Exercise 11 —The Physical Layer

Due: December 3/4, 2015

The lowest level of the OSI model's hierarchy is the *physical layer* which involves "transmitting raw bits over a communication channel." Our goal for this week will be to learn enough about the physical layer to appreciate the ways in which the physics of the communication media used places constraints on and provides opportunities for computer communication.

The readings for the week will come from our textbook (Peterson and Davie) and from texts by Tanenbaum and Walrand. I will put copies of the required material from Tanenbaum and Walrand online in PDF form.

Unfortunately, while Peterson and Davie admit the existence of the physical layer in Chapter 1, they largely ignore it in the rest of their text. Sections 2.1 and 2.2 of their book are all the coverage they provide of this layer. To compensate for this, I would like you to read some sections from Tanenbaum, and sections from two different editions of a text by Walrand. The readings from Tanenbaum discuss transmission media and introduce the use of Fourier analysis to understand transmitted signals. The readings from Walrand reinforce the materials from Tanenbaum and include details on several widely used schemes for encoding information for transmission.

I would like you to read §2.1 through §2.2 of Tanenbaum (pages 85-99). In addition, I would like you to read most of §3.1 and §3.3 from the first edition of Walrand's text (pages 69 - 79 and 97 - 108). Finally, §7.1.4 (pp. 209-213) of the second edition of Walrand's text provides descriptions of some standard encoding schemes.

Exercises

1. Sketch the encoding for the bit stream: 00011101 using

   (a) bipolar modulation (i.e. NRZ),

   (b) on-off keying (OOK),

   (c) manchester encoding,

   (d) non-return to zero with inversion (NRZI) in conjunction with the 4B/5B code, and

   (e) frequency shift keying.

   What advantages does each scheme possess? As you answer this question, think about you might compare and contrast these various encoding schemes orally during our meeting.

2. Walrand presents an analysis that predicts the frequency spectrum found in the signal produced when an alternating sequence of 0's and 1's is encoded using frequency shift keying (pp. 106-107 of 1st edition reading). He concludes that the resulting signal has a bandwidth of $f_1 - f_0 + 5R$ where f_0 and f_1 are the two carrier frequencies used and R is the rate at which binary symbols are being transmitted. The "5" is his formula is the result of the claim that the bulk of the power of the signal is found in the first five components of its Fourier series (for which he actually never provides much justification).

   I would like you to perform a similar analysis for one of the other broadband modulation schemes, phase shift keying. Determine the spectrum for the signal generated when the binary sequence

01010101... is encoded using PSK. Using Walrand's assumption that the first five terms of the Fourier series determine the bandwidth required for the signal, what bandwidth will this signal require? How does this compare to FSK?

(Warning: Years ago I took a course using a text with quite a few typos, or, as one of my classmates like to say "a lot of good exercises that are not labeled as such." The derivation on pp. 106-107 of Walrand is a bit like that. The overall structure of the derivation is right and your derivation will follow it closely, but there are definitely a few glitches along the way. I guess, in fact, that part of the point of this problem is to encourage to read the text critically rather than to assume it is always correct.)

3. Both Tanenbaum and Walrand touch on the importance of Fourier analysis to the understanding of the transmission of signals through various channels. You might have noticed that while Tanenbaum and Walrand both claim to be talking about "Fourier analysis", their explanations of what Fourier analysis is appear contradictory. Tanenbaum claims that Fourier showed that any periodic function f(t) can be written as a sum of sines and cosines (page 90):

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(2\pi nft) + \sum_{n=1}^{\infty} b_n \cos(2\pi nft)$$

Walrand, on the other hand, states that Fourier analysis is based on the fact that any signal can be expressed as a sum of sine waves where a sine wave is defined to be a function of the form (page 99 in 1st edition reading):

$$s(t) = A\sin(2\pi ft + \theta)$$

To make matters worse, in his first example of such a "sine wave" decomposition, Walrand uses cosines instead of sines! (see equation 3.12 in Walrand's text).

(a) First, show that it is fair for Walrand to use "cosine waves" instead of "sine waves". That is, show how to express any cosine wave as a sine wave.

(b) Suppose that we restrict Walrand's scheme by limiting our attention to sums of sine waves with frequencies that are multiples of some base frequency. That is, we claim that any signal can be expressed as a sum of the form:

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} A_n \sin(2\pi nft + \theta_n)$$

Show that any sum, of this form can be rewritten in Tanenbaum's form.

(c) Again restricting your attention to sums of sine waves whose frequencies are multiples of some base frequency, show that any function written in Tanenbaum's form can be written as a sum of sine waves (i.e. no cosines allowed).

Hint: Almost all you will need to do these problems is the identity

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

and the values of the sine and cosine functions at key values such as $\sin 0 = 0$, $\cos 0 = 1$, $\sin(\pi/2) = 1$ and $\cos(\pi/2) = 0$. For part (c), you may want to remember the arctangent function (although it isn't really necessary).

4. One reason Fourier analysis is important to communications is that it can be used to understand how signals are distorted, even if transmitted on a channel that was somehow completely free from noise.

   This comes from the fact that "transmission lines do not distort sine waves; they only delay and attenuate them" proven by Walrand (pages 99 and 100). Walrand's proof depends on the use of complex number (for which he appropriately apologizes). Now that you know all the ways in which a sum of sine waves can be written (from the preceding problem) it is fairly easy to show this result without using complex numbers. So, show that in a linear, time-invariant system with input

   $$x(t) = A\sin(2\pi ft)$$

   the output will be a sine wave with the same frequency. Note, it is fine if you actually show that the output is either a simple sine wave of the form

   $$y(t) = B\sin(2\pi ft + \theta)$$

   or a cosine wave of the same form or a sum of a sine and cosine:

   $$y(t) = B_s\sin(2\pi ft) + B_c\cos(2\pi ft)$$

   Hint: Use the formula for the sine of a sum from the previous problem's hints to calculate the output $y'(t)$ produced in response to the input $x'(t) = x(t{+}a)$. Use the linearity of the transmission channel to express $y'(t)$ in terms of the still unknown output function y(t). Now, consider y'(t) for the two cases $a = -t$ and $a = \frac{1}{4f} - t$. This will give you two equations that you can solve for y(t). The final expression should have the desired form, although it will be expressed in terms of two unknown (but constant) values of y(t), namely $y(0)$ and $y(\frac{1}{4f})$. For this problem, you may also find it helpful to recall that $\sin -x = -\sin x$ and $\cos -x = \cos x$.

5. As explained in both texts, different rays of light take different paths through a fiber and therefore take different amounts of time to reach the receiver. If η_1 and η_2 are the indices of refraction of the core and cladding of a fiber respectively, then the slowest rays make an angle θ with the axis of the fiber, where $\cos\theta = \eta_2/\eta_1$. Determine a formula for the time it would take the fastest rays (i.e. the rays propagating parallel to the fiber) to propagate a distance L. Determine the distance the slowest rays travel along the fiber in the time it takes the fast rays to travel the distance L. Assuming the transmission rate is R, how wide is a bit when it is first transmitted? How wide is a bit by the time it has propagated a distance L? How do these results relate to formula (3.2a) in Walrand?