Timely Detection of Heavy Hitters in External Memory

Michael A. Bender
Stony Brook University

J. W. Berry
Sandia National Laboratories

Martin Farach-Colton
Rutgers University

Rob Johnson
VMware Research

Thomas Kroeger
Sandia National Laboratories

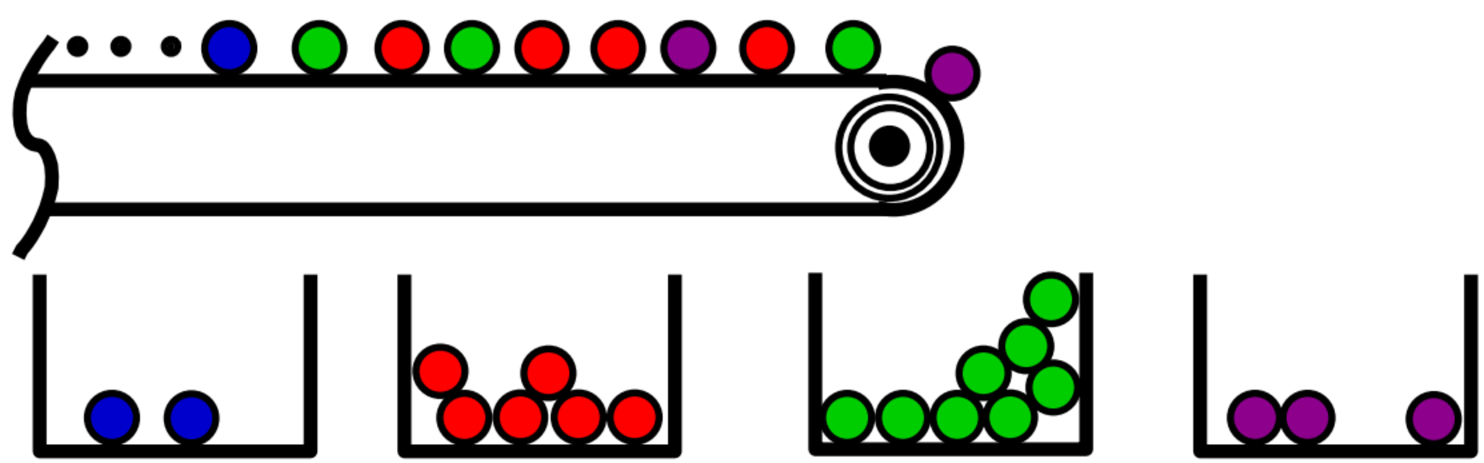
Prashant Pandey
Carnegie Mellon University

Cynthia Phillips
Sandia National Laboratories

Shikha Singh
Williams College

The Heavy Hitter Problem

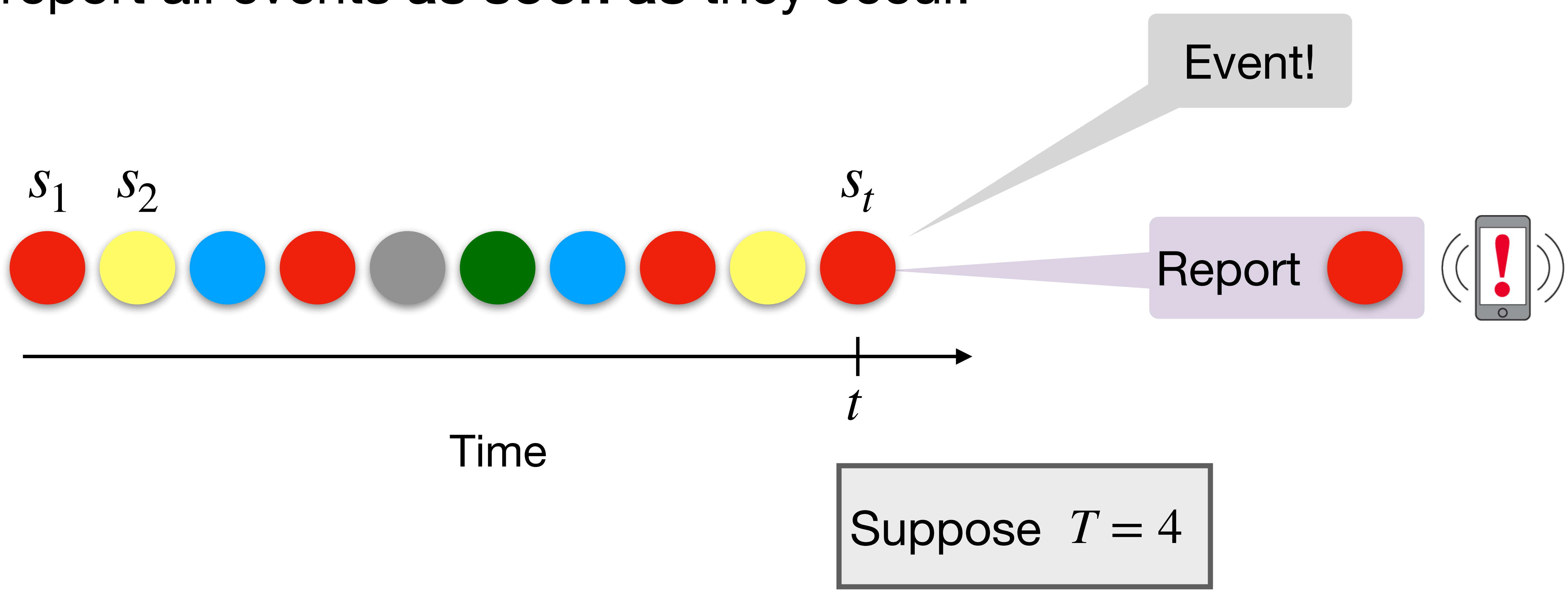
- Also called **the frequent items problem**
- Stream of N elements arrive over time
- A heavy hitter is an element that occurs at least ϕN times
- Usually reported at the end of the stream
- **Hard in small space**: exact solution requires Ω(N) words



[Cormode 05]

Timely Heavy Hitters: Online Event Detection Problem (OEDP)

- Stream of elements arrive over time
- An **event** occurs at time t if s_t occurs exactly $T = \phi N$ times in $(s_1, s_2, \ldots, s_t)$
- In the **online event detection problem (OEDP)**, we want to report all events **as soon as** they occur.



OEDP Requirements

- Stream is **large & high-speed** (millions/sec)

HIGH THROUGHPUT (fast inserts)

OEDP Requirements

Every insert is also a query!

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- Events are **high-consequence real-life events**

Immediate reporting (ONLINE)

NO ERRORS (esp. false negatives)

OEDP Requirements

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HIGH THROUGHPUT (fast inserts)

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Immediate reporting (ONLINE)

NO ERRORS (esp. false negatives)

- Very small reporting threshold ***T*** << **N** (stream size)

Be scalable to SMALL THRESHOLDS

Firehose Streaming Benchmark

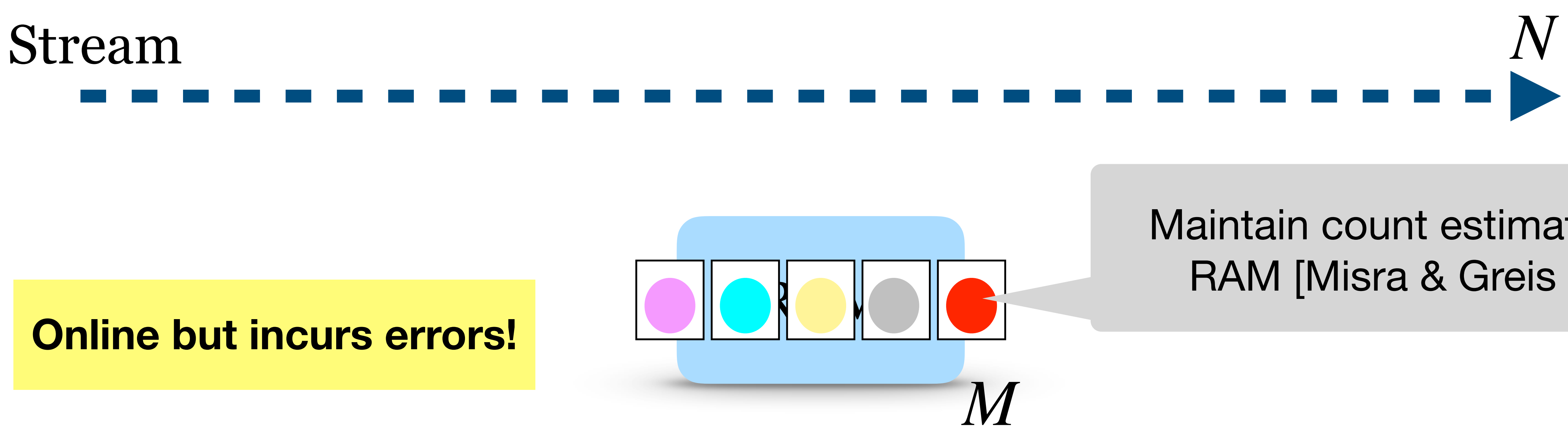
- Department of Defense (DoD) and Sandia designed the Firehose benchmark for this setting [https://firehose.sandia.gov/]
- The high-speed input stream consists of (key, value) pairs
- On the **24th** occurrence of key appears, some function of its values must be reported immediately
- Most difficult part of this is determining **when the 24th instance of a key arrives**

Event

T = 24 = o(1)
Stream size ~ 1 TB

One-Pass Streaming Has Errors

- Exact one pass solution requires $\Omega(N)$ space
- Approximate solutions trade off accuracy for space [Alon et al. 96, Berinde et al. 10, Bhattacharyya et al. 16, Bose et al. 03, Braverman et al. 16, Charikar et al. 02, 05, Demaine et al. 02, Dimitropoulos et al. 08, Larsen et al. 16, Manku et al. 02., Misra and Gries. 82, etc.]

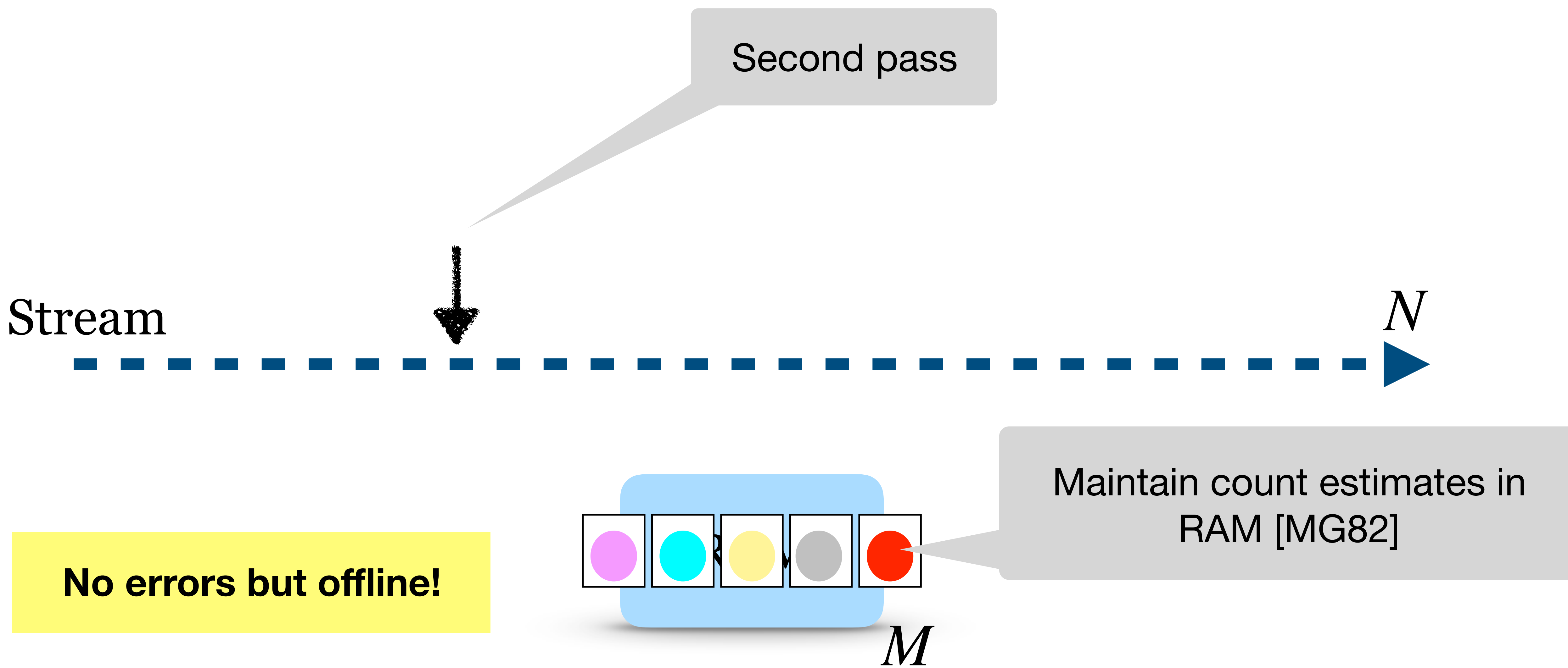


Maintain count estimates in RAM [Misra & Greis 82]

Online but incurs errors!

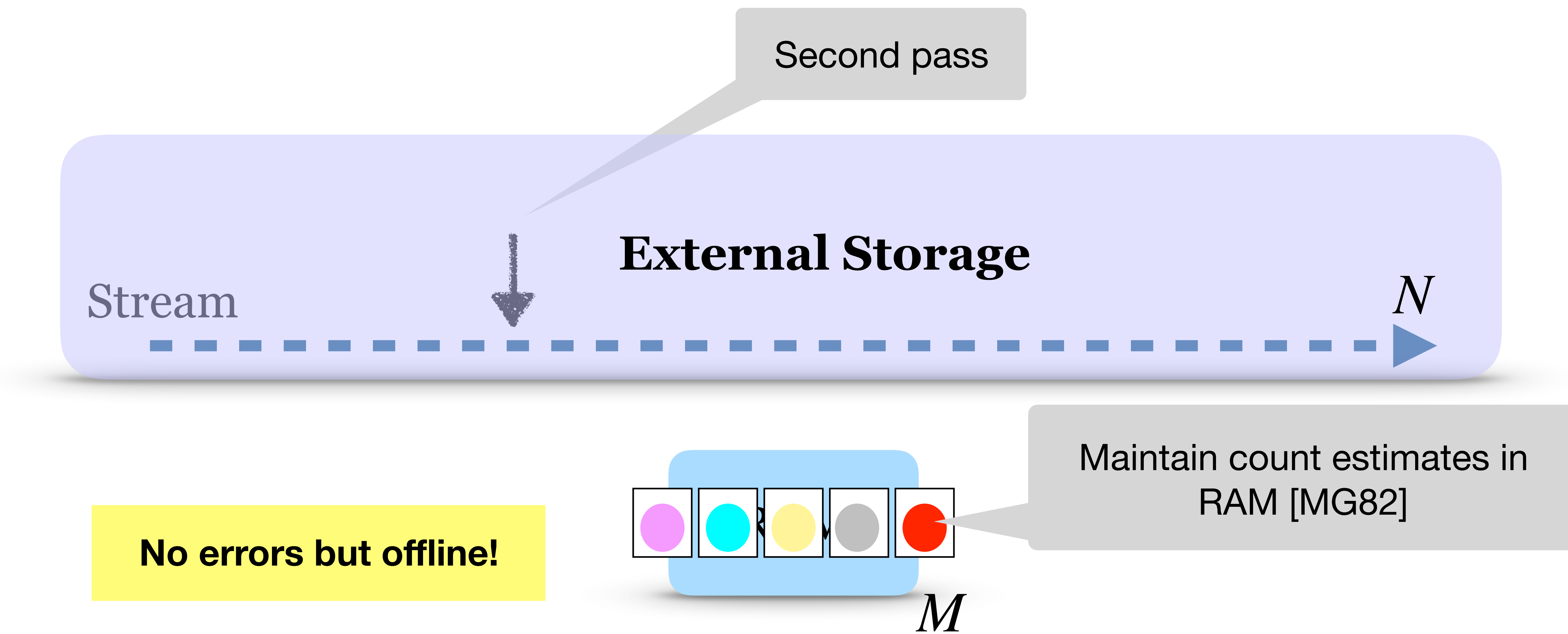
Two-Pass Streaming Isn't Real Time

- A second pass over the stream can get rid of errors



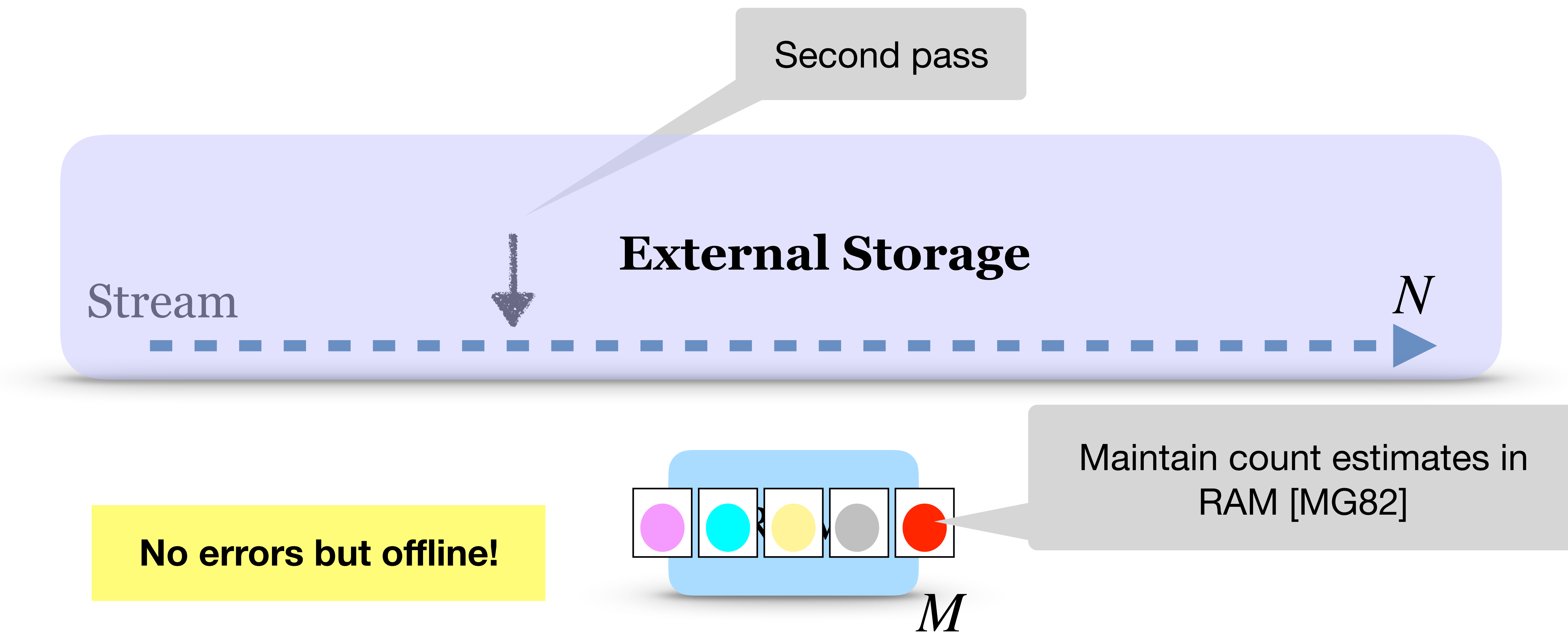
Exact Solutions Need Large Space

- A second pass over the stream can get rid of errors
- To do a second pass, you need to store your data somewhere

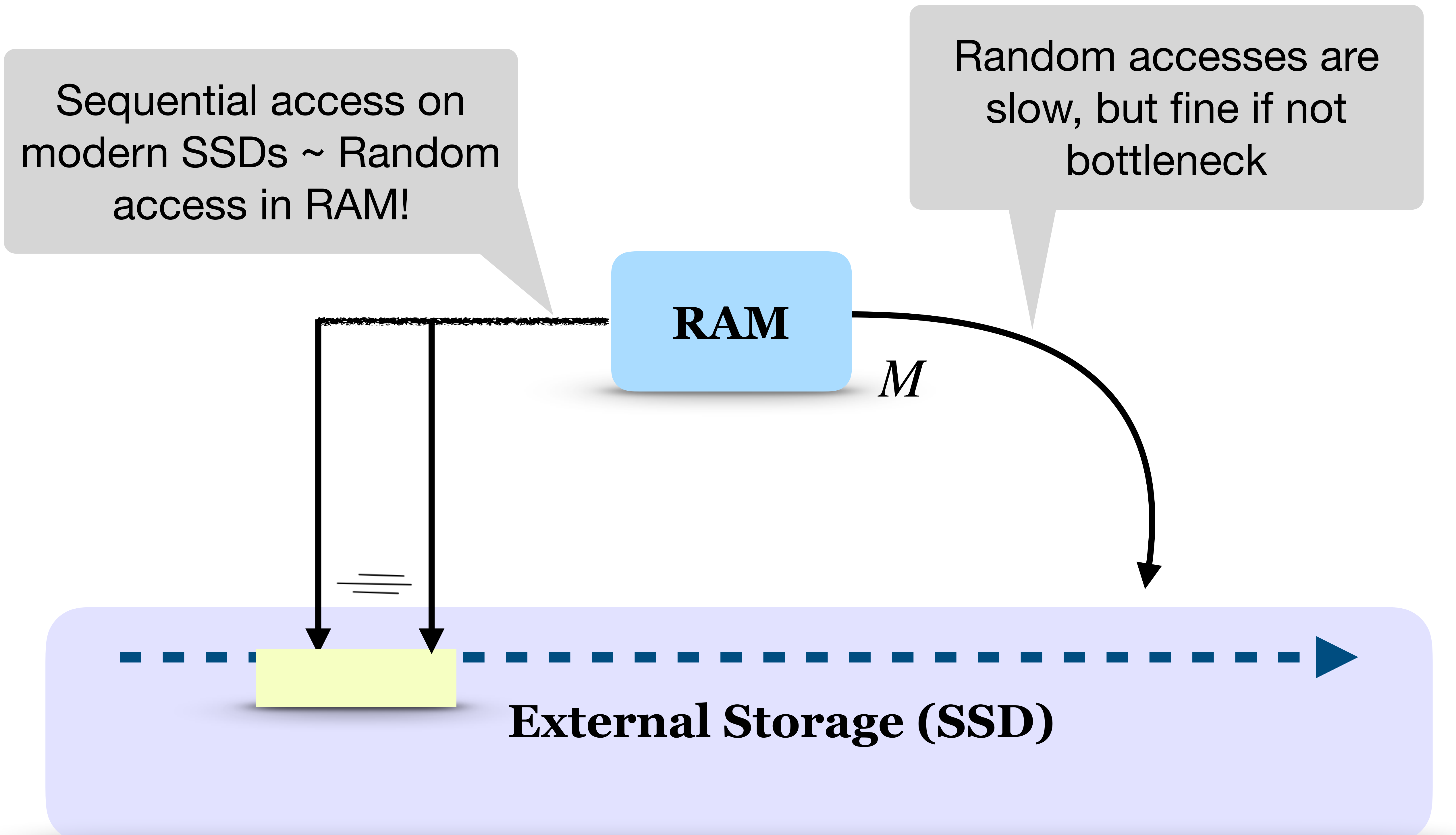


If Data is Stored: Why Not Access It?

- A second pass over the stream can get rid of errors
- To do a second pass, you need to store your data somewhere
- Why wait for second pass?

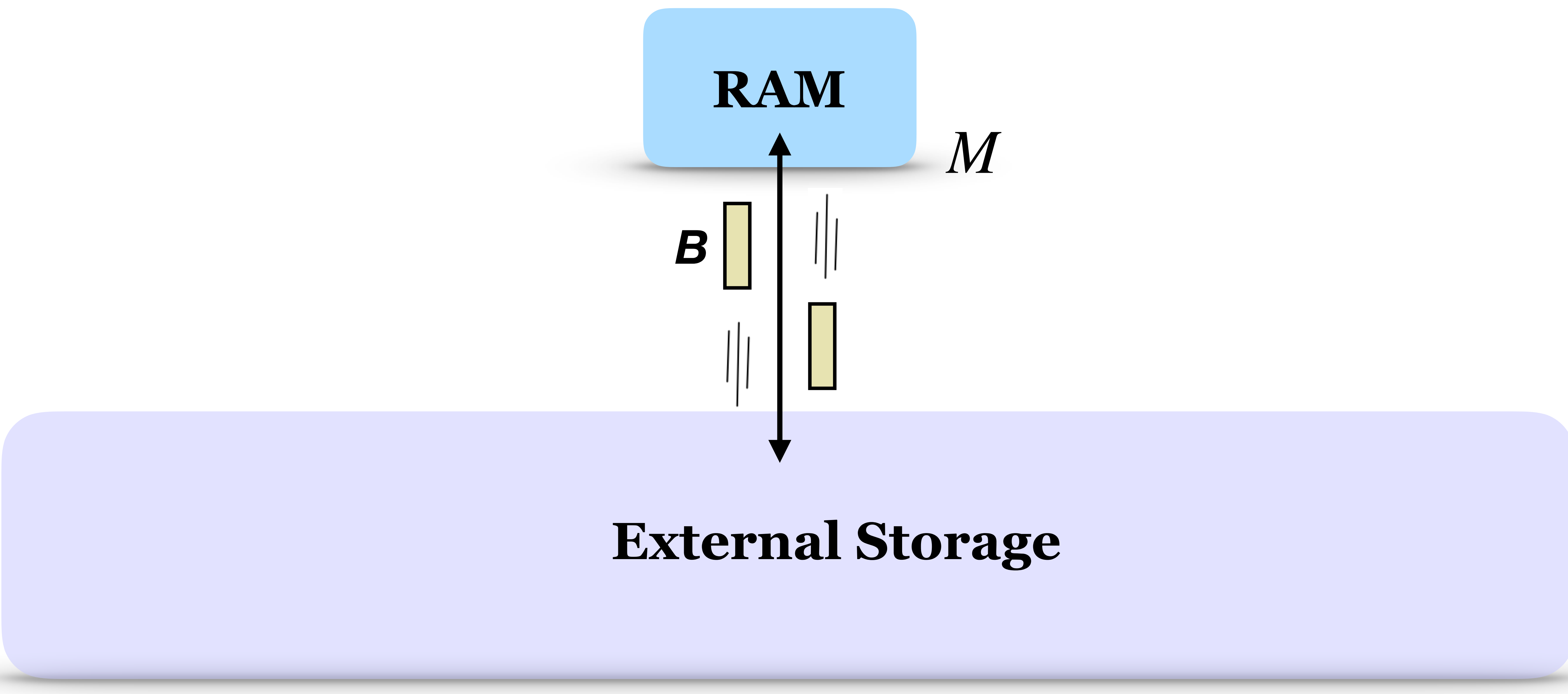


Modern External Memory: SSDs



The External-Memory Model

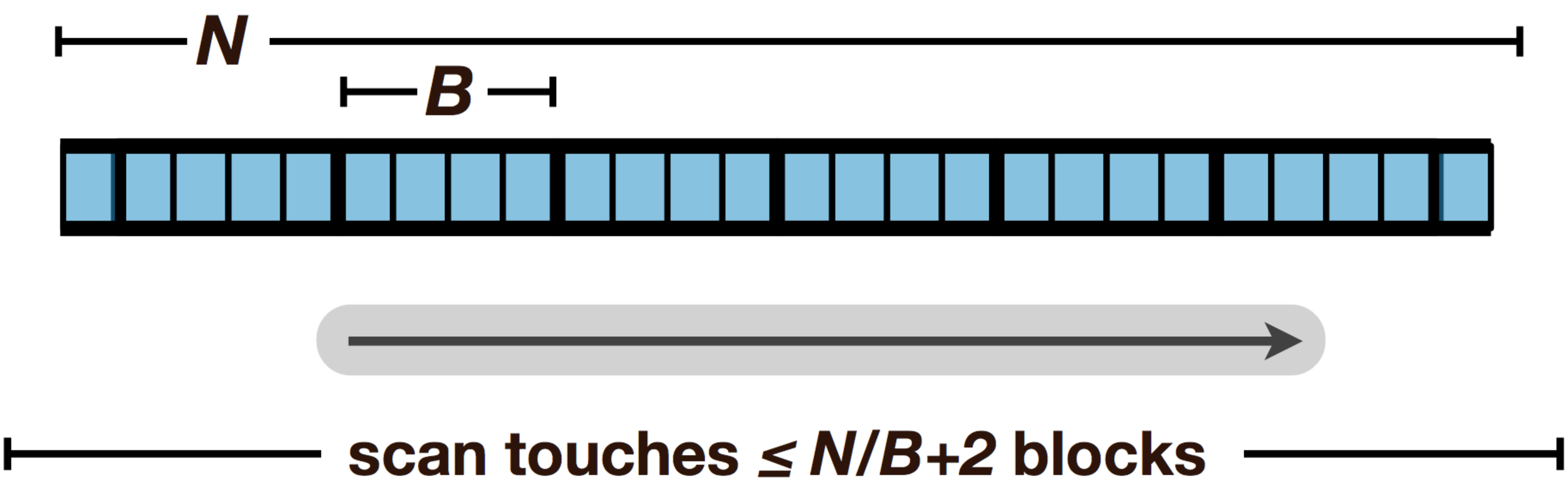
- Data is transferred between RAM and EM in blocks of size *B*
- Performances measured in # of I/Os



External-Memory Model: Review

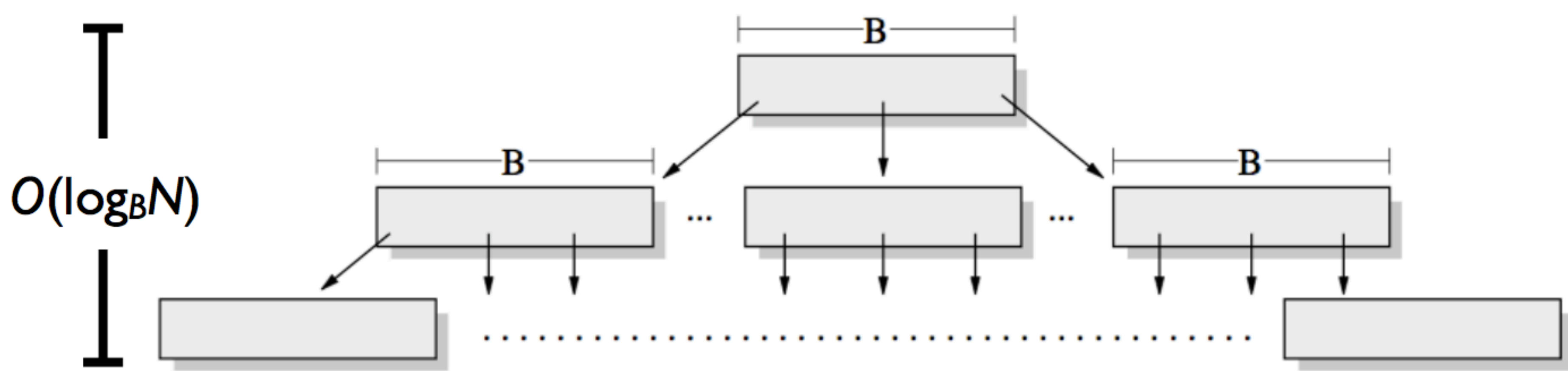
- **Question:** How many I/Os to scan an array of length N?

- **Answer:** O(N/B) I/Os.



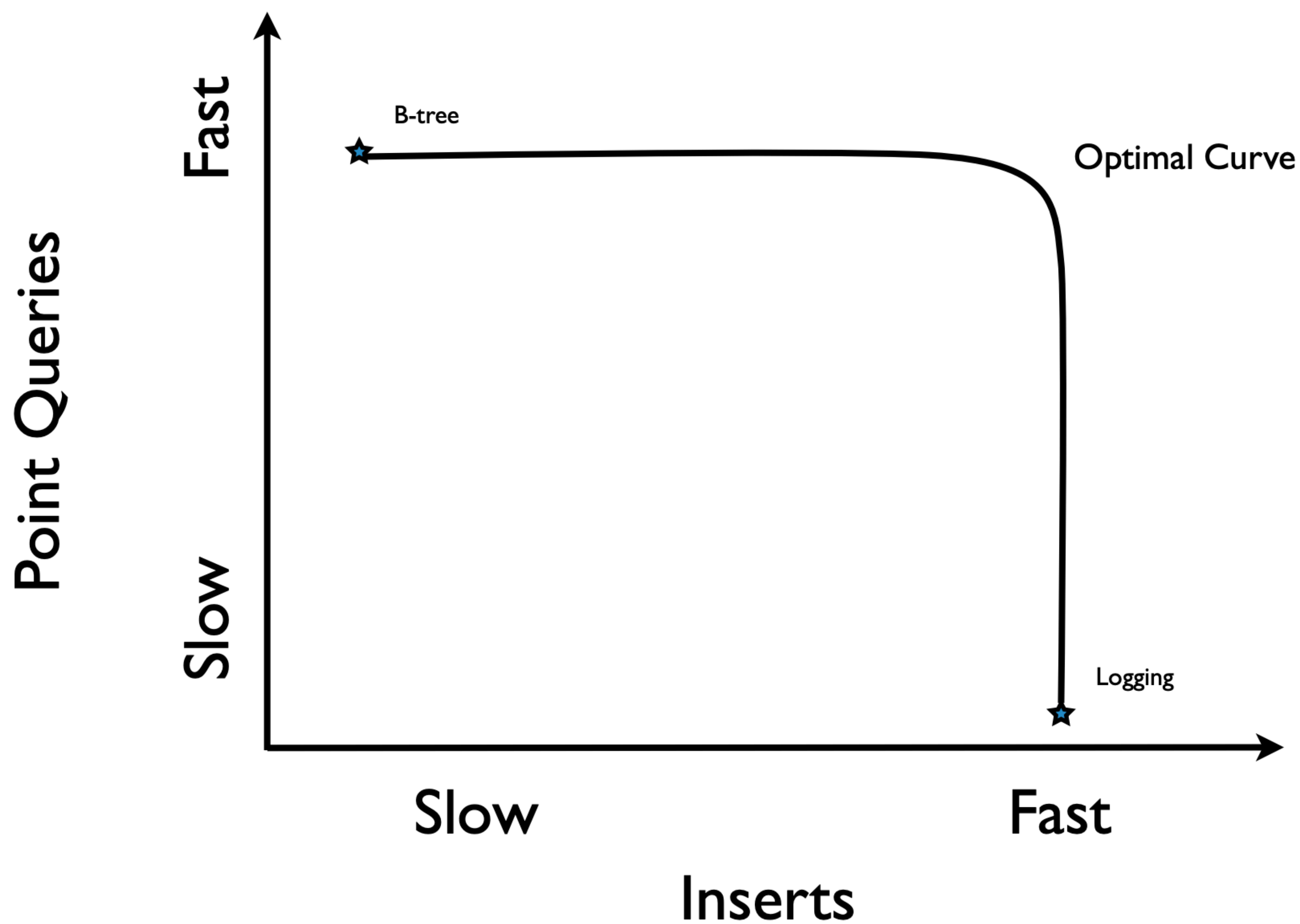
External-Memory Model: Review

- **Question:** How many I/Os for a point query or insert into a B-tree with N elements?
- **Answer:** $O(\log_B N)$



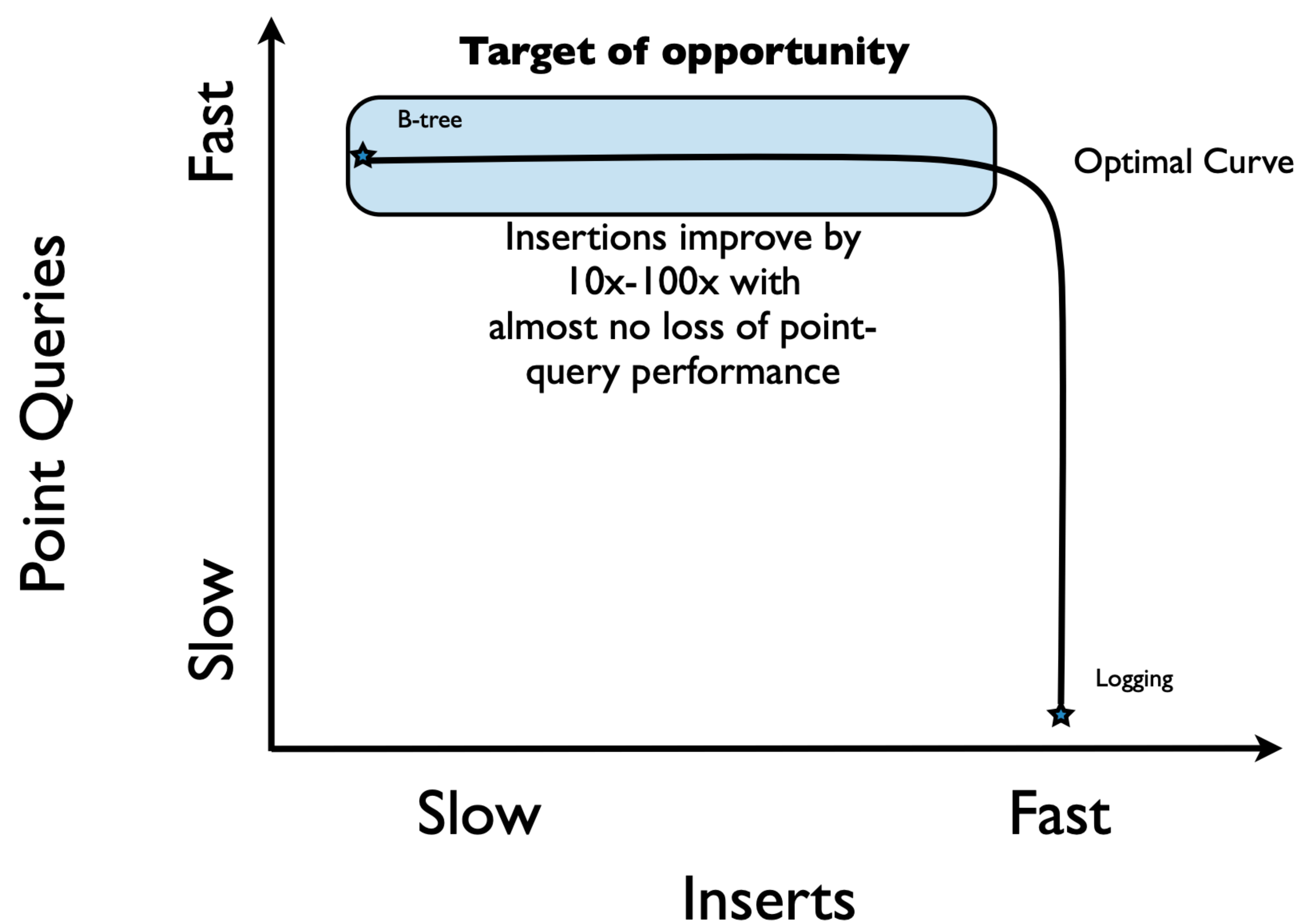
Optimal Trade-Off Curve [Brodal, Fagerberg 03]

- Logging: inserts are fast, but queries are slow
- B-trees: point queries are fast but inserts are slow



Idea Behind Write-Optimization

- You can improve insert costs without losing out on queries



Does Write-Optimization Solve OEDP?

- Write optimized data structures like COLA, cascade filters, etc. (WODs) let you do fast inserts and B-tree like queries

Insert	Query
$O\left(\frac{\log N/M}{B}\right)$ 🙂	$\Omega(\log_2 N)$ 😐

But every insert is also a query in OEDP!

Example: WODS dictionaries like cascade filters/ COLA **do not solve the problem!** But we can use insights from WODs

What We Do

- Combine streaming and WODs techniques to solve OEDP: design cache-efficient variant of the classic HH algorithm

- Can achieve **immediate reporting with no errors** at a cost that slightly worse than best insertion cost

Optimal Insert	Optimal Query
$O\left(\frac{\log N/M}{B}\right)$	$\cancel{\Omega(\log_B N)}$

Avoid most point queries

- If **slight delay in reporting** is allowed, we present a new data structure that matches the optimal insertion cost

Our Results

- Given a stream of size *N* and $\phi N > \Omega(N/M)$, the amortized cost of solving OEDP is $O\left(\left(\frac{1}{B} + \frac{1}{(\phi - 1/M)N}\right)\log\frac{N}{M}\right)$

If $\phi N > B$ or $N > MB$, this reduces to $O\left(\frac{\log N/M}{B}\right)$

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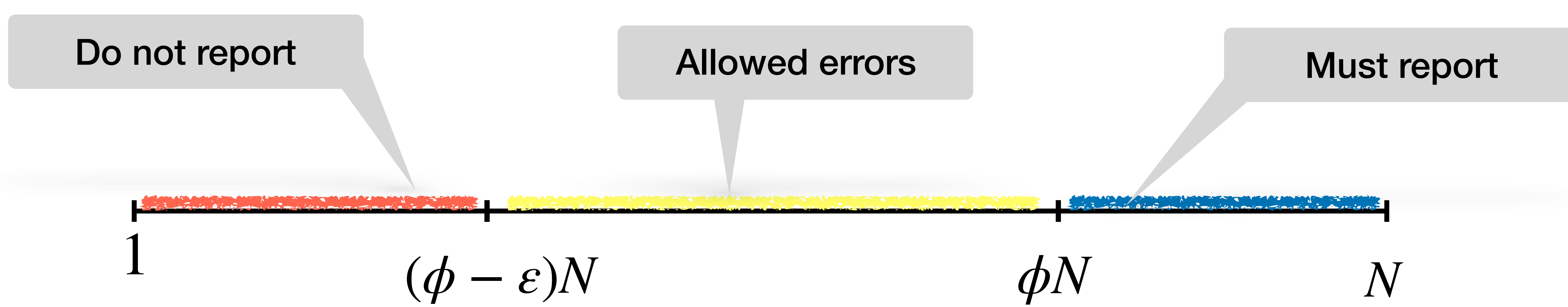
  If $\phi N > B$ or $N > MB$, this reduces to $O\left(\frac{\log N/M}{B}\right)$

- Allowing a **constant time stretch** in reporting, we can support arbitrarily small thresholds ϕ with amortized cost $O\left(\frac{\log N/M}{B}\right)$

Approximate Heavy Hitters Problem

- All items with count at least ϕN must be reported
- No item with count $< (\phi - \epsilon)N$ should be reported
- Items count in between may or may not be reported (if reported these items are false positives)



For exact, set $\varepsilon = 1/N$

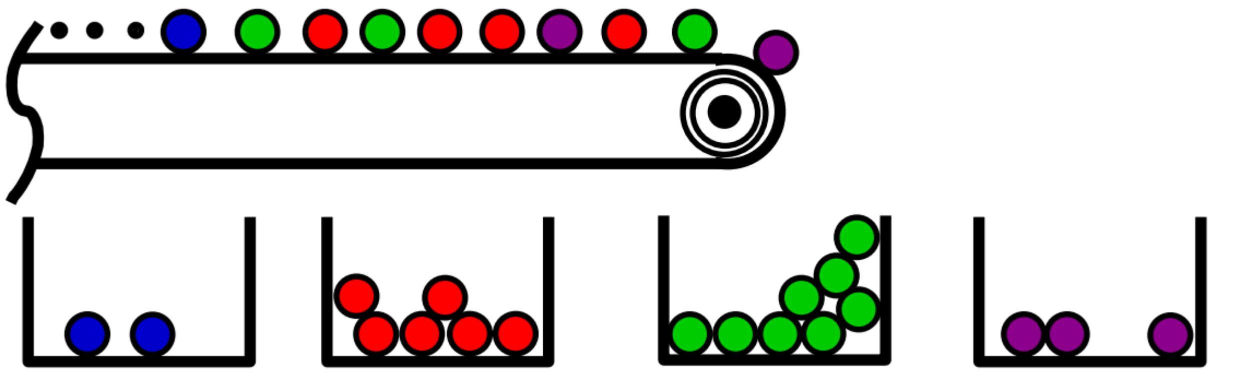
Misra Gries (MG) Algorithm

> [J.Alg 2, P208-209] Suppose we have a list of *n* numbers, representing the "votes" of *n* processors on the result of some computation. We wish to decide if there is a majority vote and what the vote is.

- Generalization of Moore 81 majority finding algorithm
- First proposed in 1982 by Misra and Gries, rediscovered twice in 2002, many improvements followed
- Finds *k* items that appear at least *N*/*k* times
- For AHH, set $k = 1/\varepsilon$

Misra Gries (MG) Algorithm

- Maintain $1/\varepsilon$ counters in memory
- When an item arrives
- if there is a counter for it, increment the counter
- if there is no counter for it
  - and there is space, add a counter and set to 1
  - otherwise, decrement all counters



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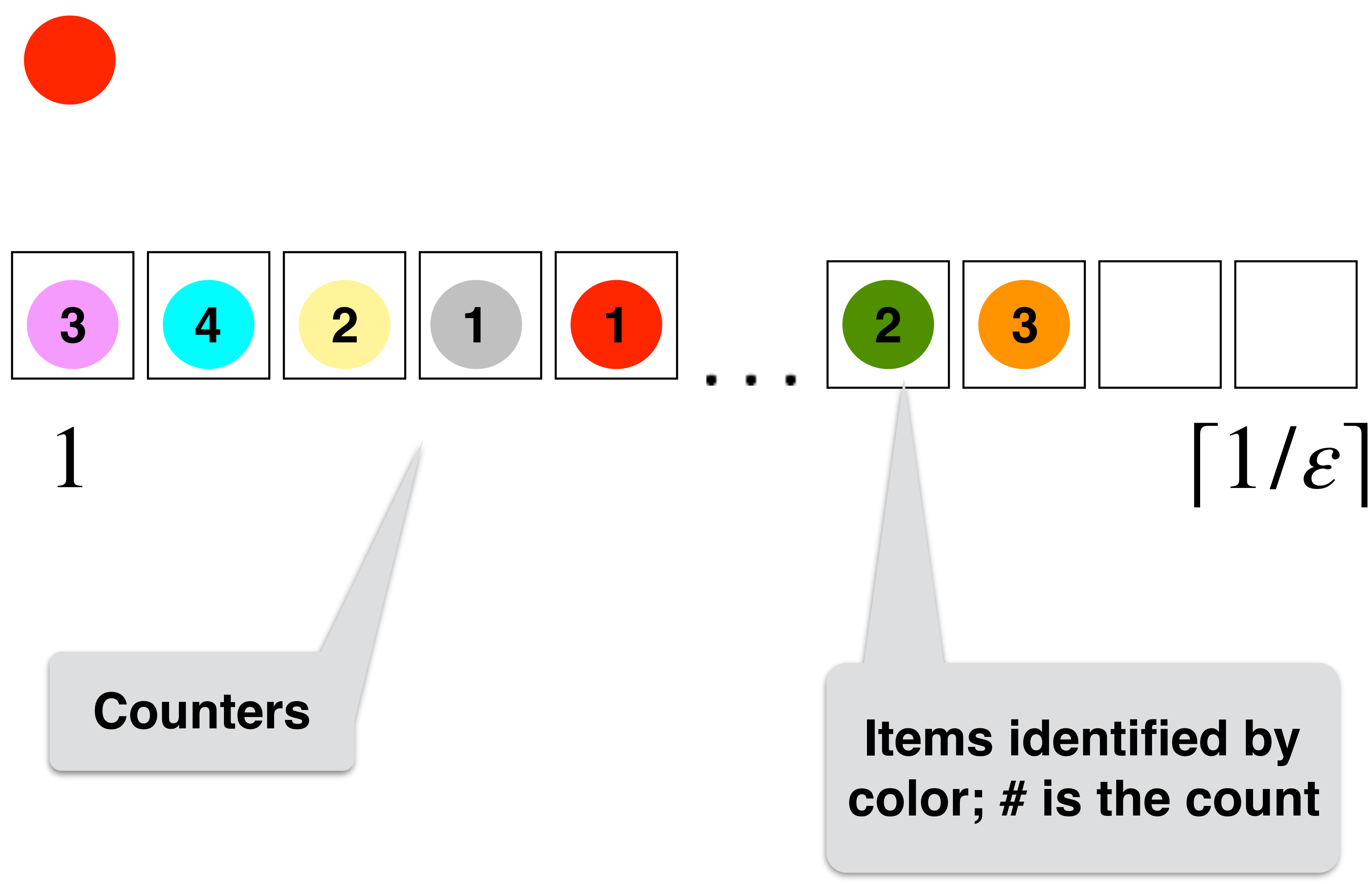

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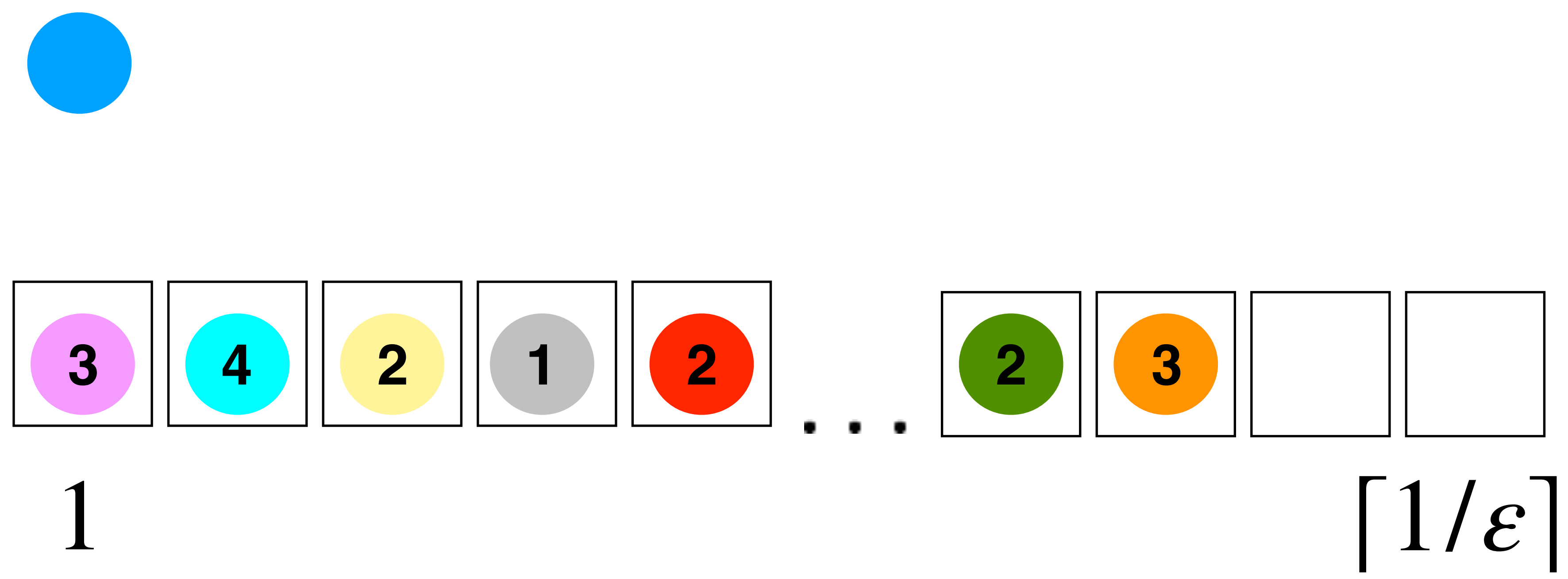

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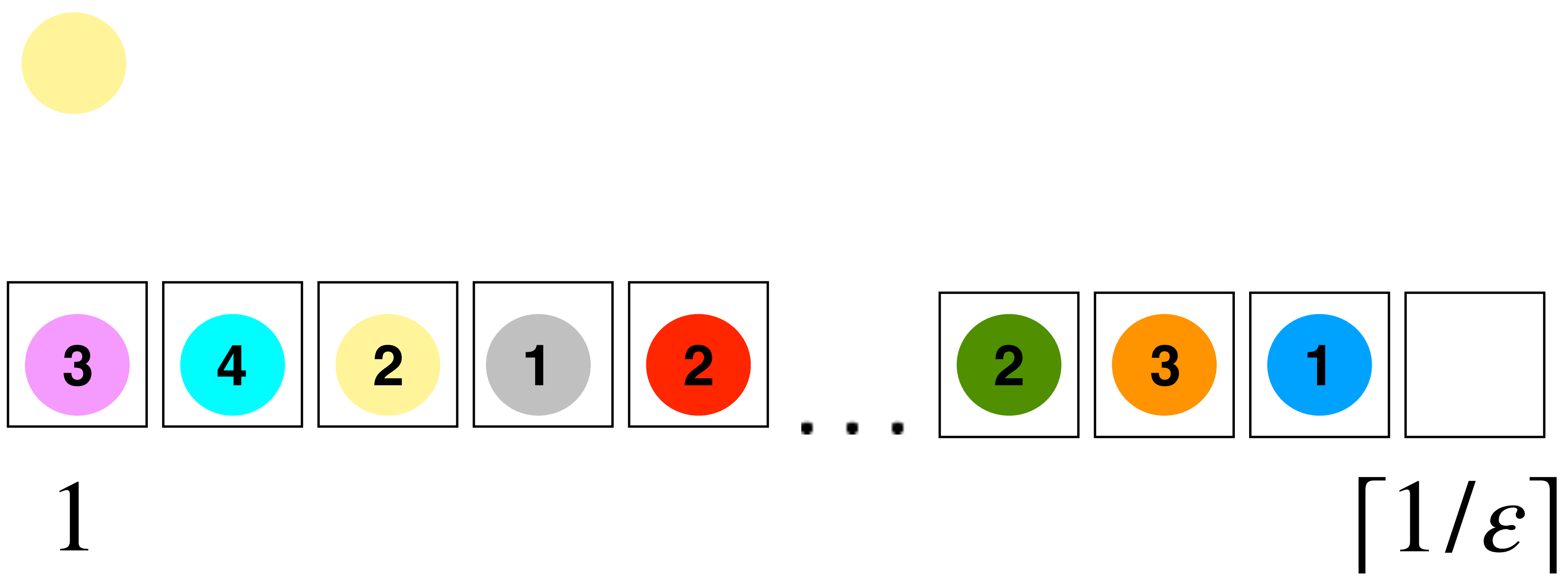

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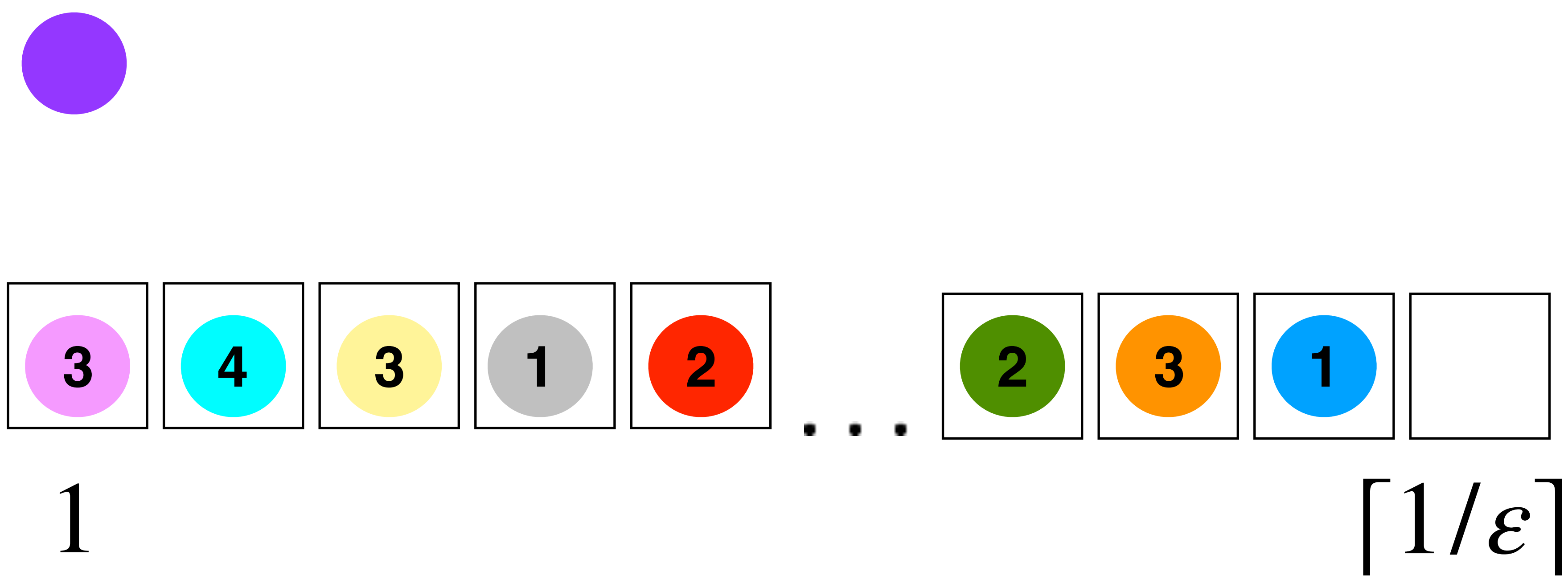

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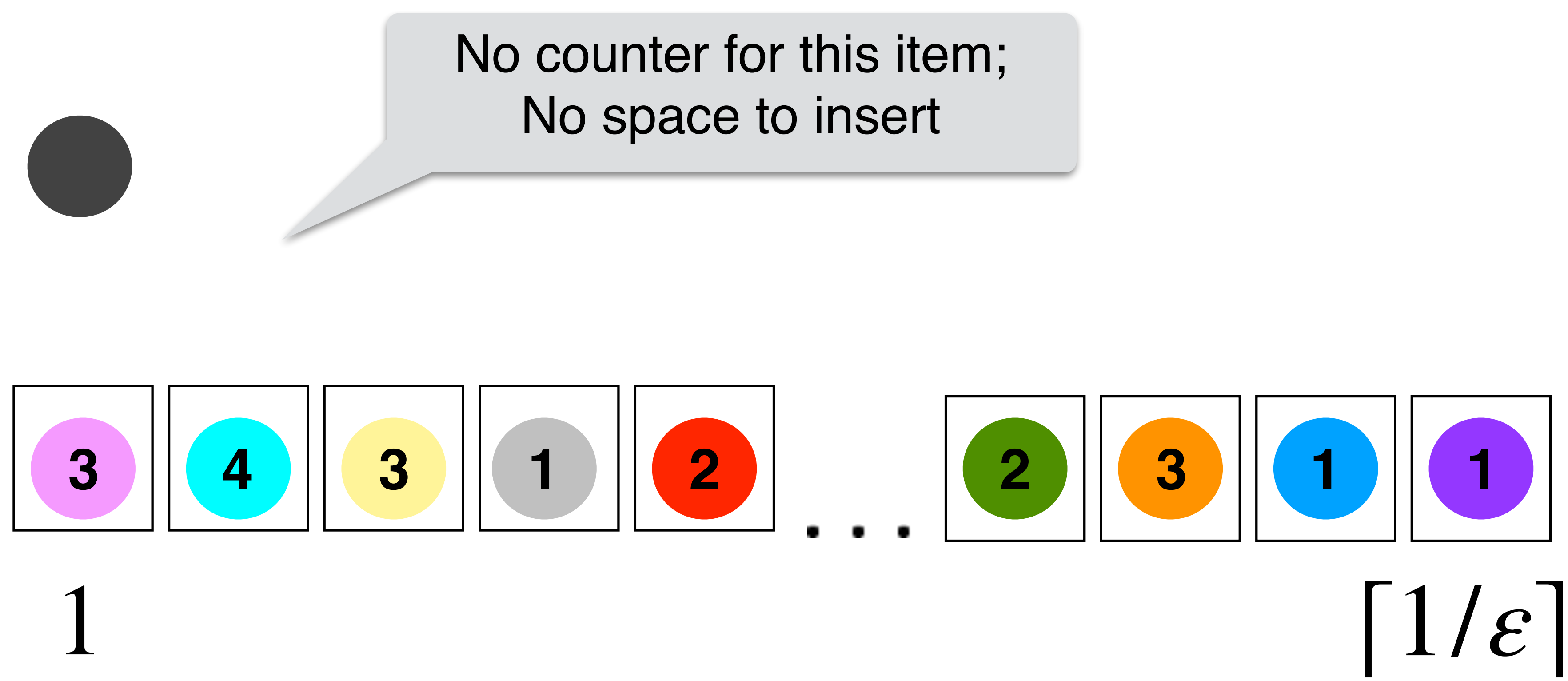

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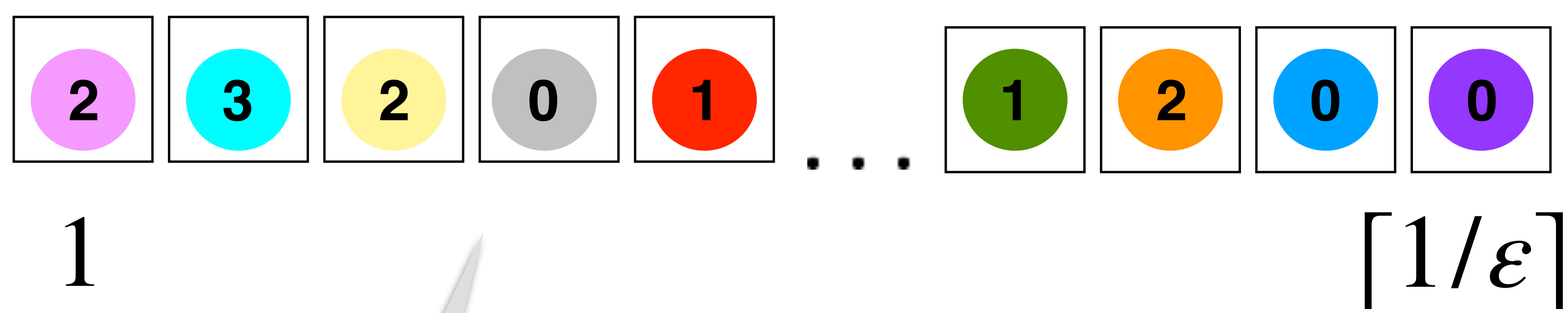
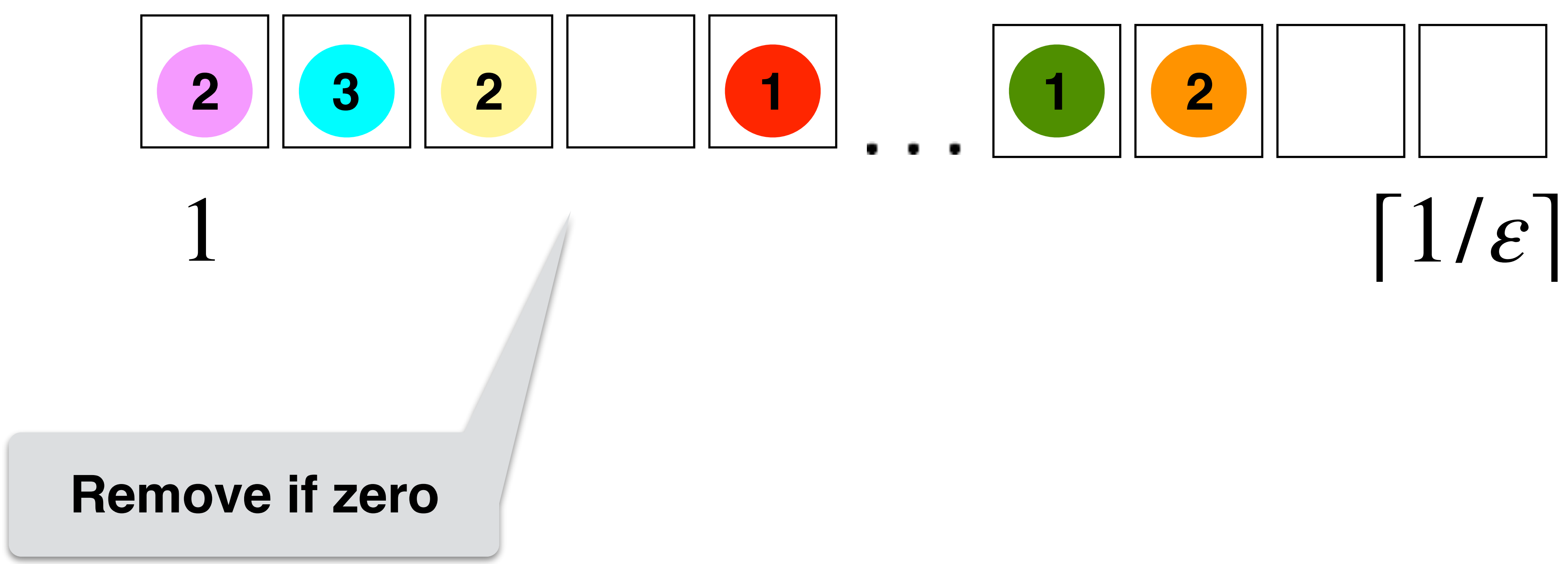


Decrement all counters

Misra Gries (MG) Algorithm



MG Algorithm Analysis

- Let $\tilde{f}$ be the count estimate for item with frequency given by MG algorithm f
- Then

$$\boxed{\tilde{f} \le f} \le \tilde{f} + N\varepsilon$$

An item's counter is incremented only when an instance of it is seen

MG Algorithm Analysis

- Let $\tilde{f}$ be the count estimate for item with frequency given by MG algorithm f
- Then

$$\tilde{f} \leq f \leq \tilde{f} + N\varepsilon$$

How many times can we lose a count of an item? Every time we lose an item count, we decrement all counters by one. Can happen only $N/(1/\varepsilon)$ times!

MG for Approximate Heavy Hitters

- Run the MG algorithm and report all items with count estimate $> (\phi - \varepsilon)N$

- Satisfies AHH guarantees
    - If $f \leq (\phi - \varepsilon)N$, since $\tilde{f} \leq f$, item not reported
    - If $f \geq \phi N$, then item is always reported because

$$\tilde{f} \geq f - N\varepsilon \geq (\phi - \varepsilon)N$$

External-Memory Misra Gries

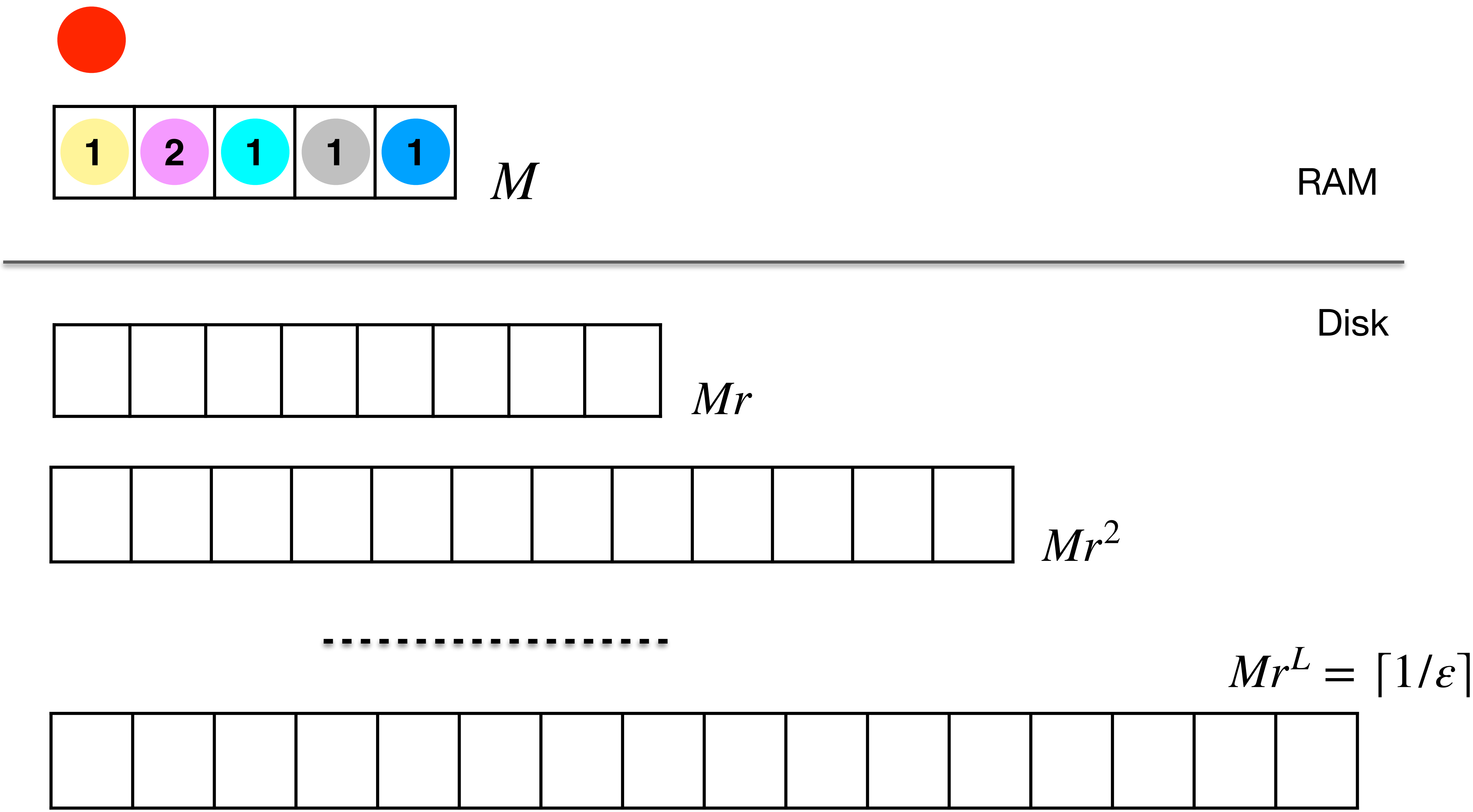
Structure

- A sequence of geometrically increasing Misra-Gries tables
- The smallest table is in memory and is of size M, the last table is of size $\lceil 1/\varepsilon \rceil$
- Total levels = $O(\log(1/\varepsilon M))$

Algorithm

- The top level receives its input from the stream
- Decrements from one level are inputs to the level below
- Decrements from the last level leave the structure

External-Memory Misra Gries



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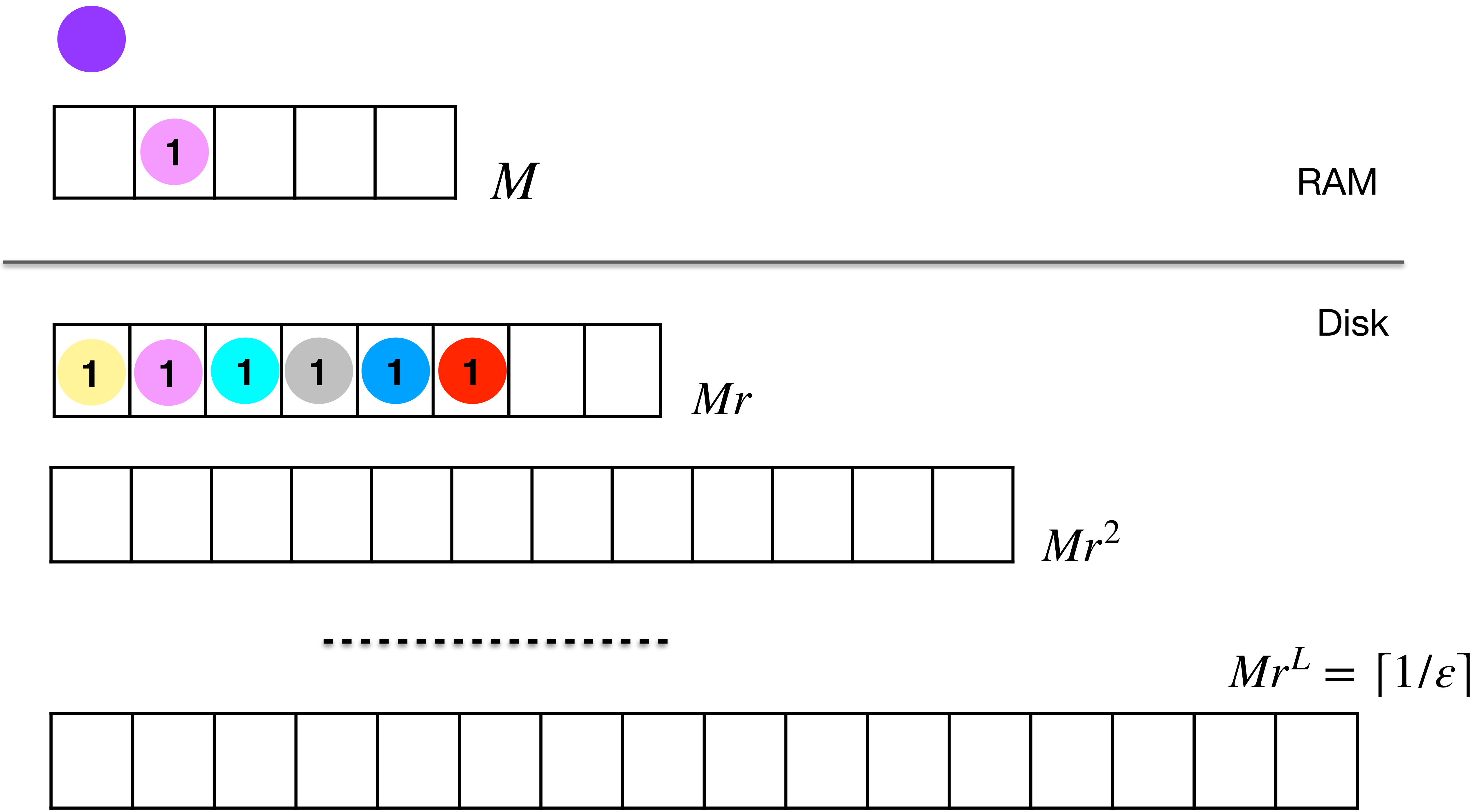

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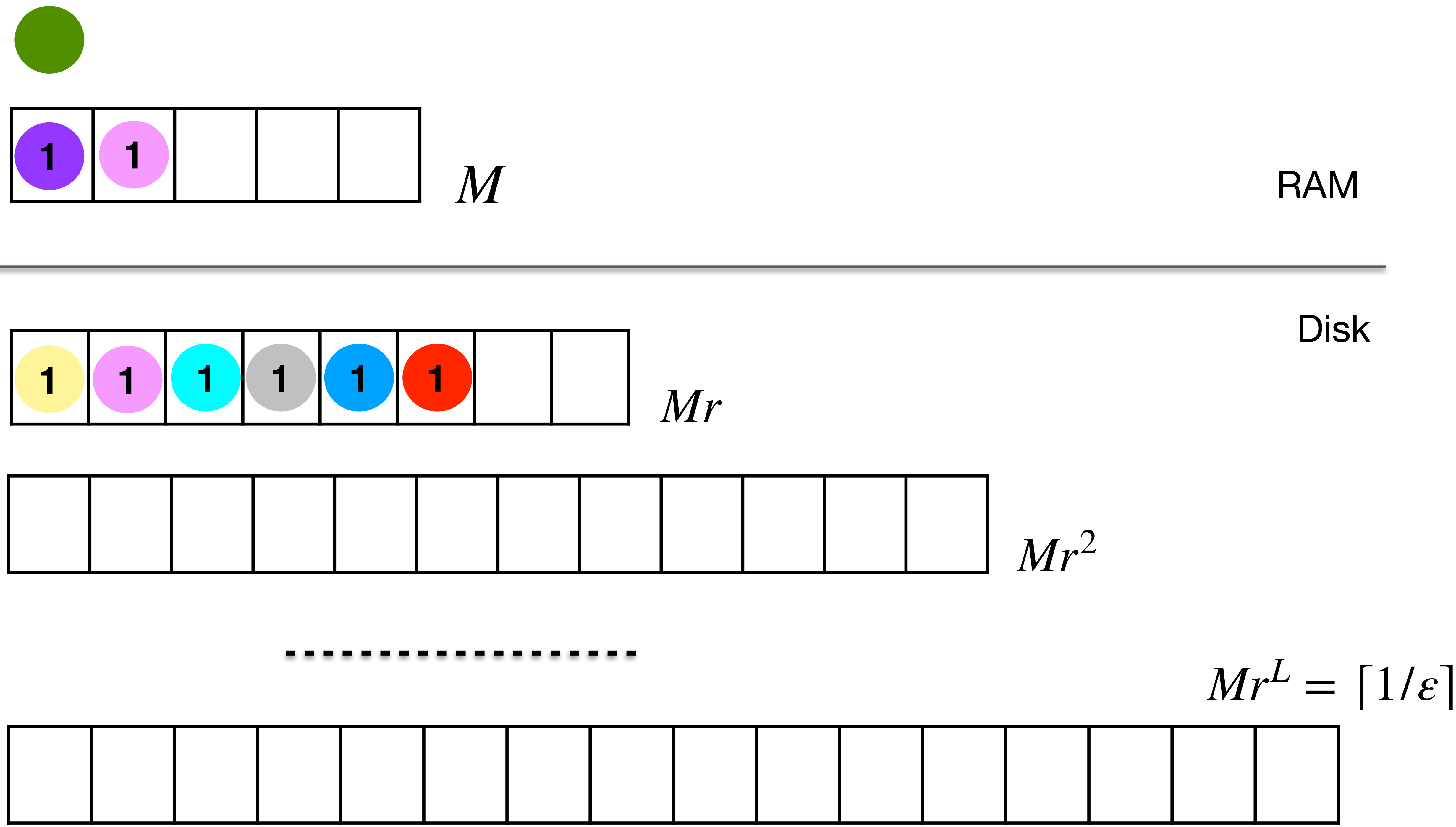

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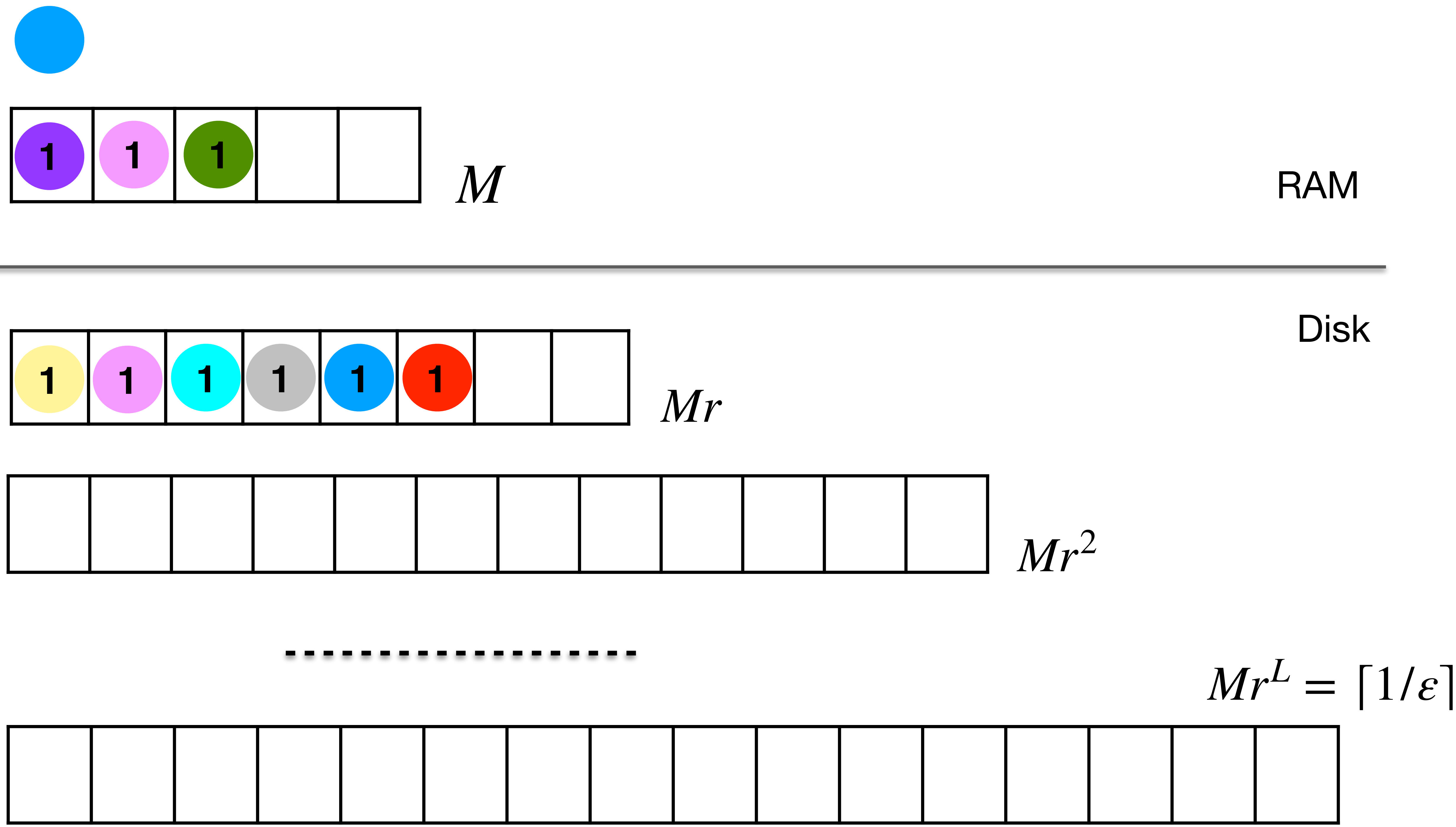

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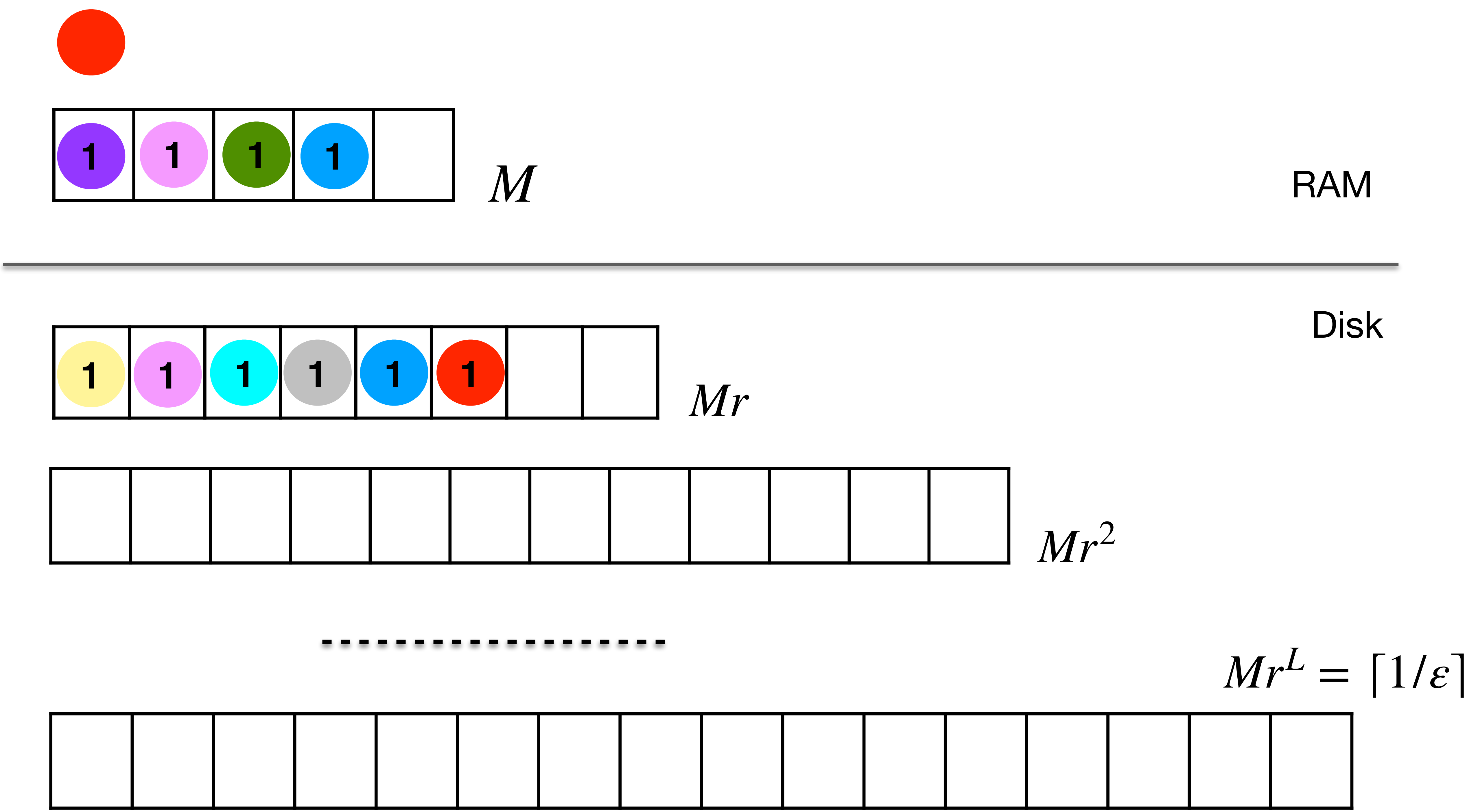

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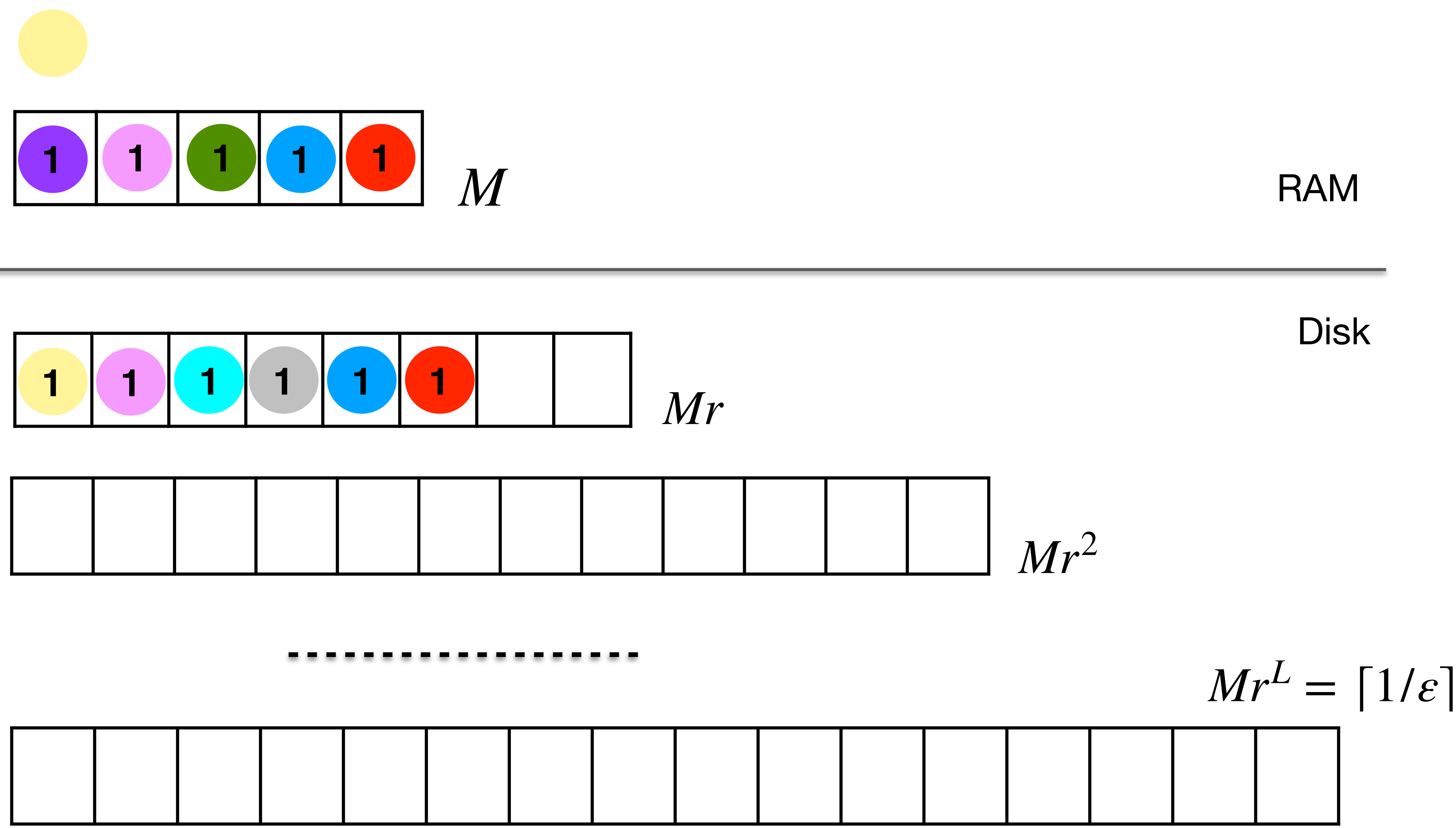

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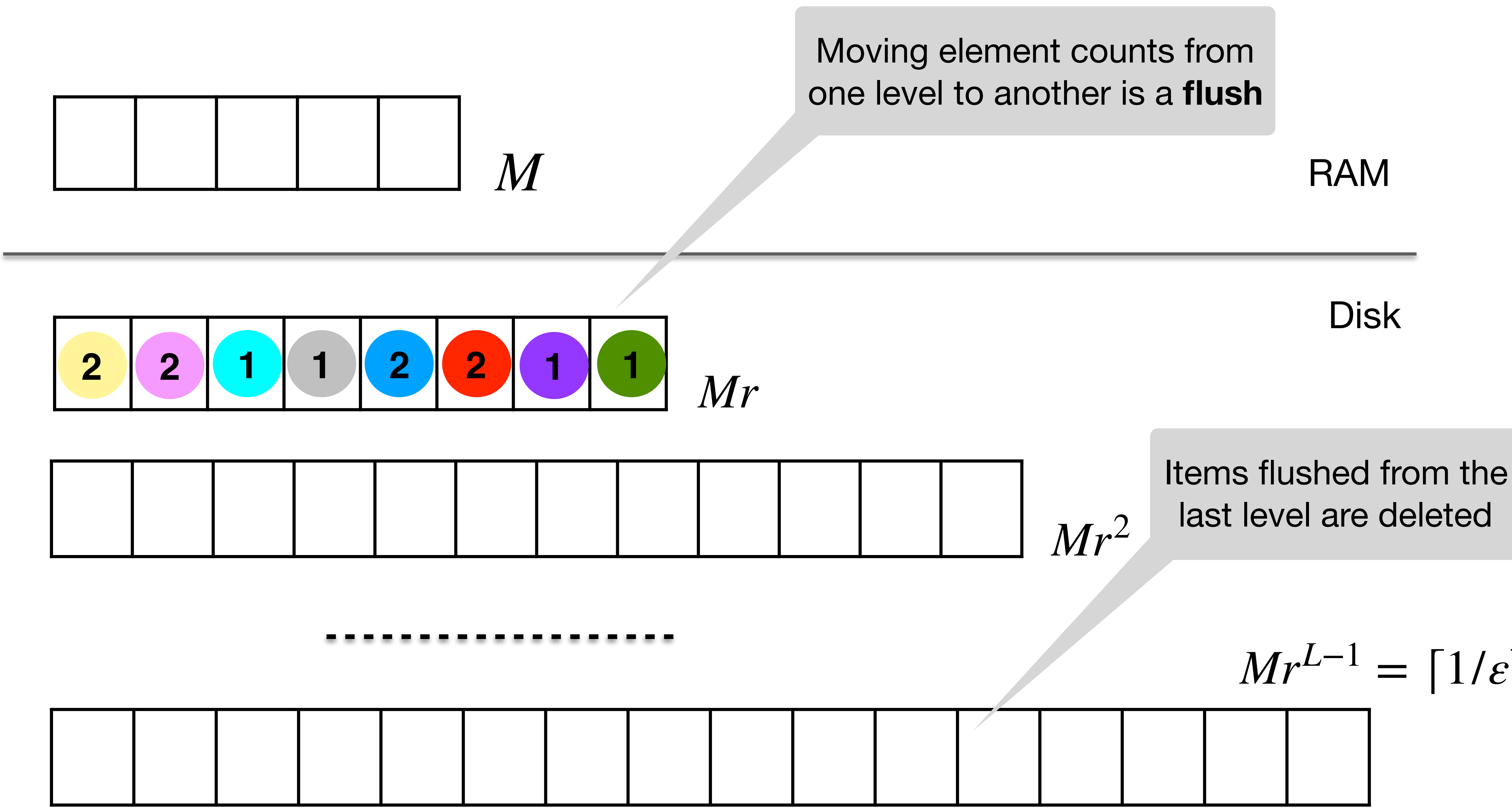


EM Misra Gries Analysis

of levels

Theorem. Amortized cost of insert in EM Misra Gries is $O\left(\frac{1}{B}\log\frac{1}{\varepsilon M}\right)$

- A flush from level i and inserting into level $i+1$ costs $O\left(\frac{r^{i+1}M}{B}\right)$ I/Os
- Each such ***flush*** moves $r^i M$ down one level
- Amortized cost of a flush $= \frac{r^{i+1}M}{r^i M}\cdot\frac{1}{B} = O\left(\frac{r}{B}\right)$
- Each element can move down at most $\log 1/(\varepsilon M)$ levels

External-Memory MG Algorithm: Takeaways

- **Supports fast inserts for small ε .**
  For the common case, when $B = \Omega(\log N)$, the cost of inserting into an external-memory MG algorithm even for small ε is << 1 I/O.

- **Does not support timely reporting.**
  Counts of items may be buried on lower levels on disk, that is, online event detection is no longer possible

Towards Online Event Detection

How do we get timely reporting?

- **OEDP.** We can pay for more I/Os to do queries
  - When to query? Querying on every insert is too expensive
- **OEDP with time stretch.** No explicit queries necessary if bounded delay is allowed
  - The algorithm can “organically” find the events

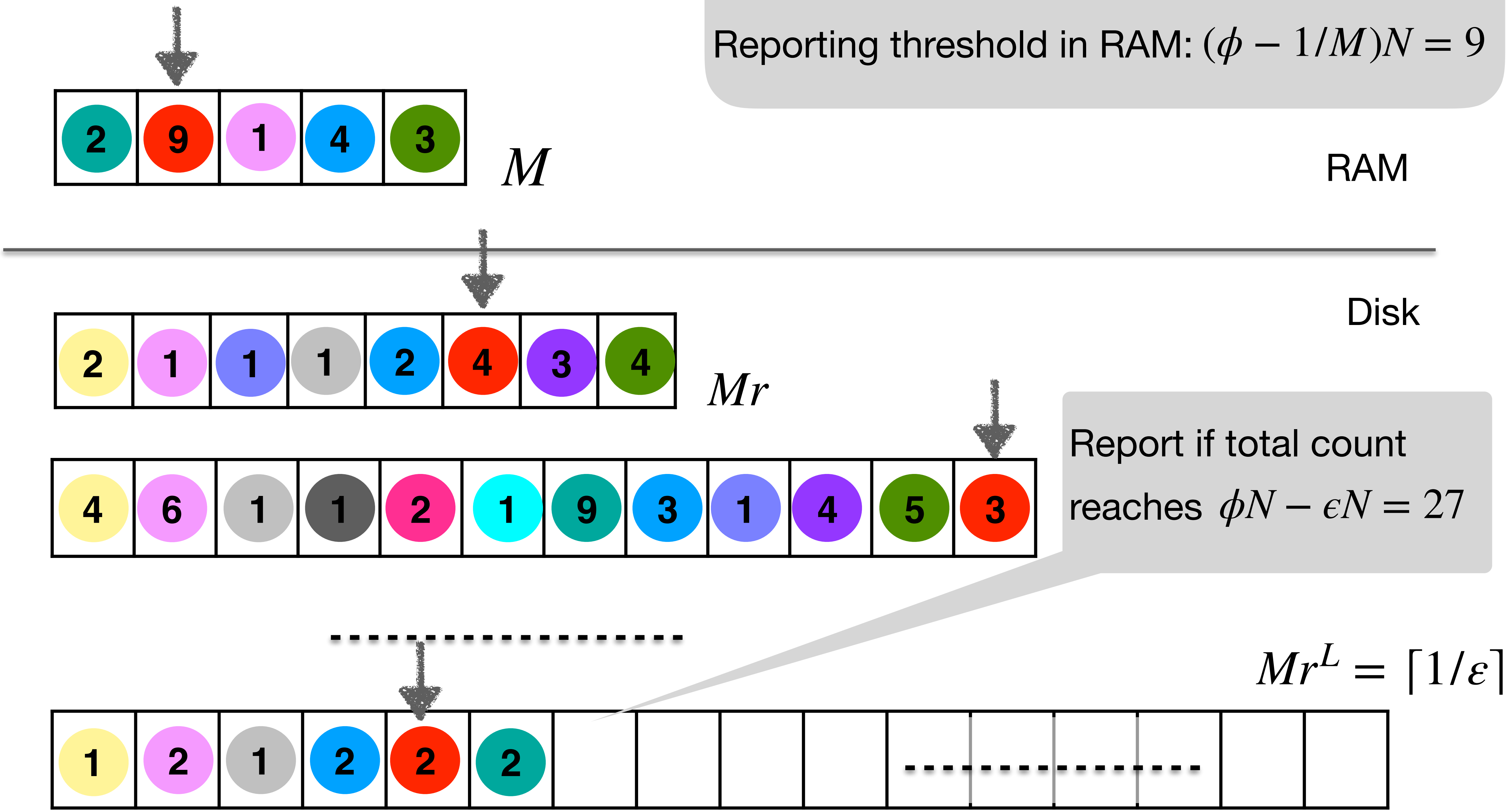
OEDP Algorithm

- Modify external-memory MG algorithm to support timely reporting
- When the in-memory count estimate of an item reaches the reporting threshold of RAM MG table $(\phi - 1/M)N$, query all levels for rest of the counts
- If consolidated count reaches overall threshold $(\phi - \varepsilon)N$ then report

OEDP Algorithm

Suppose $N/M = 31,\ \phi N = 40,\ \varepsilon N = 3$

Reporting threshold in RAM: $(\phi - 1/M)N = 9$



Report if total count reaches $\phi N - \epsilon N = 27$

OEDP Algorithm Analysis

Theorem. Given a stream of size *N*, and $\phi > 1/M + \Omega(1/N)$, the amortized cost of solving OEDP is $O\left(\left(\frac{1}{B} + \frac{1}{(\phi - 1/M)N}\right)\log\frac{1}{\epsilon M}\right)$

I/O cost of regular external-memory MG

OEDP Algorithm Analysis

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Fraction of elements that have count $\geq (\phi - 1/M)N$

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Fraction of elements that have count $\geq (\phi - 1/M)N$

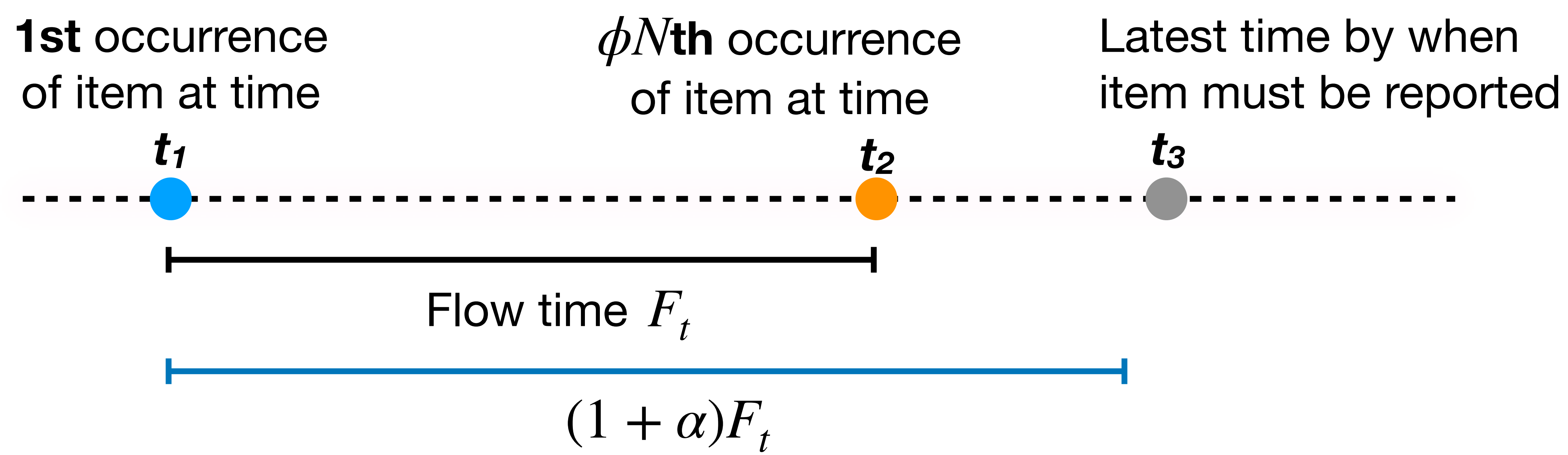
Number of levels touched to consolidate count

This can be very expensive if ϕ is close to $1/M$!

Bounded Reporting Delay

OEDP with Time Stretch

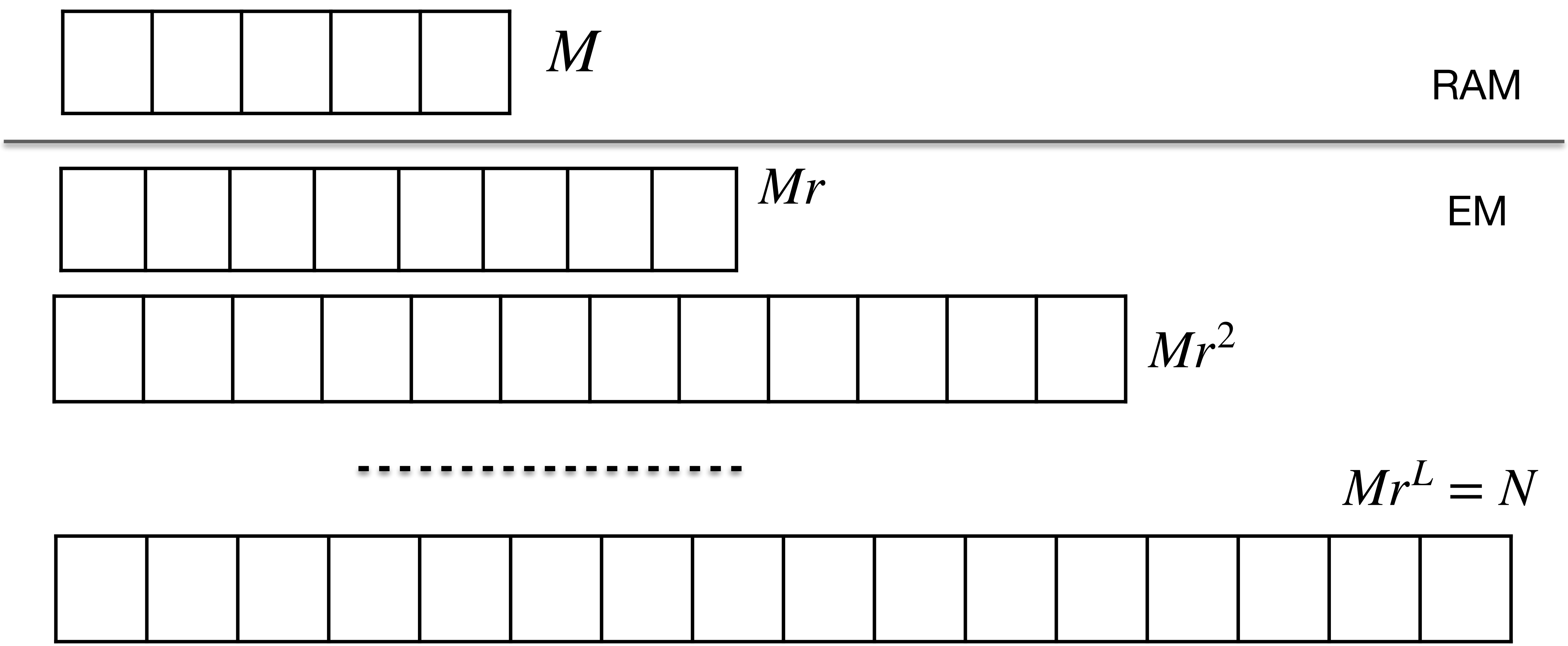
For a **time-stretch of** $1 + \alpha$, we must report an element a no later than time $t_1 + (1 + \alpha)F_t$



Key idea: the longer the flow time of a key, the more leeway we have in reporting it

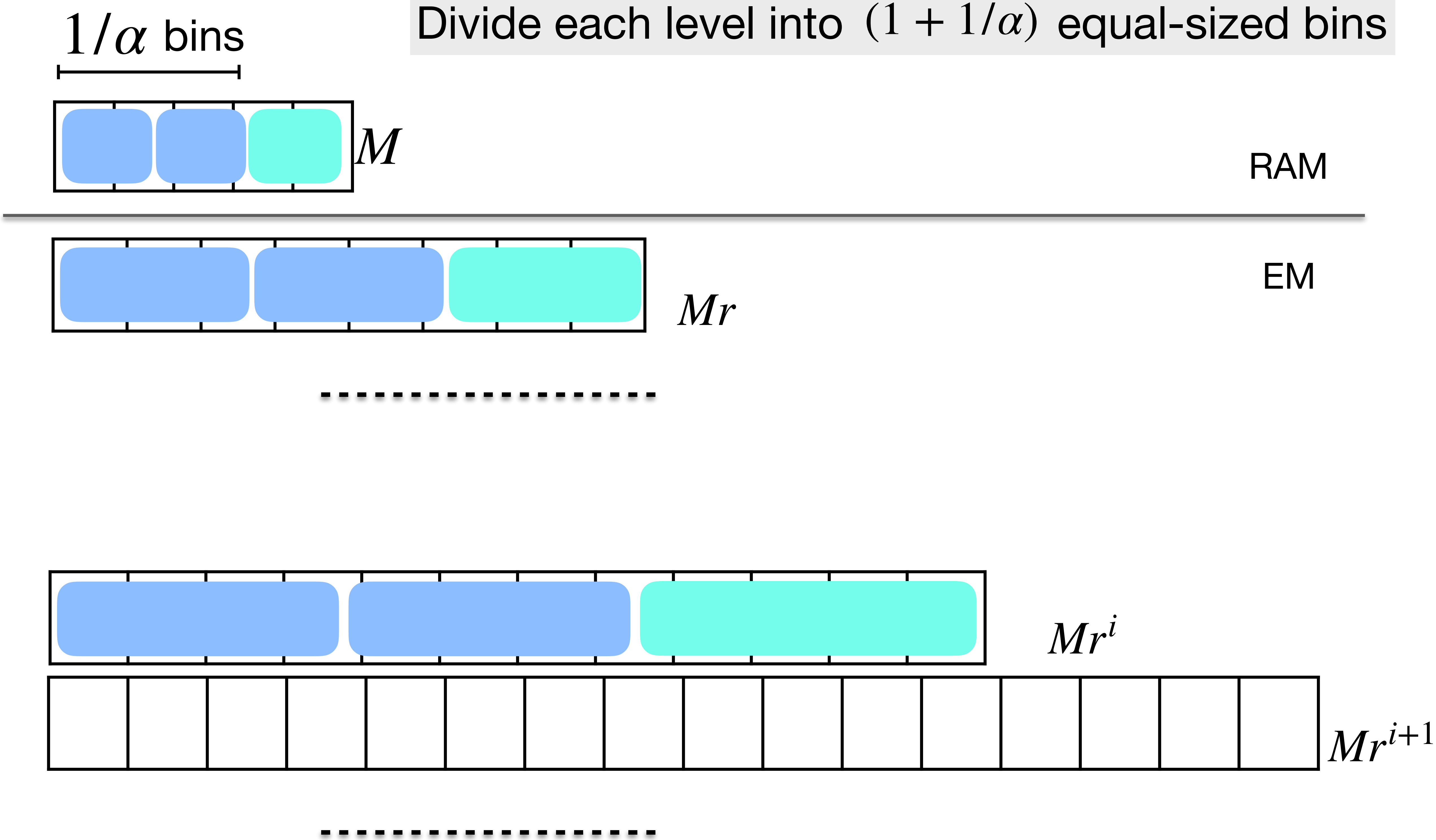
Time-Stretch Filter

- Cascade of geometrically increasing tables
- Total levels = $O(\log N/M)$



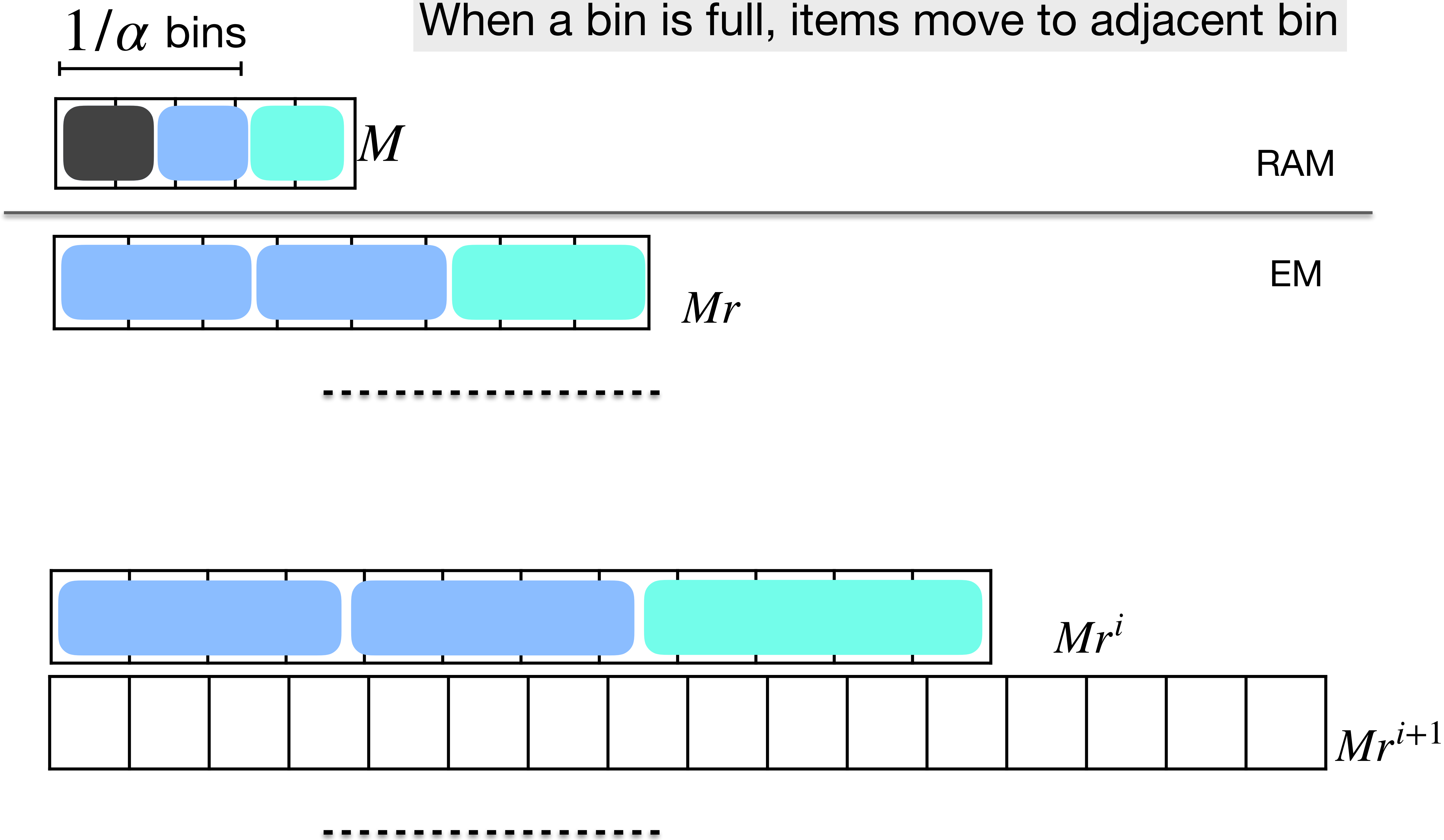
Time-Stretch Filter

Divide each level into $(1 + 1/\alpha)$ equal-sized bins



Time-Stretch Filter

When a bin is full, items move to adjacent bin



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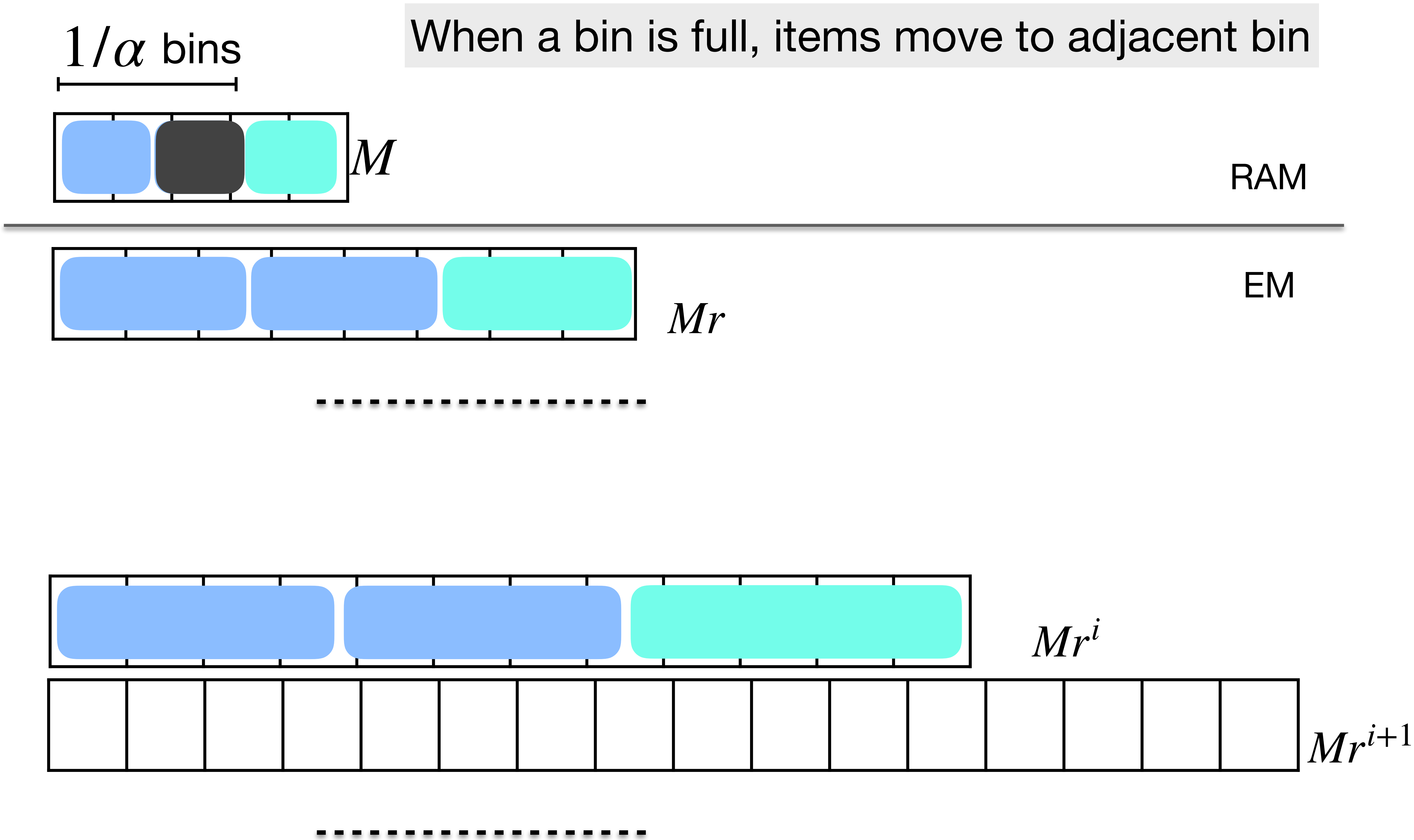

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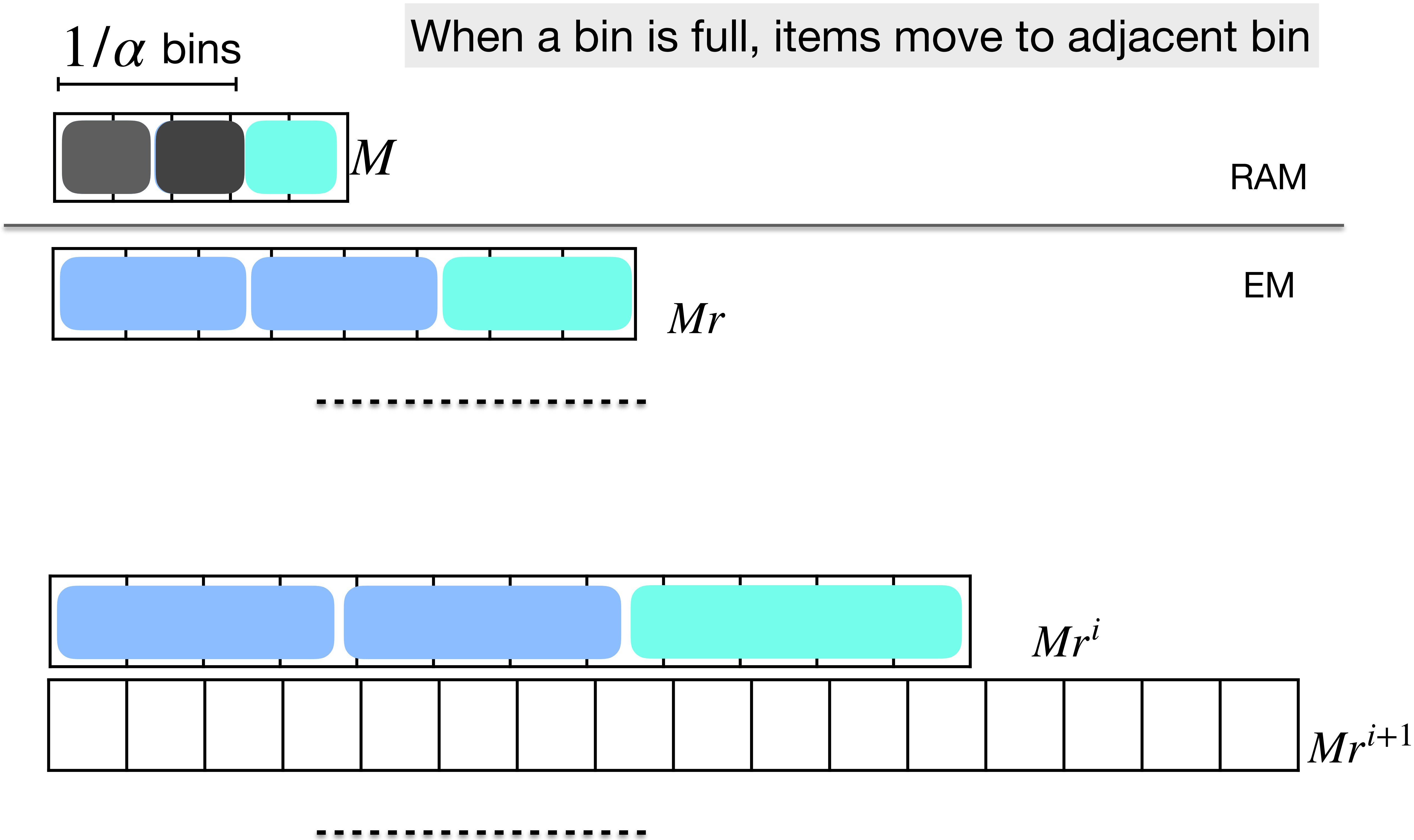

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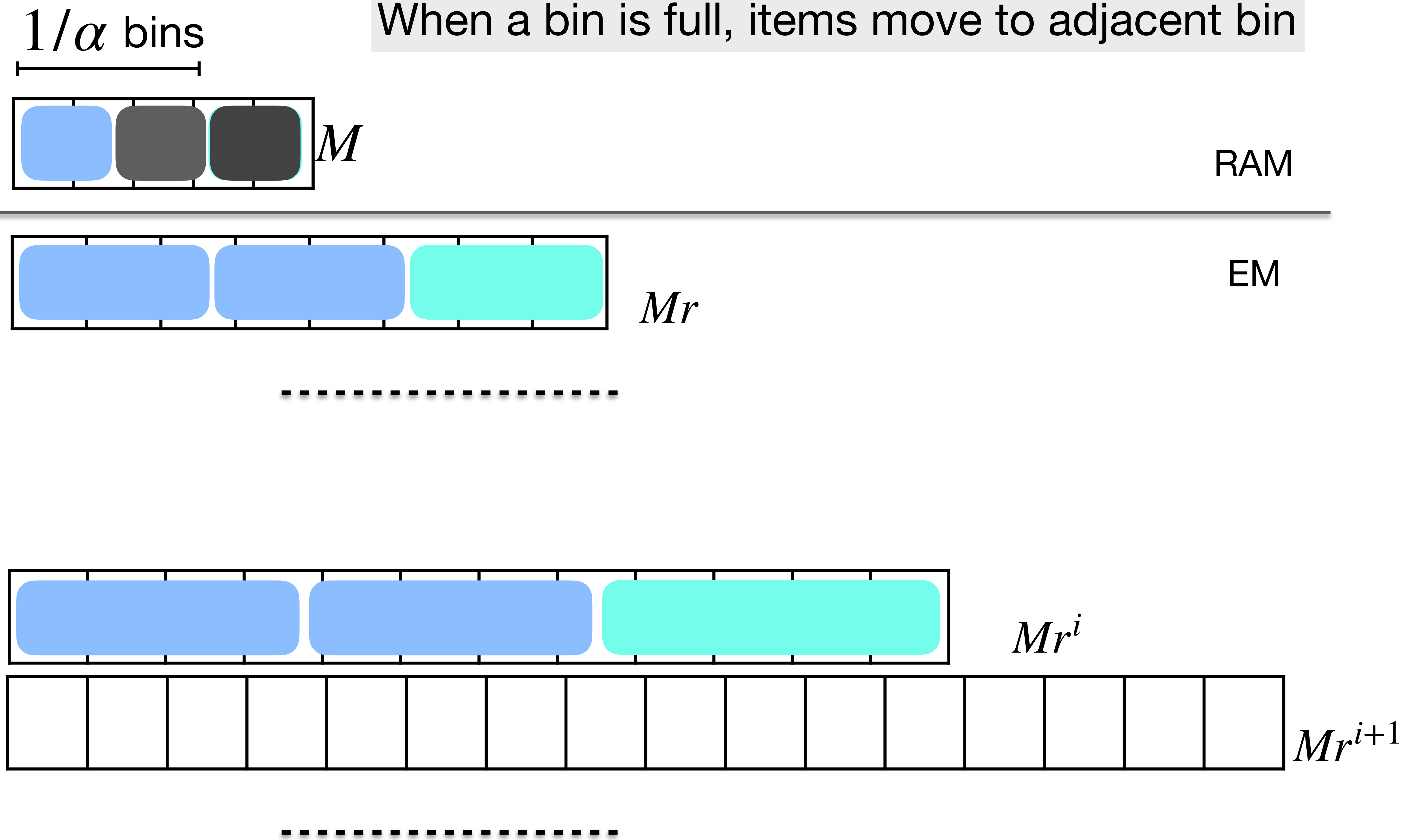
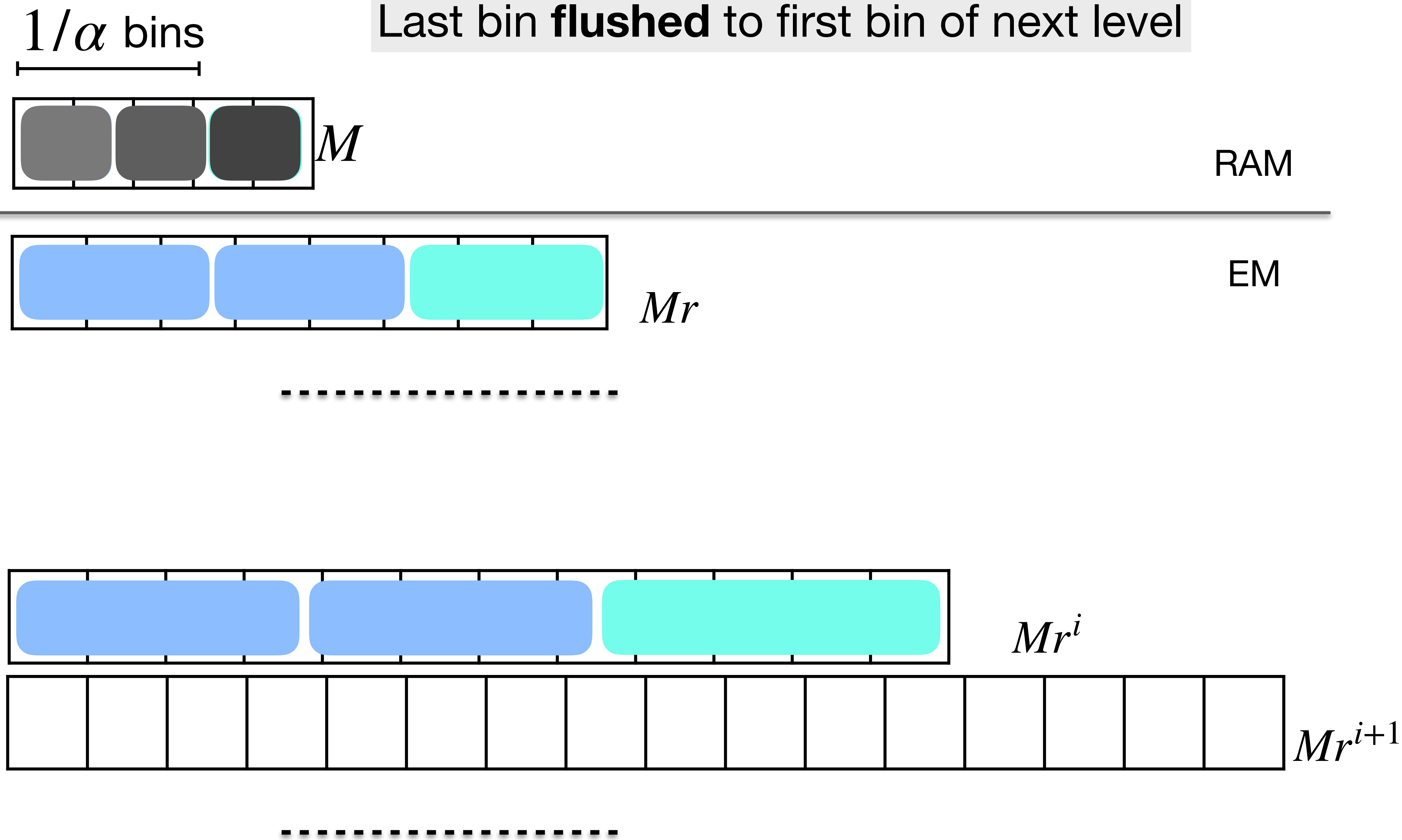


Time-Stretch Filter

Last bin **flushed** to first bin of next level



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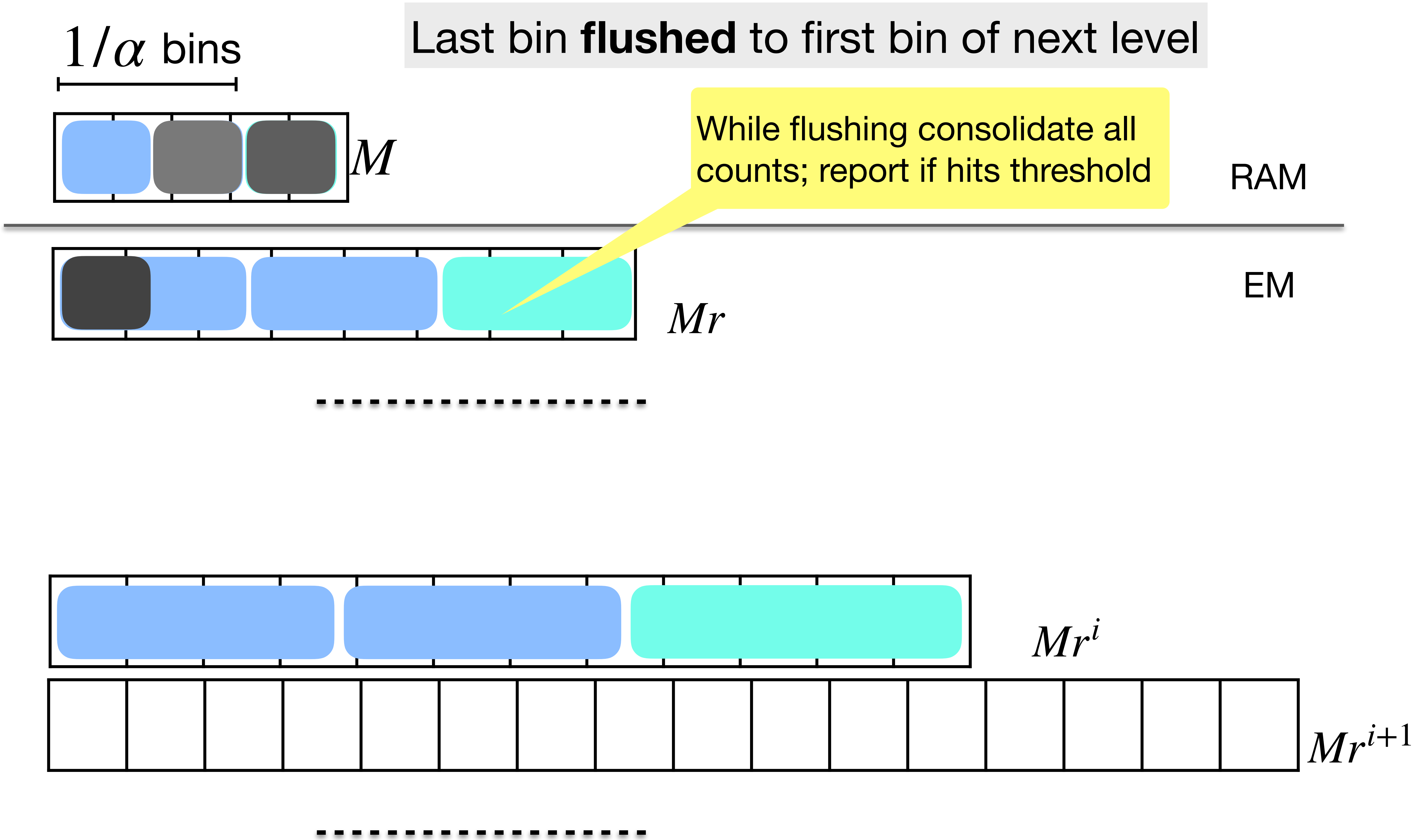

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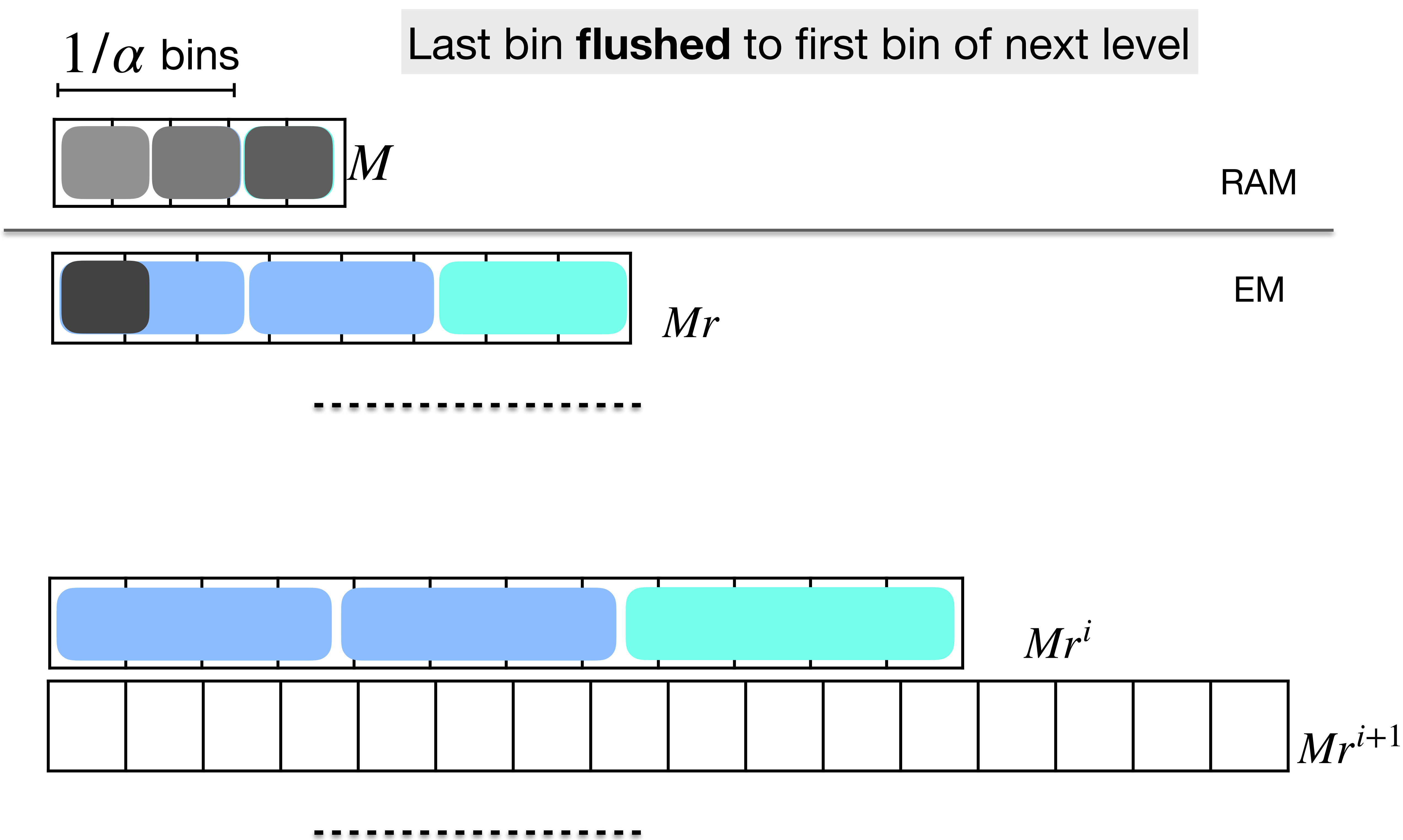

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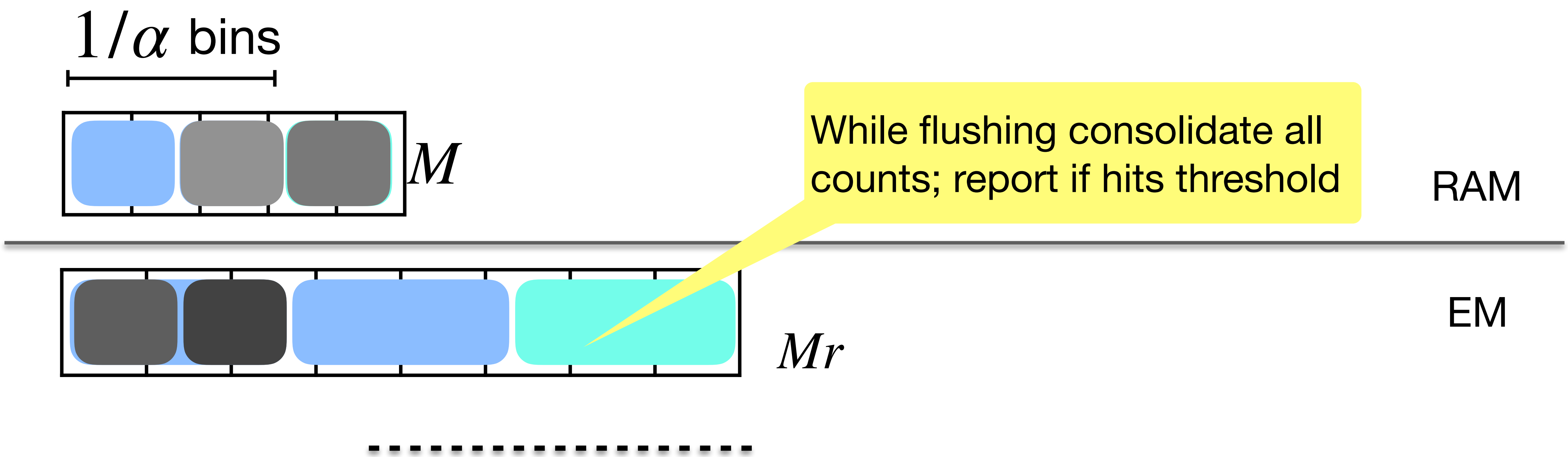
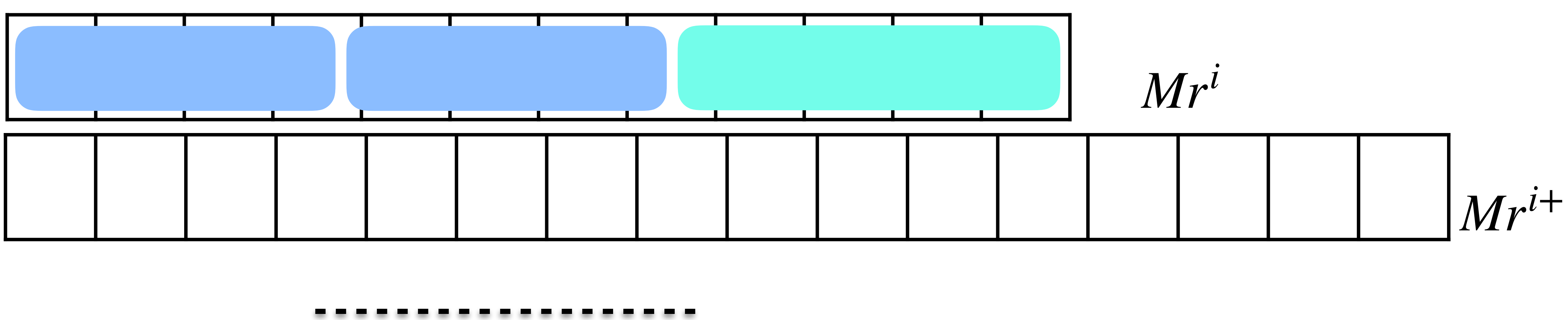
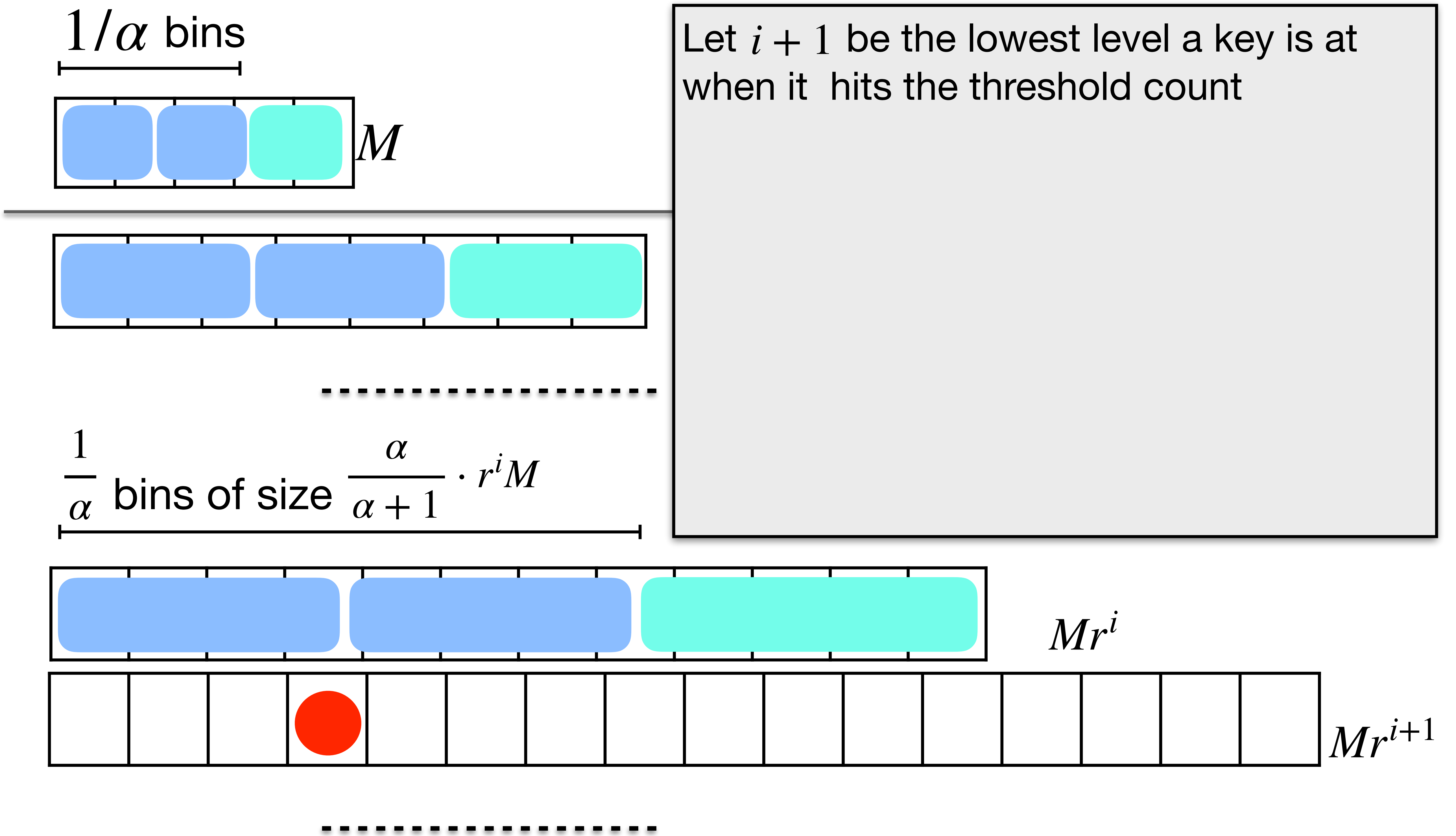


Main idea: key is not put on a deeper level until it has **"aged sufficiently"**



Time-Stretch Filter Correctness

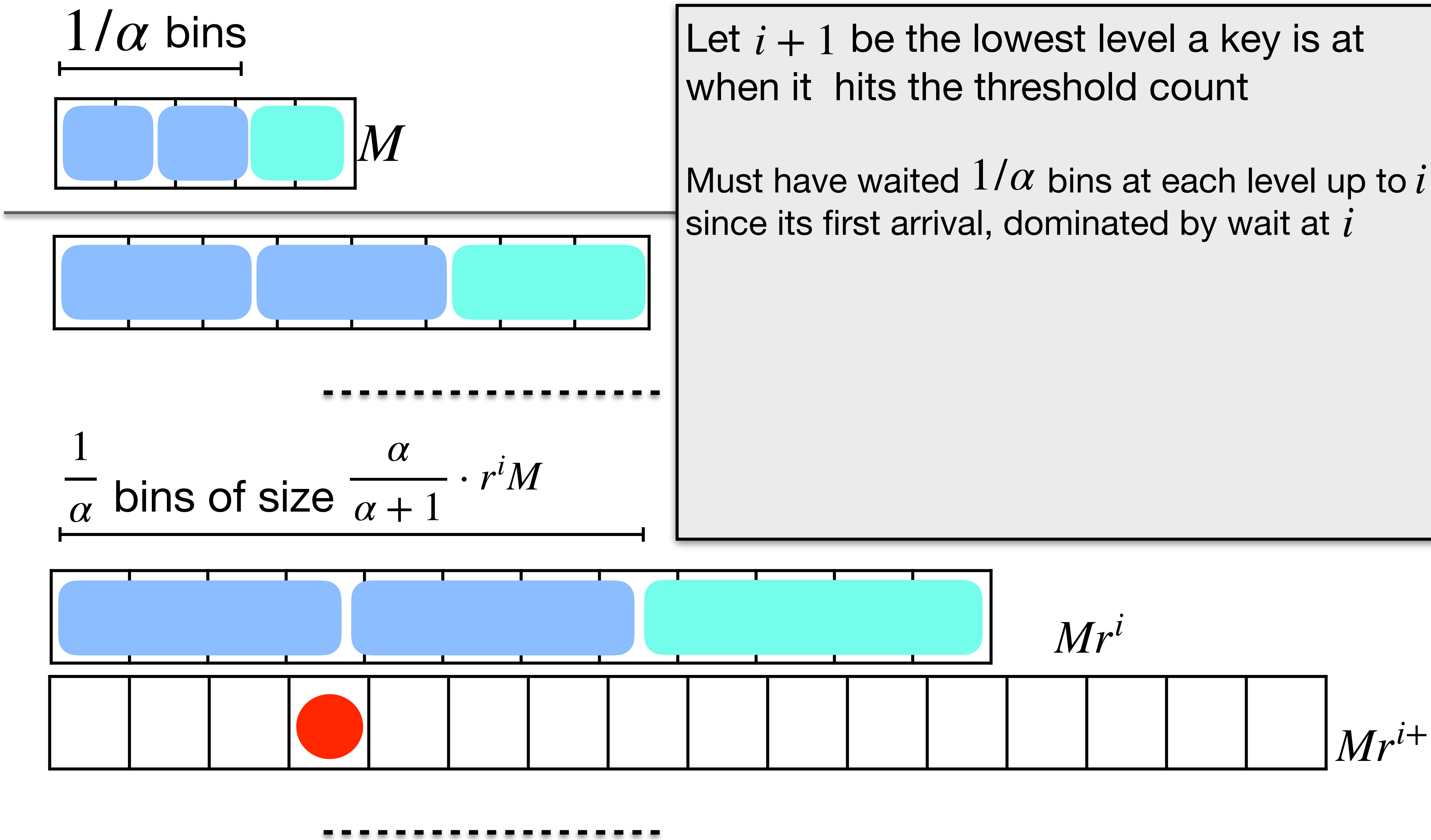


Let $i+1$ be the lowest level a key is at when it hits the threshold count

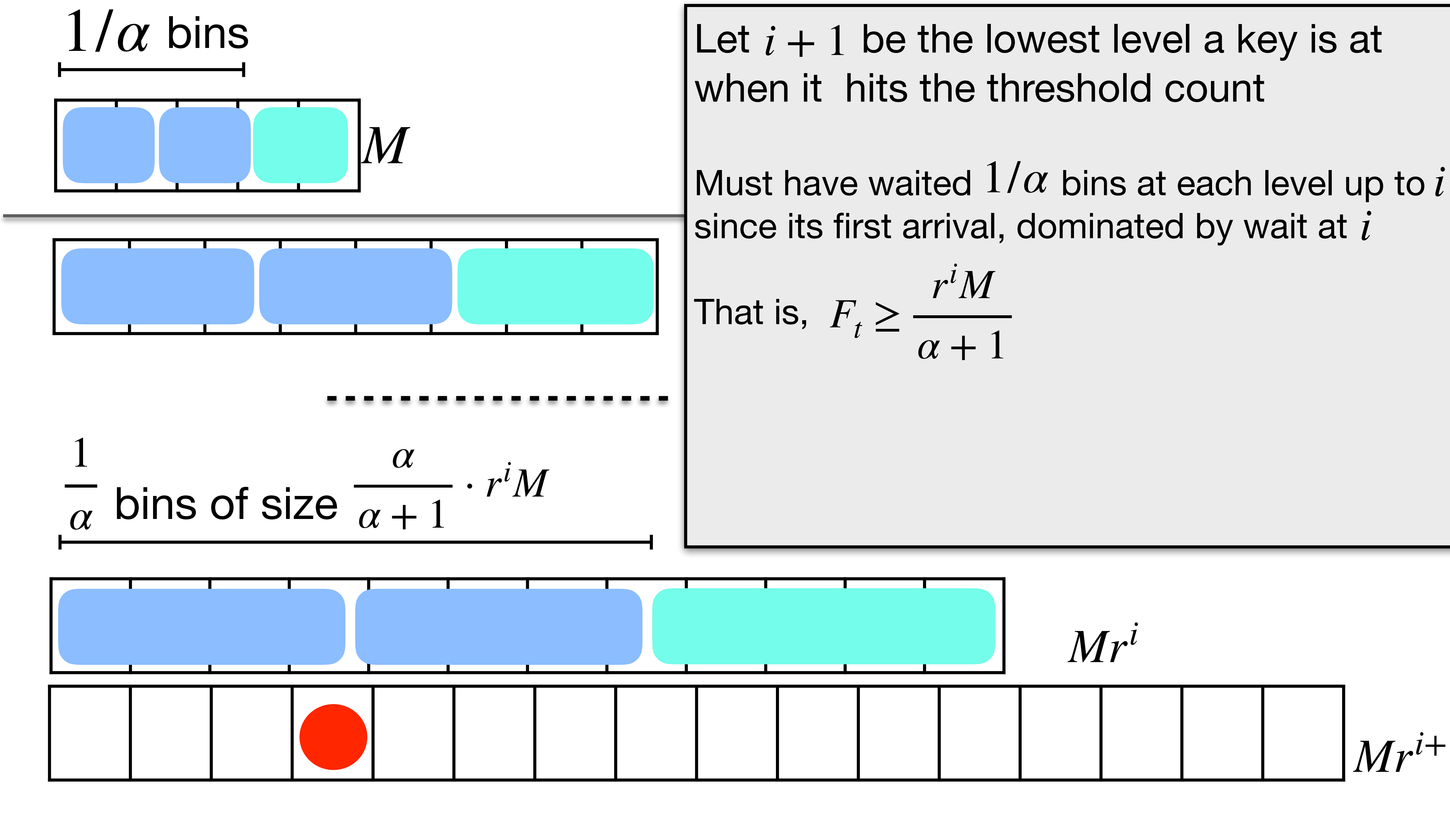
Time-Stretch Filter Correctness

Let $i+1$ be the lowest level a key is at when it hits the threshold count

Must have waited $1/\alpha$ bins at each level up to i since its first arrival, dominated by wait at i



Time-Stretch Filter Correctness



Let $i+1$ be the lowest level a key is at when it hits the threshold count

Must have waited $1/\alpha$ bins at each level up to i since its first arrival, dominated by wait at i

That is, $F_t \geq \dfrac{r^i M}{\alpha+1}$

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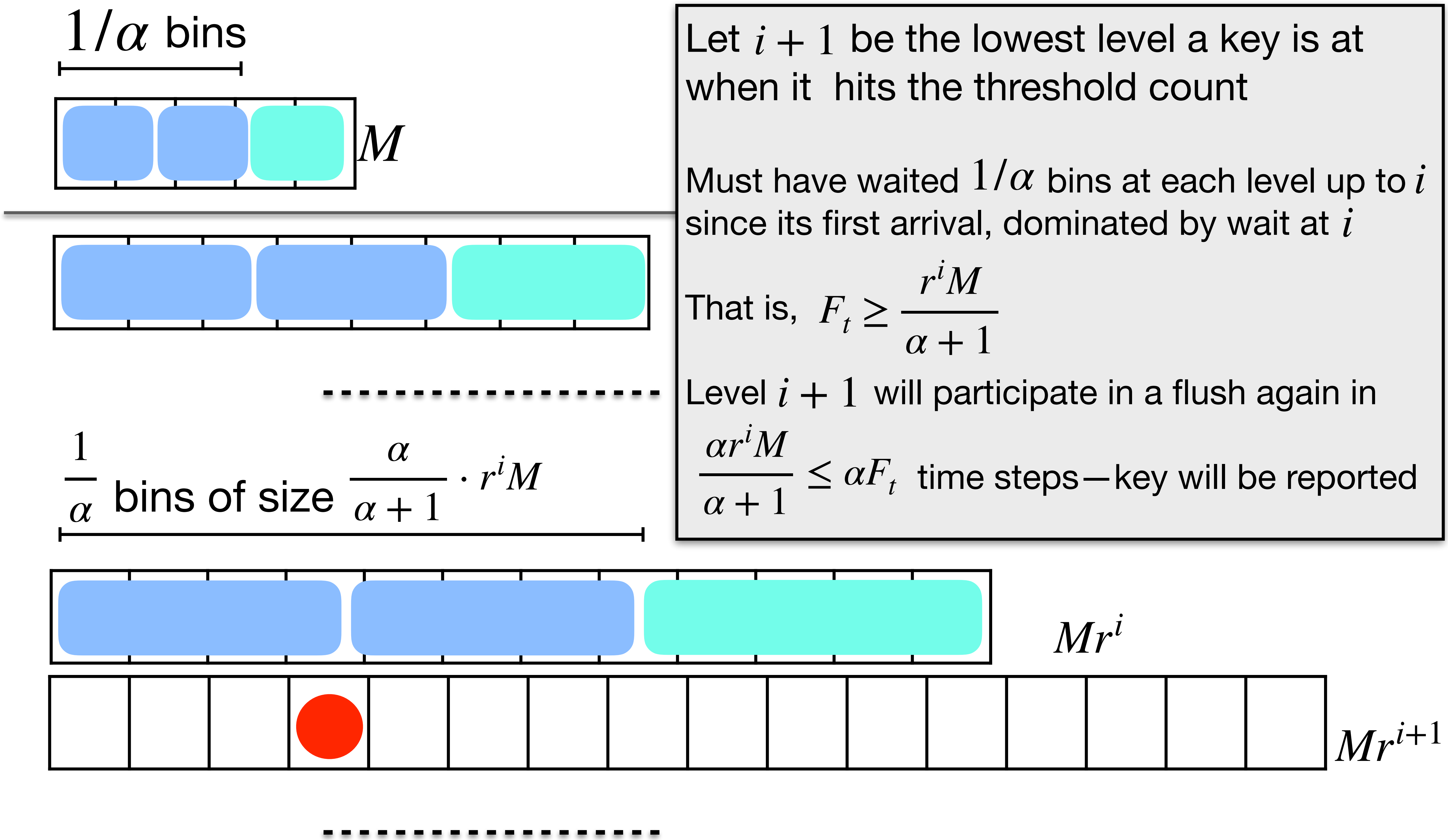
Level $i+1$ will participate in a flush again in $\dfrac{\alpha r^i M}{\alpha+1} \leq \alpha F_t$ time steps—key will be reported



Time-Stretch Filter Analysis

Theorem. Given a stream of size *N*, the amortized cost of solving OEDP with a time stretch $1+\alpha$ is $O\left(\left(\frac{1+\alpha}{\alpha}\right)\frac{1}{B}\log\frac{N}{M}\right)$

Optimal insert cost for EM & write-optimized dictionaries

Time-Stretch Filter Analysis

Theorem. Given a stream of size *N*, the amortized cost of solving OEDP with a time stretch $1 + \alpha$ is $O\left(\left(\frac{1+\alpha}{\alpha}\right)\frac{1}{B}\log\frac{N}{M}\right)$

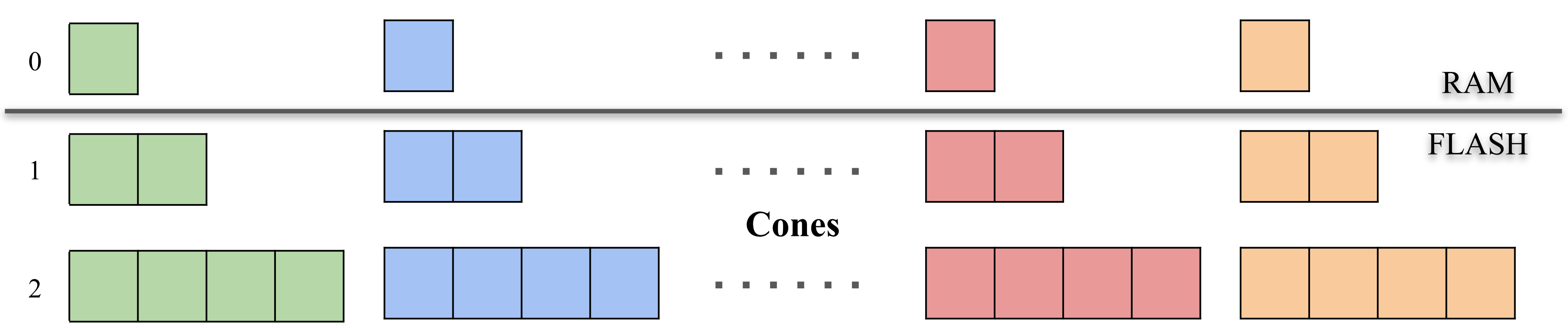
Factor lost because we only flush **a fraction of each level**; Constant loss for constant α

Almost-online reporting with no extra query cost!

Implementation/ Optimizations

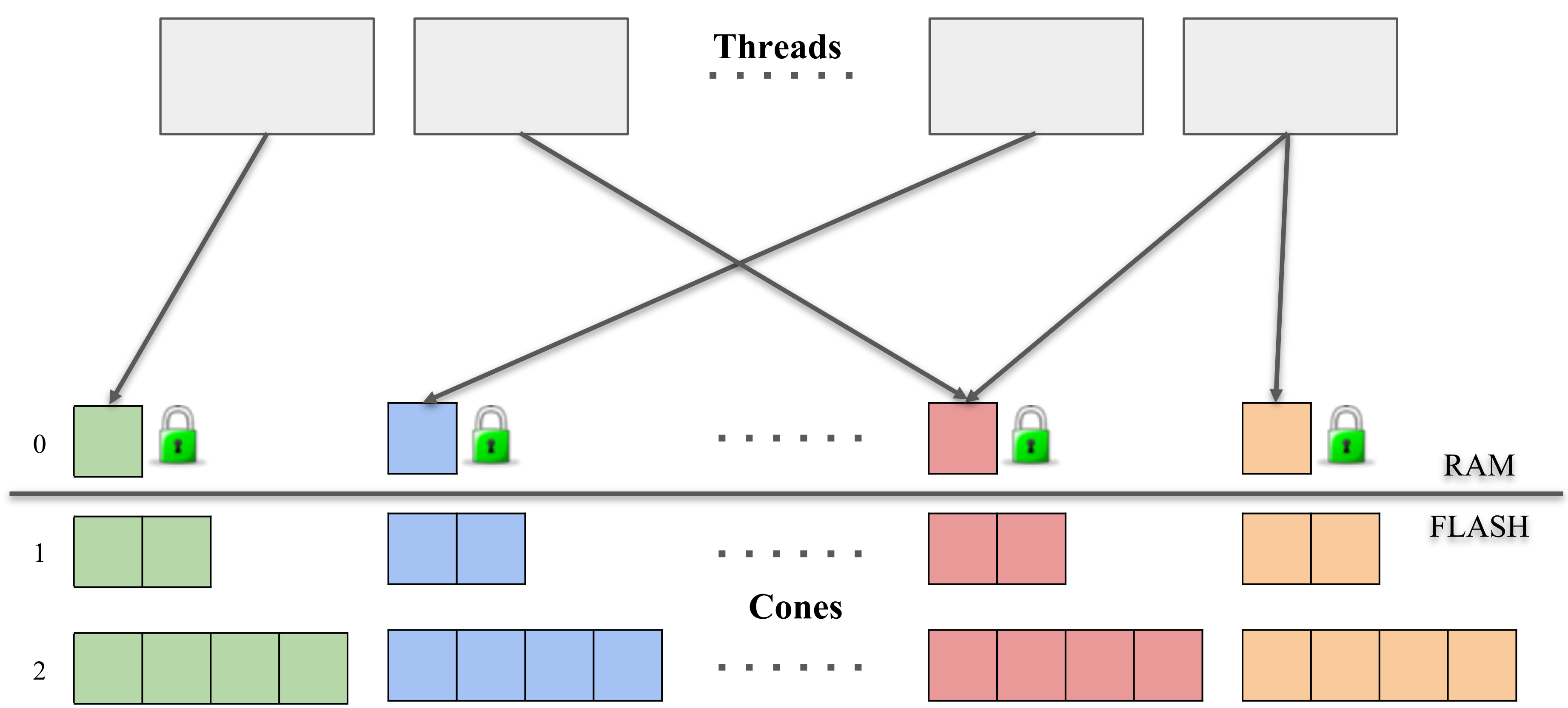
- Counts stored succinctly using **counting quotient filters** (CQFs) [Pandey et al. 17]

- Deamortize by dividing filter at each level into multiple smaller filters called *cones*



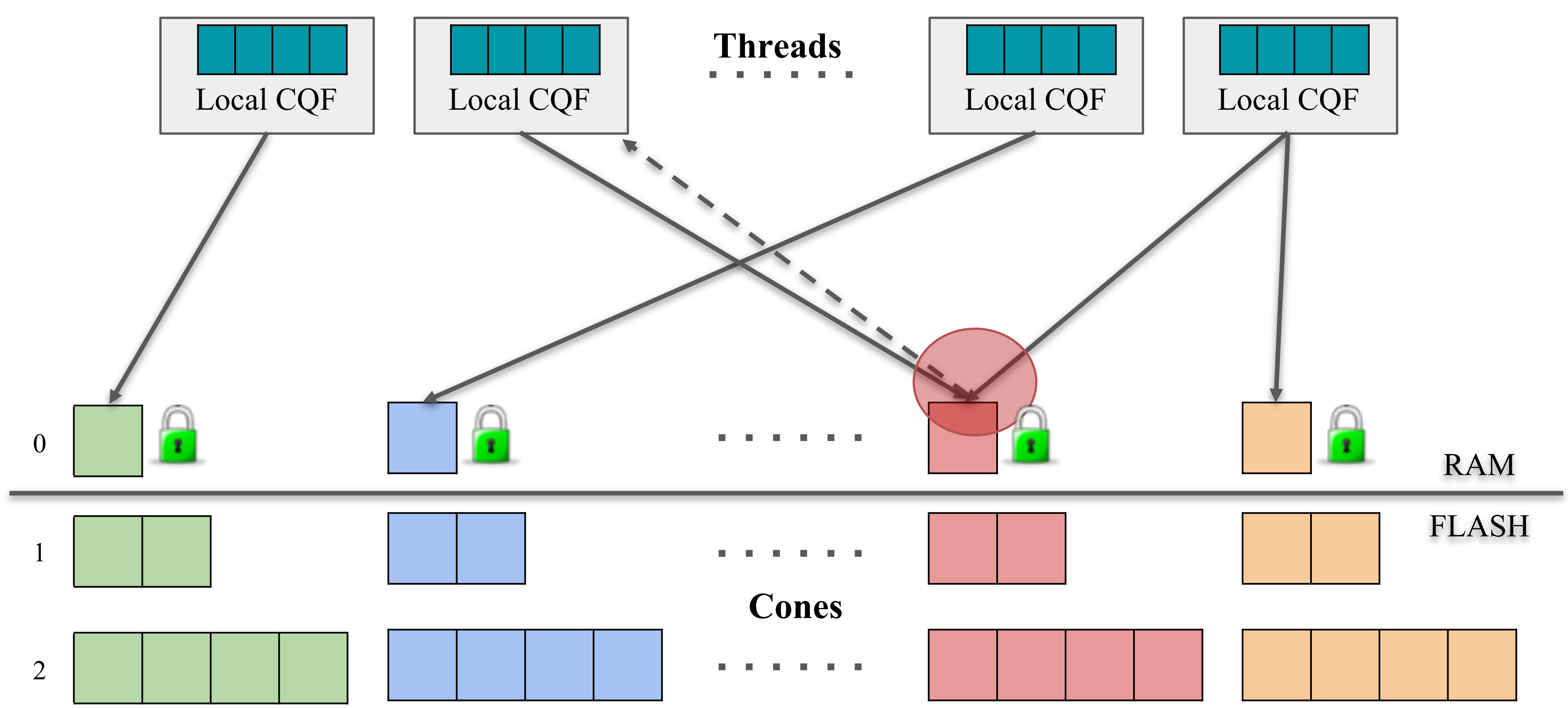
Implementation/ Optimizations

- Multi-threaded implementation
- Each thread operates by first taking a lock at the cone and then performing the insert operation

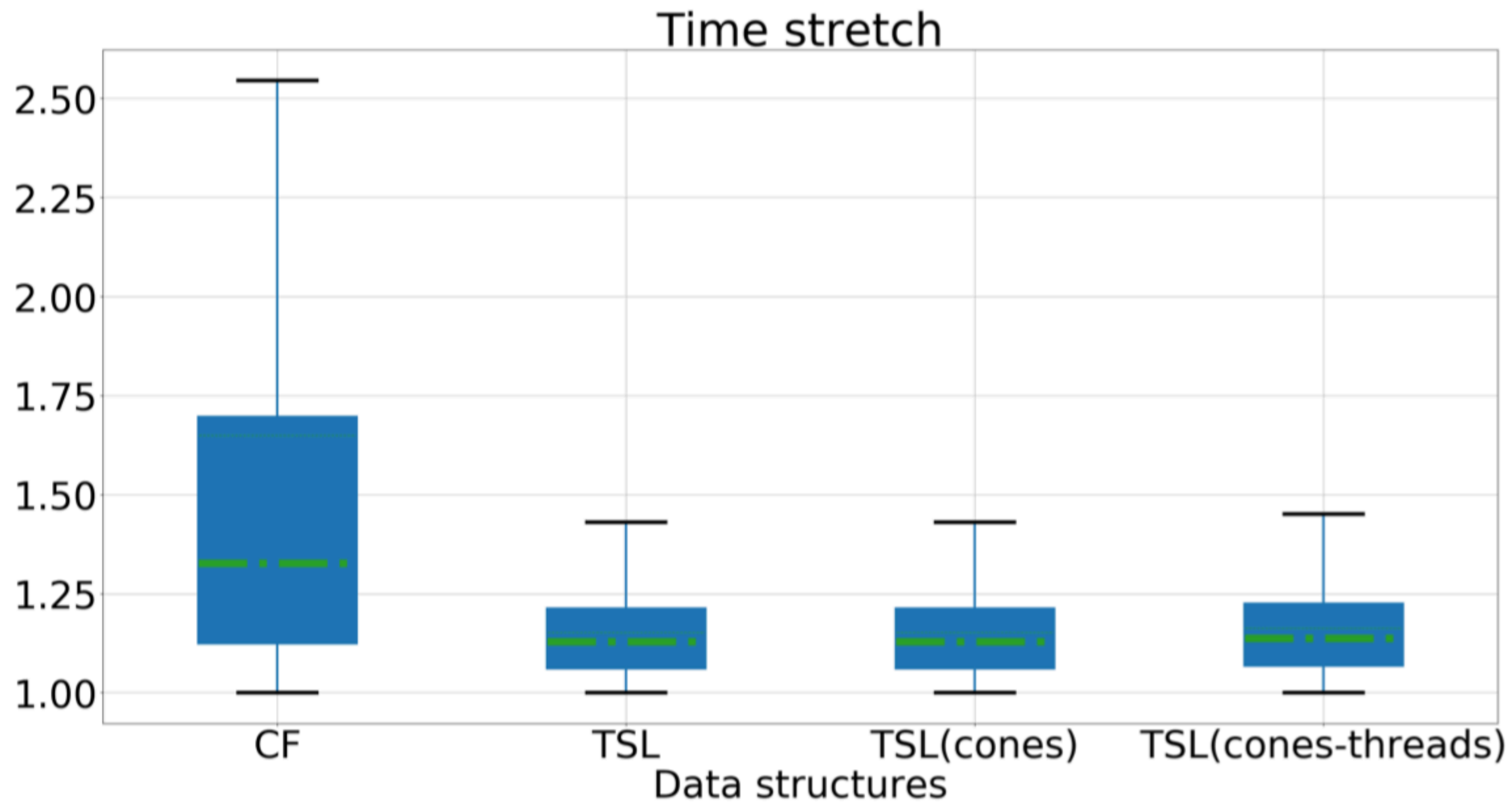


Implementation/ Optimizations

- Multi-threaded implementation
- Each thread operates by first taking a lock at the cone and then performing the insert operation
- If there is contention, the thread then inserts the item in its local buffer and continues

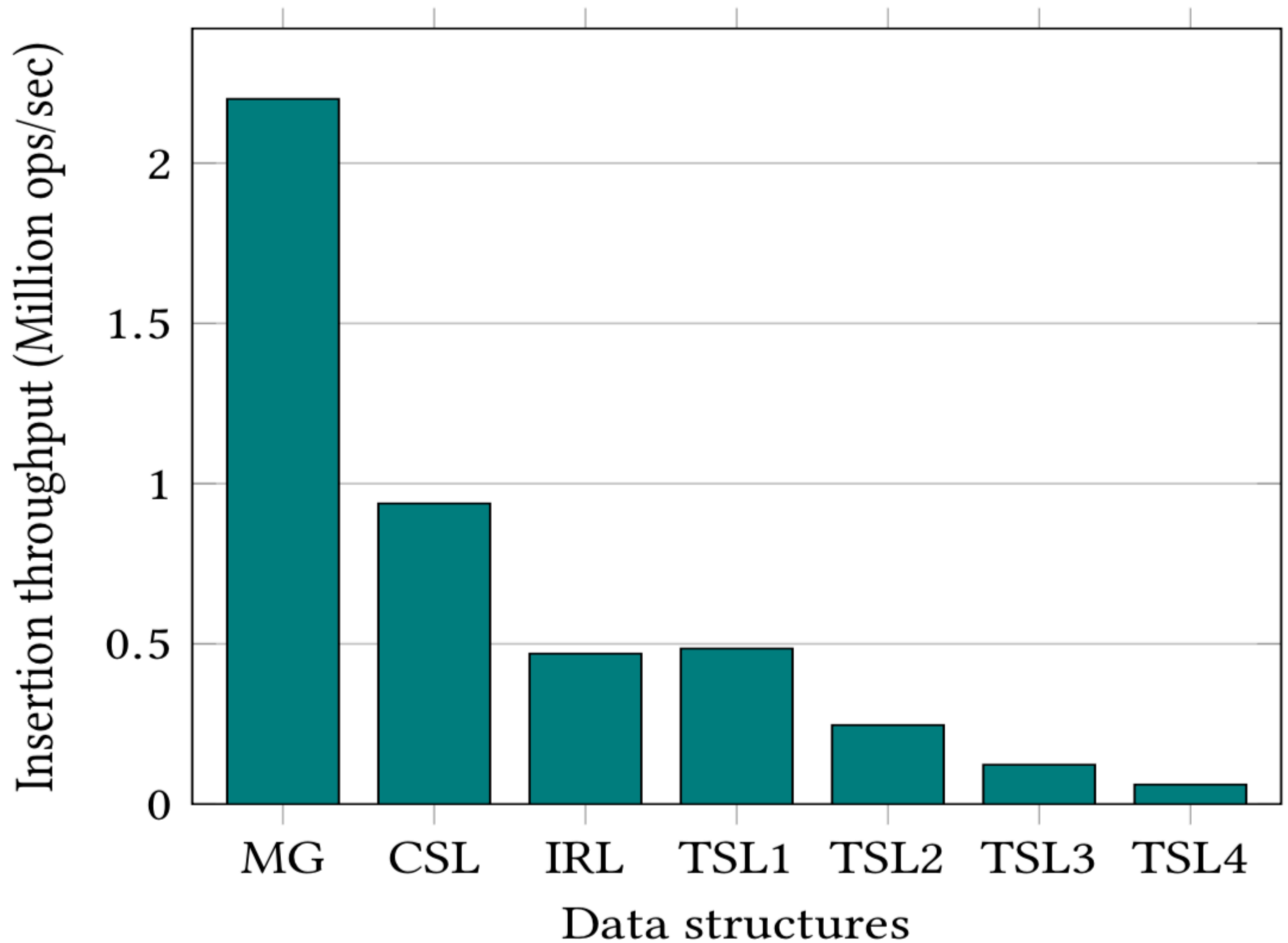


Evaluations: Time Stretch



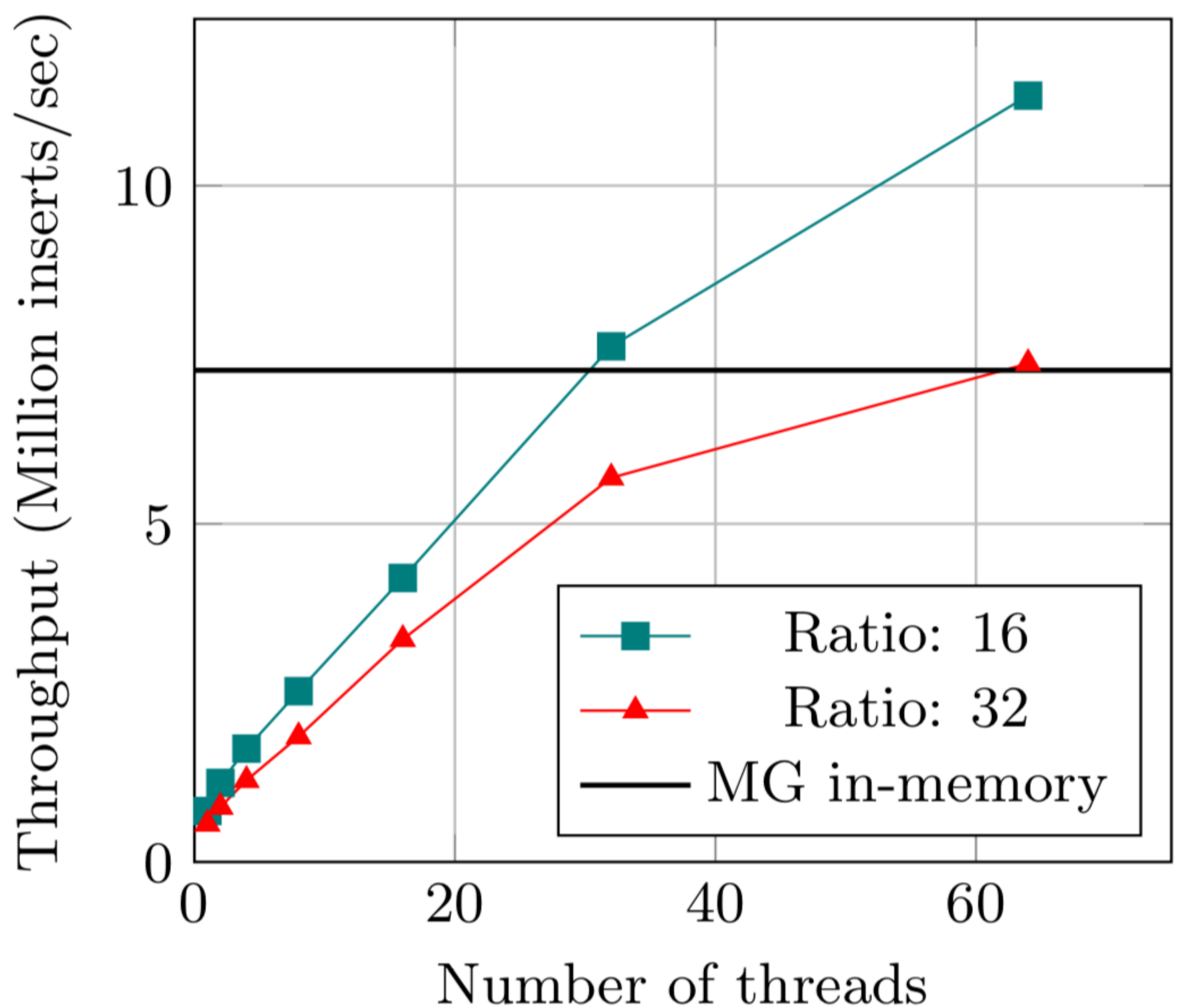
- Time-stretch filter gives improvements in timely reporting compared to out-of-the-box structures like **cascade filters [Bender et al.12]**

Evaluations: Insertion Throughput



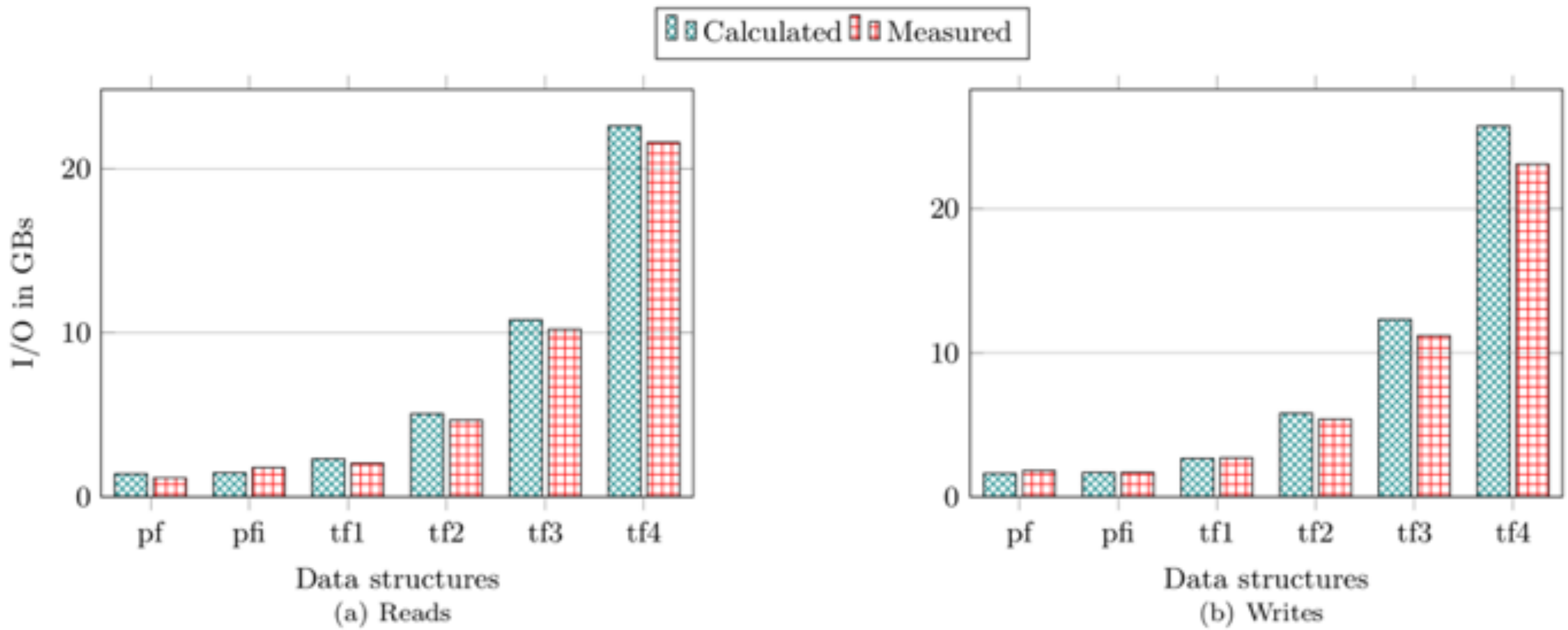
- The EM MG filter without immediate reporting has the highest throughput, followed by other variants

Evaluations: Scalability



- With higher # of threads and higher N/M, the counter-stretch filter beats throughput of regular MG algorithm

Evaluations: I/O Performance

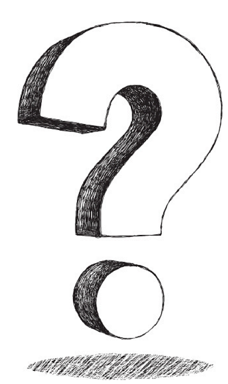


- The total I/O measured using ‘iotop’ and calculated theoretically is similar for various versions of the OEDP data structures

Conclusions & Future Directions

- Bridging the gap between streaming & external memory
- With modern SSDs and I/O techniques possible to match cache-latency-bound in-memory data structures in EM
- What other streaming problems can be **solved exactly in EM** at comparable speed?
- What is the write model for streaming in modern EM?

Streaming Model



External-memory Model

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All applications received by December 1 will receive full consideration, and review of applications will continue until the position is filled.

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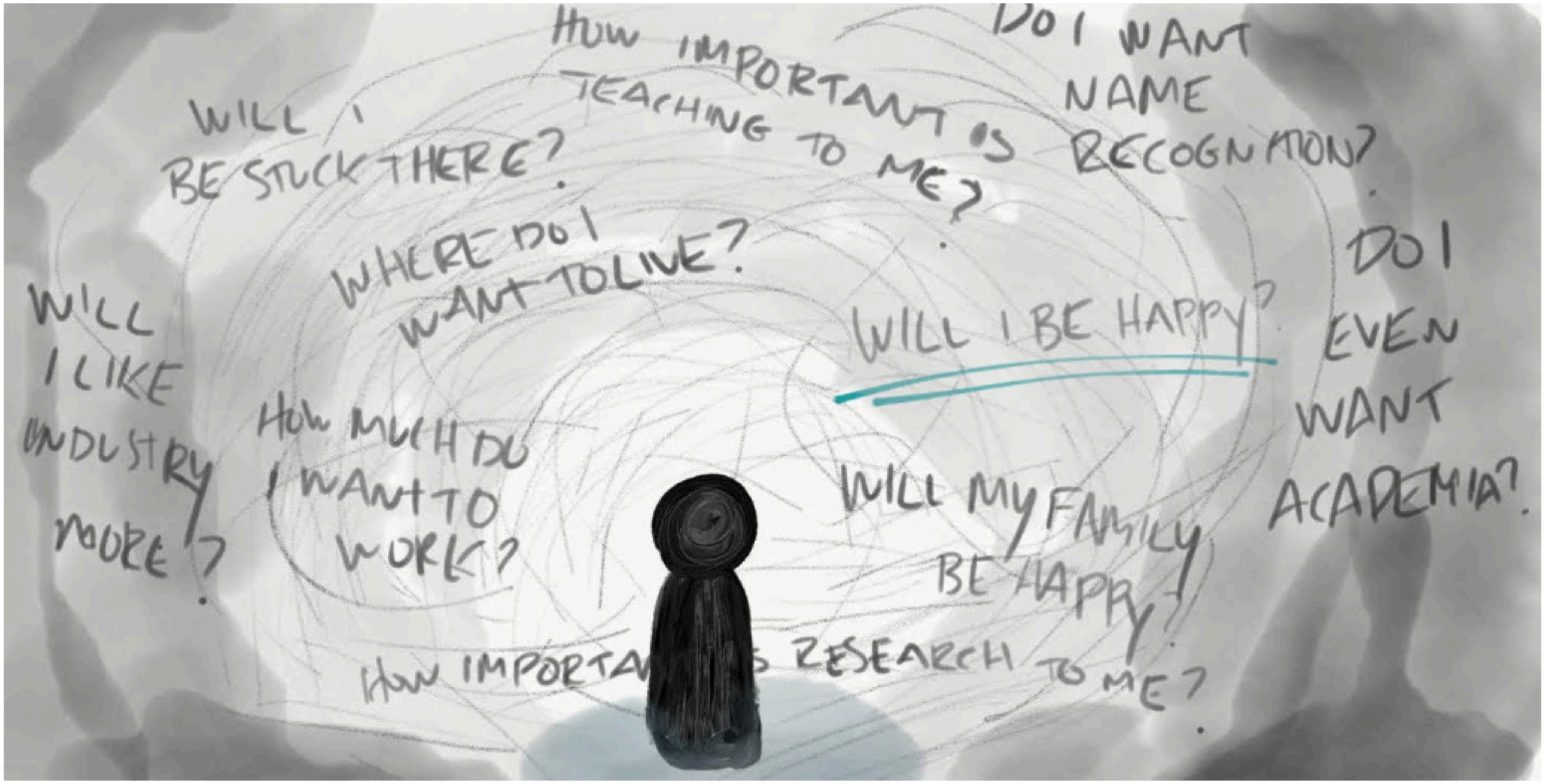


THANK
YOU