Bits, Bytes, and Integers (part II)

CSCI 237: Computer Organization
3rd Lecture, Feb 12, 2025

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Administrative Details

- Lab today and tomorrow!
- Six puzzles due next Tue/Wed at 11pm
- All puzzles due Feb 25/26 at 11pm
- Lab mostly focuses on material from today, next Monday, and next Wednesday (Ch 2 in book)
- No class on Friday (Winter Carnival!)
- First Glow homework will go out next week

Last time: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - **Representation: unsigned and signed**
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings
- Summary

Counting in Binary

- How do we convert decimal to binary?

15213 / 2 = 7606	r1
7606 / 2 = 3803	r0
3803 / 2 = 1901	r1
1901 / 2 = 950	r1
950 / 2 = 475	r0
475 / 2 = 237	r1
237 / 2 = 118	r1
118 / 2 = 59	r0

59 / 2 = 29	r1
29 / 2 = 14	r1
14 / 2 = 7	r0
7 / 2 = 3	r1
3 / 2 = 1	r1
1 / 2 = 0	r1

- For context, consider finding digits in decimal:

15213 / 10 = 1521	r3
1521 / 10 = 152	r1
152 / 10 = 15	r2
15 / 10 = 1	r5
1 / 10 = 0	r1

Counting in Binary

- 0.25_{10} → 0.01_2

  .25 × 2 = .5 r0
  .5 x 2 = 1.0 r1 ↓

- For context, consider finding digits in decimal:

  .25 × 10 = 2.5 r2
  .5 x 10 = 5.0 r5 ↓

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers (Ch 2.2)
  - **Representation: unsigned and signed**
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
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Review: General Boolean Algebras

- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  --------      --------      --------      --------
  01000001      01111101      00111100      10101010
```

- All of the Properties of Boolean Algebra Apply

Example: Representing & Manipulating Sets

- Representation
  - Width w bit vector represents subsets of {0, …, w–1}
  - $a_j = 1$ if $j \in A$
    - 01101001 { 0, 3, 5, 6 } = A
    - *76543210*
    - 01010101 { 0, 2, 4, 6 } = B
    - *76543210*
- Operations

▪	& Intersection	A & B =01000001	{ 0, 6 }
▪	| Union	A | B = 01111101	{ 0, 2, 3, 4, 5, 6 }
▪	^ Symmetric difference	A ^ B = 00111100	{ 2, 3, 4, 5 }
▪	~ Complement	~B = 10101010	{ 1, 3, 5, 7 }

Bit-Level Operations in C (Lab 1!)

- Operations &, |, ~, ^ available in C
  - Apply to any "integral" data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (char data type)
  - `~0x41 → 0xBE`
    - $\sim 01000001_2 \rightarrow 10111110_2$
  - `~0x00 → 0xFF`
    - $\sim 00000000_2 \rightarrow 11111111_2$
  - `0x69 & 0x55 → 0x41`
    - $01101001_2 \;\&\; 01010101_2 \rightarrow 01000001_2$
  - `0x69 | 0x55 → 0x7D`
    - $01101001_2 \mid 01010101_2 \rightarrow 01111101_2$

Contrast: Logic Operations in C

- Contrast to Logical Operators
  - `&&, ||, !`
    - View 0 as “False”
    - Anything nonzero as “True”
    - Always return 0 or 1
    - Early termination
- Examples (char data type)
  - `!0x41 → 0x00`
  - `!0x00 → 0x01`
  - `!!0x41 → 0x01`
  - `0x69 && 0x55 → 0x01`
  - `0x69 || 0x55 → 0x01`

Contrast: Logic Operations in C

- Contrast to Logical Operators
  - `&&, ||, !`
    - View 0 as "Fals
    - Anything
    - Alway
    - Early t
- Examples
  - `!0x41`
  - `!0x00`
  - `!!0x41`
  - `0x69 && 0x55  →  0x01`
  - `0x69 || 0x55  →  0x01`

Watch out for && vs & (and || vs |)…

One of the more common mistakes in beginner C programming!

Shift Operations

- Left Shift: `x << y`
  - Shift bit-vector **x** left **y** positions
    - Throw away extra bits on left
    - Fill with 0's on right
- Right Shift: `x >> y`
  - Shift bit-vector **x** right **y** positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- Undefined Behavior
  - Shift amount < 0 or ≥ word size

Argument x	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument x	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

Encoding Integers

Unsigned (>=0)

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Binary to unsigned

Two's Complement (Signed)

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Binary to two's complement

```
short int x =  15213;
short int y = -15213;
```

Sign Bit

- In C, `short` is 2 bytes long

	Decimal	Hex	Binary
x	15213	**3B 6D**	**00111011 01101101**
y	-15213	**C4 93**	**11000100 10010011**

- Sign Bit
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative

Two's-complement: Simple Example

```
          -16   8    4    2    1
10 =       0    1    0    1    0      8+2 = 10

          -16   8    4    2    1
-10 =      1    0    1    1    0      -16+4+2 = -10
```

To negate, flip the bits and add 1!

Two's-complement Encoding Example (Cont.)

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Numeric Ranges

- Unsigned Values
  - *UMin* = 0

    000…0
  - *UMax* = $2^w - 1$

    111…1
- Two's Complement Values
  - *TMin* = -2^{w-1}

    100…0
  - *TMax* = $2^{w-1} - 1$

    011…1
- Negative 1?

Values for *W* = 16

	Decimal	Hex	Binary
UMax	**65535**	**FF FF**	**11111111 11111111**
TMax	**32767**	**7F FF**	**01111111 11111111**
TMin	**-32768**	**80 00**	**10000000 00000000**
-1	**-1**	**FF FF**	**11111111 11111111**
0	**0**	00 00	00000000 00000000

Decimal Values for Different Word Sizes

	W (in bits)			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

- Observations
  - $|TMin|$ = $TMax + 1$
    - Asymmetric range
  - $UMax$ = $2 * TMax + 1$

- **C Programming**
  - #include <limits.h>
  - Declares constants, e.g.,
    - ULONG_MAX
    - LONG_MAX
    - LONG_MIN
  - Values platform specific

Unsigned & Signed Numeric Values

X	B2U(*X*)	B2T(*X*)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

- **Equivalence**
  - Same encodings for nonnegative values
- **Uniqueness**
  - Every bit pattern represents unique integer value
  - Each representable integer has unique bit encoding
- **⇒ Can Invert Mappings**
  - $U2B(x) = B2U^{-1}(x)$
    - Bit pattern for unsigned integer
  - $T2B(x) = B2T^{-1}(x)$
    - Bit pattern for two's comp integer

Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - **Conversion, casting**
  - Expanding, truncating
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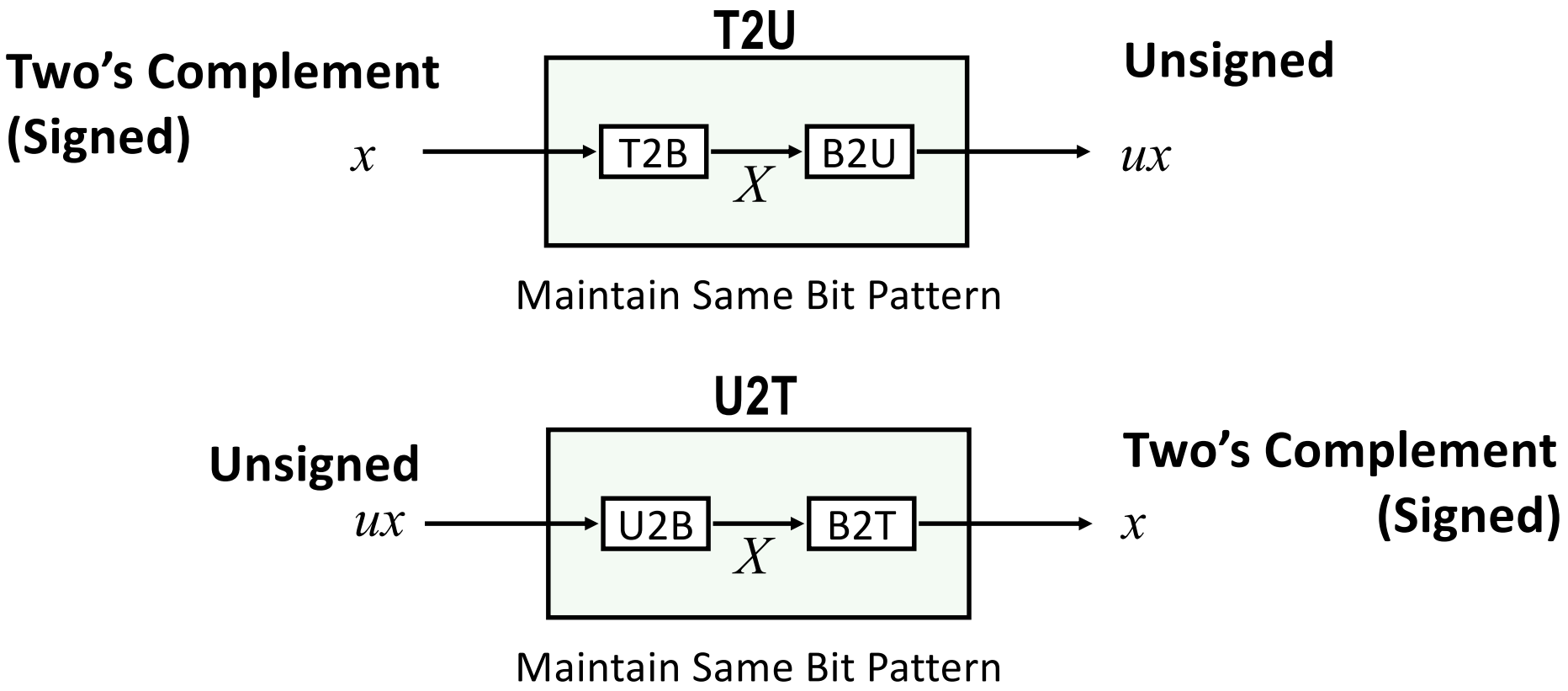
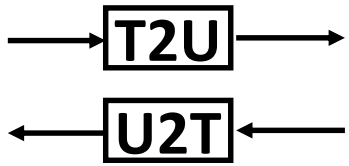
Mapping Between Signed & Unsigned



- Mappings between unsigned and two's complement numbers:
  Keep bit representations and reinterpret value

Mapping Signed ↔ Unsigned

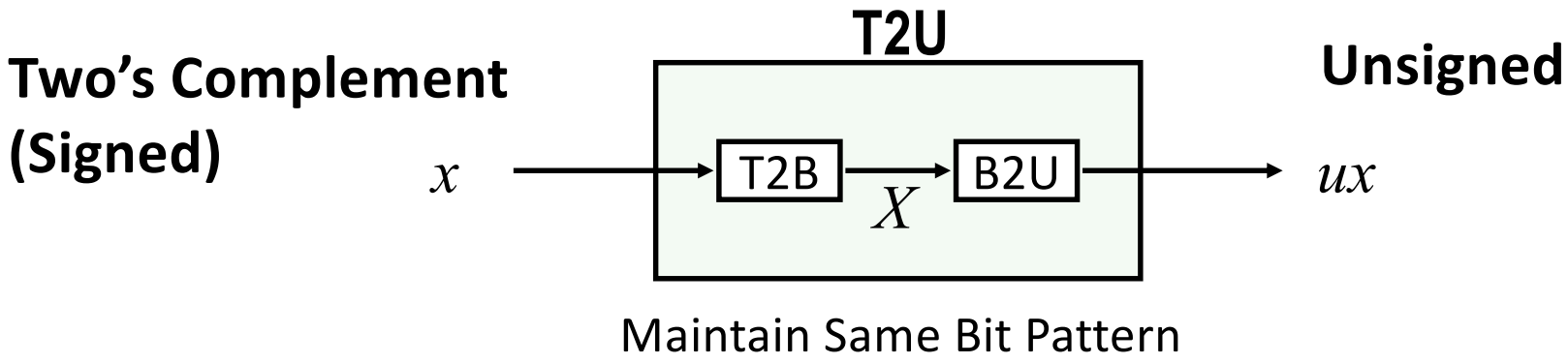
Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

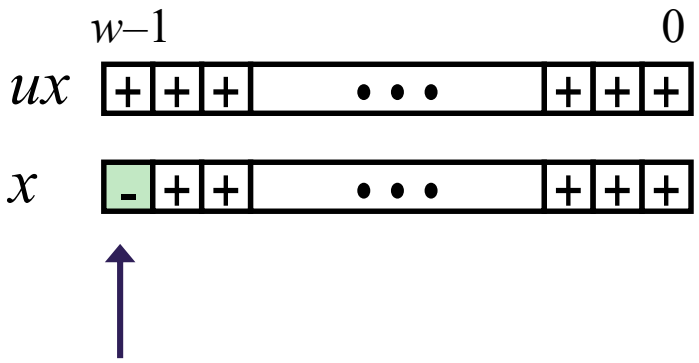


Mapping Signed ↔ Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3	=	3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6		10
1011	-5	+/- 16	11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

Relation between Signed & Unsigned





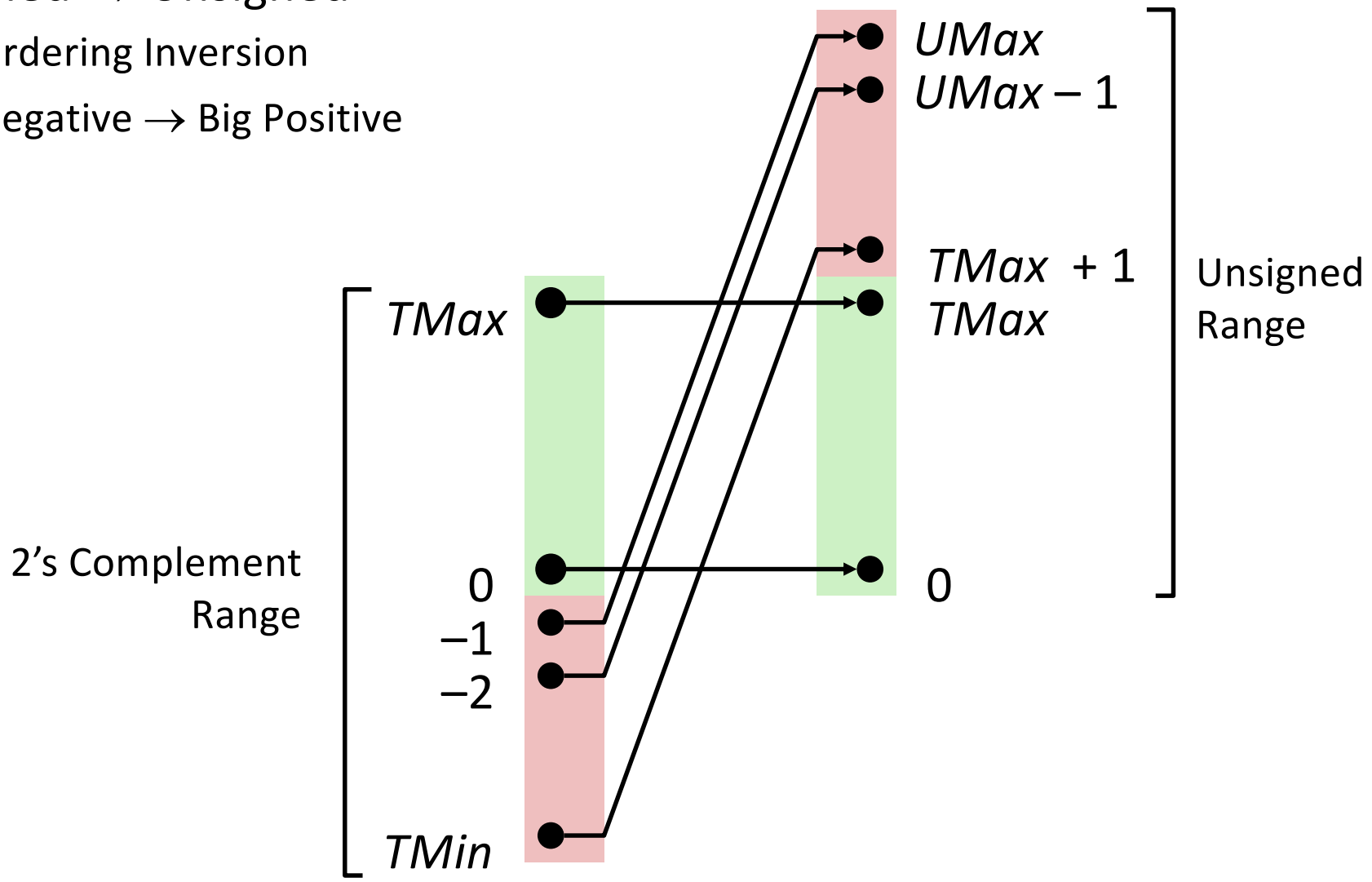
Large negative weight

becomes

Large positive weight

Conversion Visualized

- Signed → Unsigned
  - Ordering Inversion
  - Negative → Big Positive



Signed vs. Unsigned in C

- Constants
  - By default constants are considered to be **signed** integers
  - Only unsigned if they have "U" as suffix

    `0U, 4294967259U`
- Casting
  - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty; //signed by default
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  - Implicit casting also occurs via assignments and procedure calls

```
tx = ux;
uy = ty;
```

```
int fun(unsigned u);
uy = fun(tx);
```

Casting Surprises

- Expression Evaluation
  - If there is a mix of unsigned and signed values in single expression, ***signed values are implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples for *W* = 32: **TMIN = -2,147,483,648 TMAX = 2,147,483,647**

Constant$_1$	Constant$_2$	Relation	Evaluation
0	0U	==	unsigned
-1	0	<	signed
-1	0U	>	unsigned
2147483647	-2147483647-1	>	signed
2147483647U	-2147483647-1	<	unsigned
-1	-2	>	signed
(unsigned)-1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int) 2147483648U	>	signed

Summary
Casting Signed ↔ Unsigned: Basic Rules

- Bit pattern is maintained
- But pattern is reinterpreted
- Can have unexpected effects: adding or subtracting 2^w

- Expression containing signed and unsigned values
  - Signed `int` is cast to `unsigned`!!

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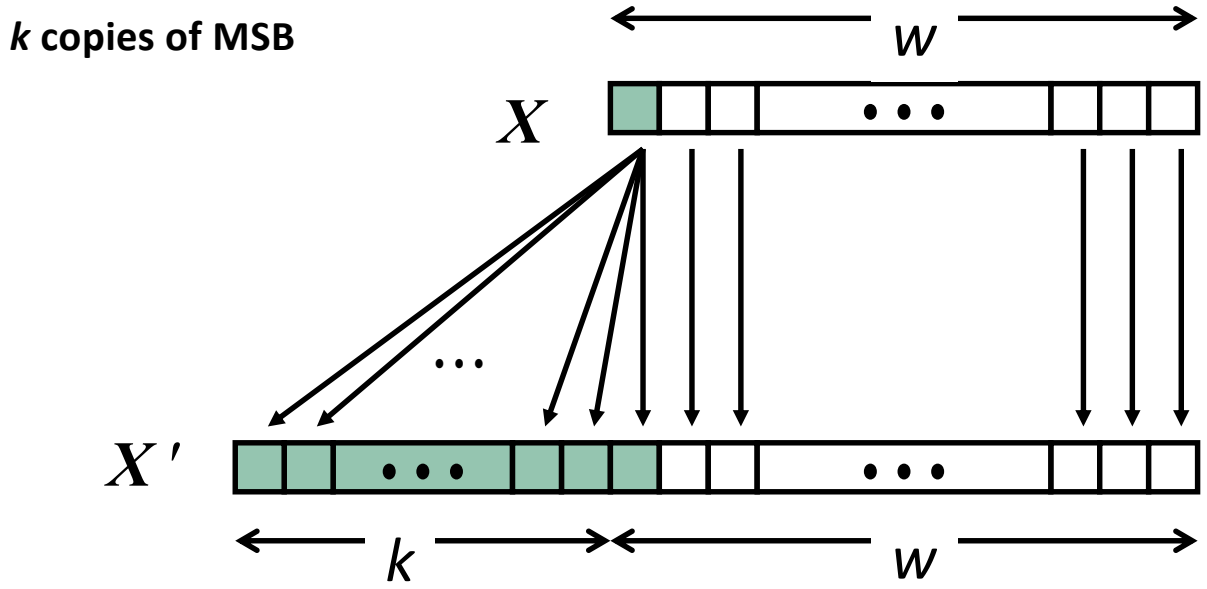
Sign Extension

- Task:
  - Given *w*-bit signed integer *x*
  - Convert it to *w*+*k*-bit integer with same value
- Rule:
  - Make *k* copies of sign bit:
  - $X' = x_{w-1}, \ldots, x_{w-1}, x_{w-1}, x_{w-2}, \ldots, x_0$



Sign Extension: Simple Example

Positive number

	-16	8	4	2	1
10 =	0	1	0	1	0

	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

Negative number

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-32	16	8	4	2	1
-10 =	1	1	0	1	1	0

Larger Sign Extension Example

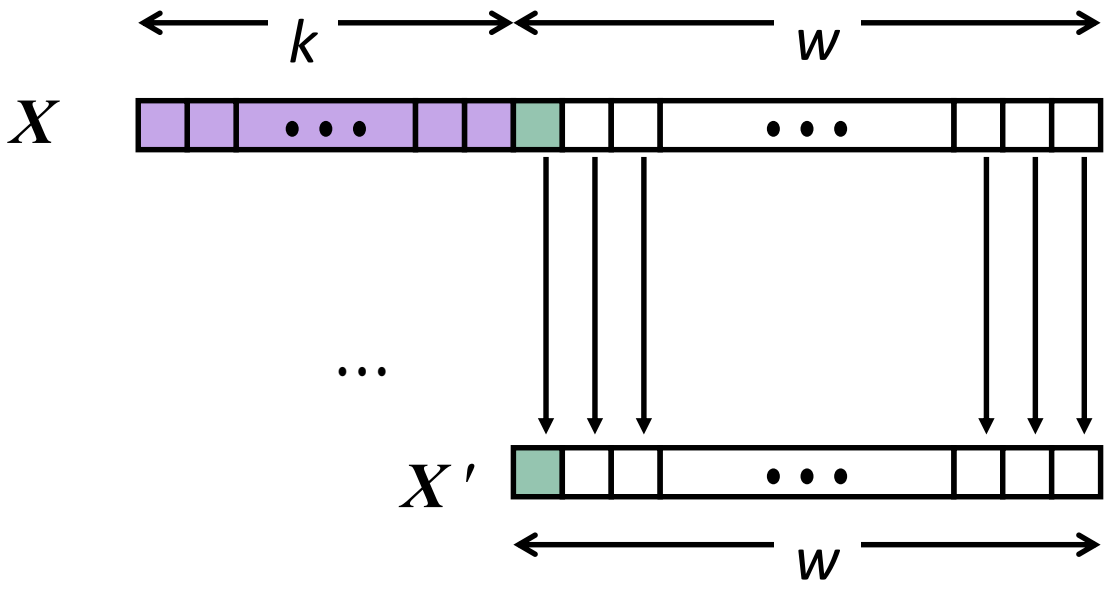
```
short int x =  15213;
int      ix = (int) x;
short int y = -15213;
int      iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Converting from smaller to larger integer data type
- C automatically performs sign extension

Truncation

- Task:
  - Given k+*w*-bit signed or unsigned integer *X*
  - Convert it to *w*-bit integer X' with same value for "small enough" X
- Rule:
  - Drop top *k* bits:
  - $X' = x_{w-1}, x_{w-2}, \ldots, x_0$



Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

2 mod 16 = 2

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

-6 mod 16 = 26U mod 16 = 10U = -6

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

10 mod 16 = 10U mod 16 = 10U = -6

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

-10 mod 16 = 22U mod 16 = 6U = 6

Summary:
Expanding, Truncating: Basic Rules

- Expanding (e.g., short int to int)
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result

- Truncating (e.g., unsigned to unsigned short)
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small numbers yields expected behavior