about fiber. (The following figures are meant more to be suggestive of the scale of things than precisely grounded in reality.)

- The spectrum of visible light falls roughly between $2.5x10^{15}$ and $5x10^{15}$Hertz. Thus, the range of frequencies available for multiplexed signals in this range is about $2.5x10^{15}$ hertz.
- When transmitting at one Gigahertz, the span of the first five terms of a frequency modulated signal would be about 10^{10} Hertz.
- The spectrum of visible light could be divided into roughly $2.5x10^5$ channels each capable of carrying a Gigahertz.

4. Now, think about what would happen if instead of using two frequencies, we used four frequencies to encode 2 bits per baud.

- If we choose the new frequencies so that they fall between the two original frequencies, the range of the spectrum required for the signal will not increase!
- Thus, in this case, the bandwidth available limits the baud rate, but not necessarily the bit rate.
- What limits the bit rate is noise on the line (and in the receiver) which limits how closely together the frequencies used can fall before we can no longer distinguish them.

Dispersion

1. One of the homework problems was designed to lead you to recognize an important physical limitation on the transmission rate possible through optical fiber.

- I'm referring to the problem that asked you to think about the difference in propagation speed between the light rays that travel "straight" through a fiber and those that bounce back and forth between the boundaries of the core and cladding.

- The problem was intended to make you realize that the light pulses sent through the fiber get wider as they travel. This is called *dispersion*.

  At the point where the pulses have spread so much that the pulses on either side of a given pulse completely overlap the given pulse, it will become basically impossible to decode the intended signal.
- As a result, in such fiber there is an upper bound on the product of the data rate and the length of the fiber.
- Graded-index fiber and single mode fiber are used to reduce this effect.

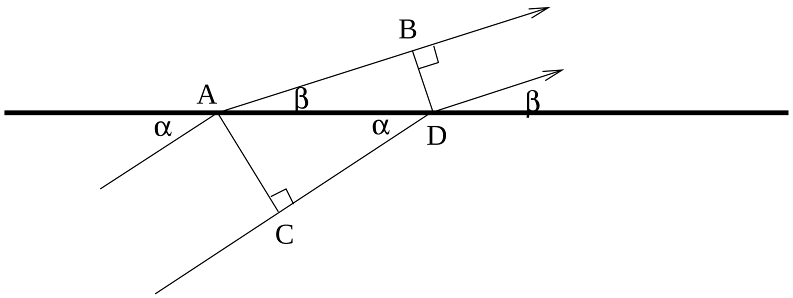
2. In addition to spreading the pulse out, the different paths light rays can follow in fiber will distort the "squareness" of the light pulses.

   - The intensity at the leading edge will be reduced as only the fastest rays of the leading edge will remain "leading".

3. By investigating the source of this effect a bit more closely, we will realize that a form of dispersion will also effect electrical signals and optical signals in single mode fiber.

4. The problem I assigned involved an angle that depended on the indices of refraction of the core and cladding of the fiber. Let's do a little physics to make sure we understand what the index of refraction of a material really is.

   - The diagram below represents two parallel rays of light being refracted at the boundary between two materials with different indices of refraction, η_1 and η_2.



   - Assuming that η_1 is the index of refraction of the material through which the light passes first, then the angles can be determined using *Snell's Law*:

$$\frac{\eta_2}{\eta_1} = \frac{\cos\alpha}{\cos\beta}$$

   - To appreciate this formula, one must understand what the constant known as the index of refraction really is.
     - When we speak of the speed of light, "c", we are really talking about the speed of light in a vacuum.
     - When light (or other electromagnetic signals) travel through other materials they travel at speeds less than c.
     - The index of refraction of a material is the number by which one must divide "c" to obtain the speed of light in a particular material.
   - In the diagram, the point C represents the location of the leading edge of one of the rays at the point that the other ray reaches the boundary (at A). The points B and C similarly represent the positions of the two rays at a slightly later point in time.
   - From the diagram it is clear that

$$\overline{CD} = \overline{AD}\cos\alpha$$

   and

$$\overline{AB} = \overline{AD}\cos\beta$$

   - Therefore:

$$\frac{\overline{CD}}{\overline{AB}} = \frac{\cos\alpha}{\cos\beta}$$

   - If t is the time it takes one ray to travel from A to B, then $\overline{AB}$ must equal t times the speed at which the ray is traveling. This speed is c/η_2. Similarly, the speed of the ray going from C to D is c/η_1.
   - Replacing the lengths of CD and AB then yields:

$$\frac{\eta_2}{\eta_1} = \frac{\cos\alpha}{\cos\beta}$$

5. Now that you understand how refraction behaves in a fiber, let's get back to an example of refraction with which you are probably more familiar: the behavior of white light passing through a prism.

   - As you know, passing white light through a prism separates the light showing the colors of the spectrum.
   - This occurs because different frequencies of light are refracted differently.
   - This implies that the index of a refraction is not really a constant associated with a material, but a function of both the material and the frequency of the light passing through.

- Finally, since the index of refraction is just the ratio of the speed of light in a material to its speed in a vacuum, this implies that different frequencies of light travel at different speeds (unless in a vacuum).

6. Now, we can put this together with Fourier analysis to realize the horrible truth.
   - When we send anything other than a pure sine wave, we are actually sending a wave that is composed of many different frequencies.
   - These different sine wave will travel at different speeds.
   - As a result, they will spread apart as they travel just like the rays of light taking different paths through a fiber.
7. The horrible truth: All baseband and modulated signals suffer from dispersion.

Attenuation and Channel Characterization

1. For one of the problems on the second homework assignment, I've asked you to examine an interesting property of sine waves.
2. Dispersion has the effect of distorting the shape of most waves transmitted through a wire or fiber.
   - What starts as a nice square wave tends to turn into a rounded little bump.
3. This sort of distortion does not affect sine waves.
   - If a pure sine wave is transmitted through a noiseless channel, the signal received at the other end will be a sine wave of the same frequency.
   - The amplitude of the wave will be reduced (a process known as attenuation, but the shape (sine) and frequency will not change!
4. As you will learn through the homework problem, this behavior is a consequence of the *linearity* of transmission systems.
5. If the transmitted signal is a sine wave, two pieces of information are needed to predict the received signal.
   - The phase of the received signal will depend on the total propagation time. This, as we just discussed, will depend on the signal frequency.
   - The amplitude of the signal will depend on the degree to which it is attenuated as it passes through the transmission medium. This is also dependent on the waves frequency.
6. One can characterize the behavior of a transmission medium by determining two functions:
   (a) A function that maps a sine wave's frequency into its propagation speed (or alternately the phase shift that will be observed when it is received).
   (b) A function that maps a sine wave's frequency into the factor by which it will be attenuated.

   Given an arbitrary signal to be transmitted through the medium one can then apply these functions to the transmitted signal's Fourier series to determine its form on arrival.