CSCI 136
Data Structures & Advanced Programming

Lecture 20

Spring 2018

Profs Bill & Jon

Last Time

- Iterators Recap
- Iterating over Iterators

Today

- Trees!
  - General Idea and Uses
  - Terminology
  - Some examples
    - Expression trees
  - Introduction to `structure5` `BinaryTree` class

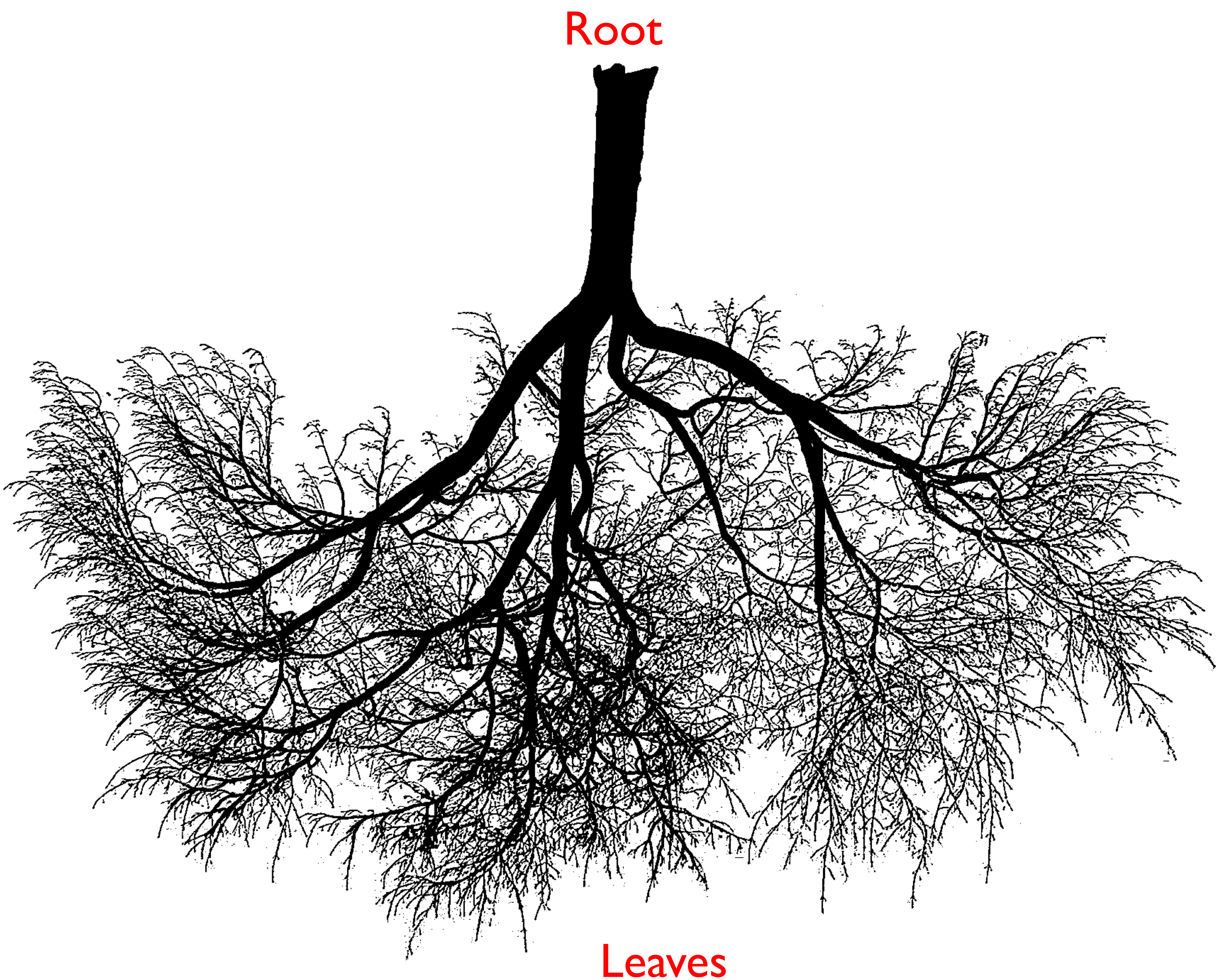
  - `BinaryTree` class implementation details
  - Proofs and theory
  - Traversing trees

Introducing Trees

- Our structures have had a linear organization
  - Stacks, queues
  - Even ordered vectors, ordered lists, arrays, vectors, lists are visualized linearly
- By linear we essentially mean that each element has at most one successor and at most one predecessor…

Branching Out: Trees

- A tree is a data structure where elements can have multiple successors (called children)
- But still only one predecessor (called parent)

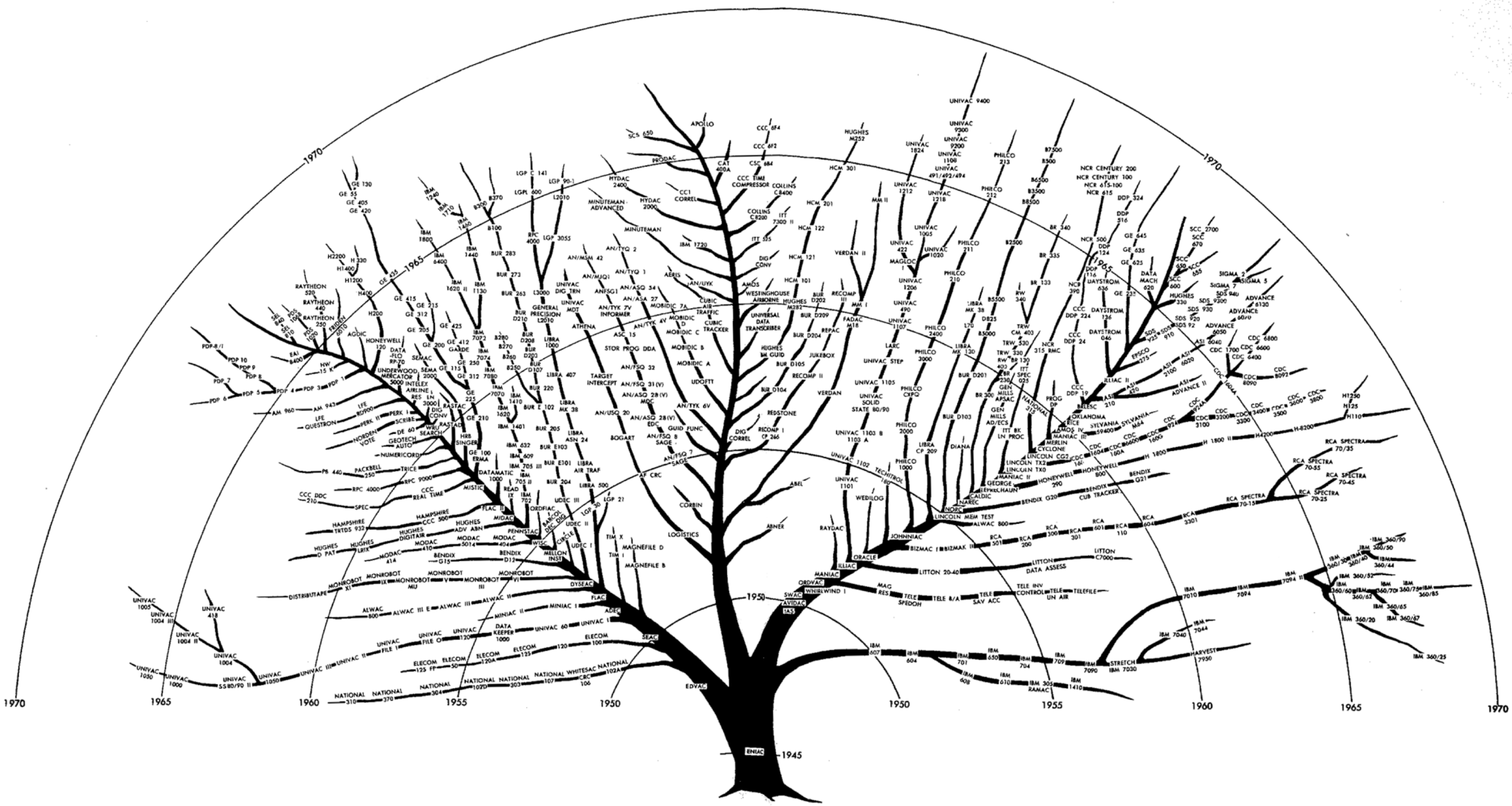


Root
Leaves

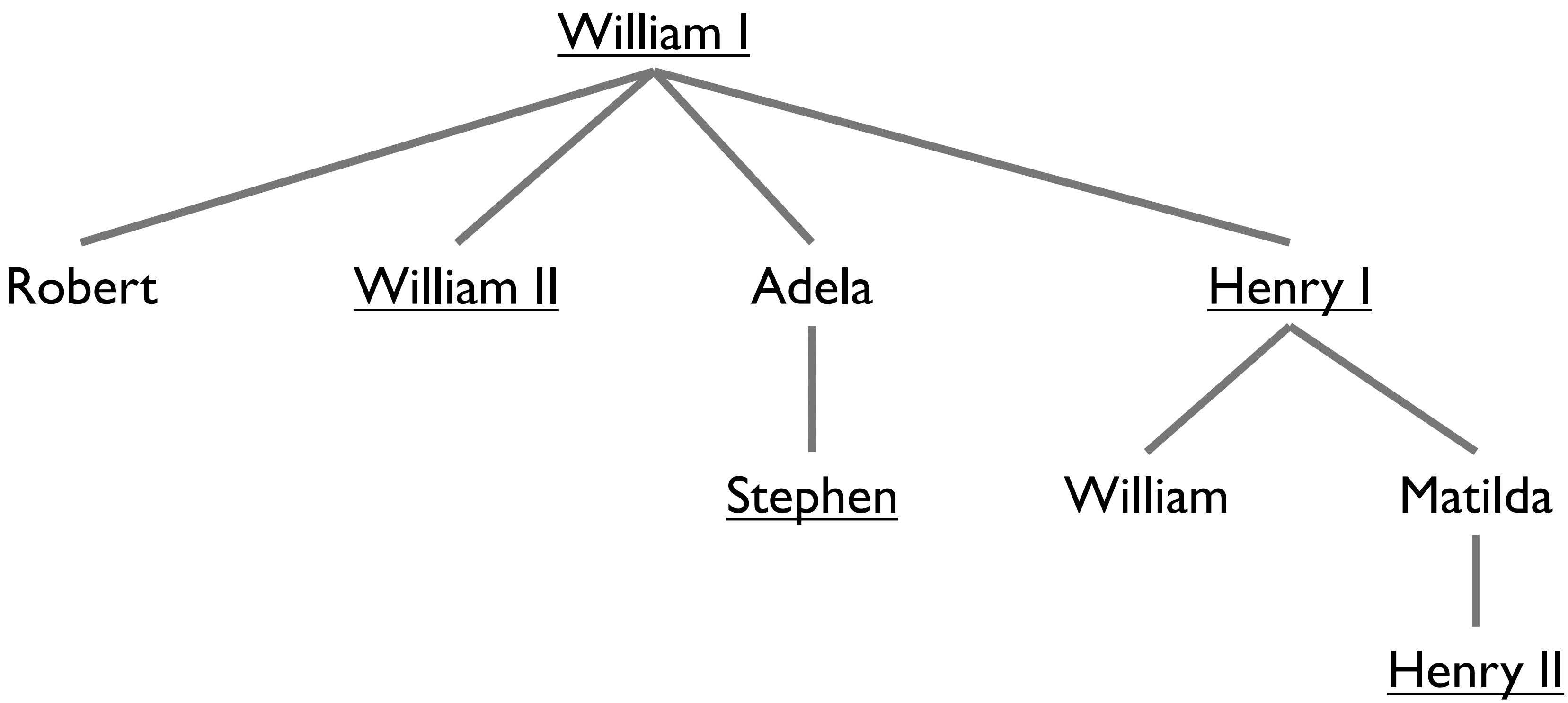
Tree Logic (Natalie Jereminjenko) at Mass MoCA

https://berkshireonstage.com/2016/07/08/get-a-sneak-peek-at-mass-mocas-phase-iii-expansion-its-a-game-changer/rs32047_joe_4-copy-copy-lpr-2-838x627/

"Computer Tree"



House of Normandy, Battle of Hastings, 1066

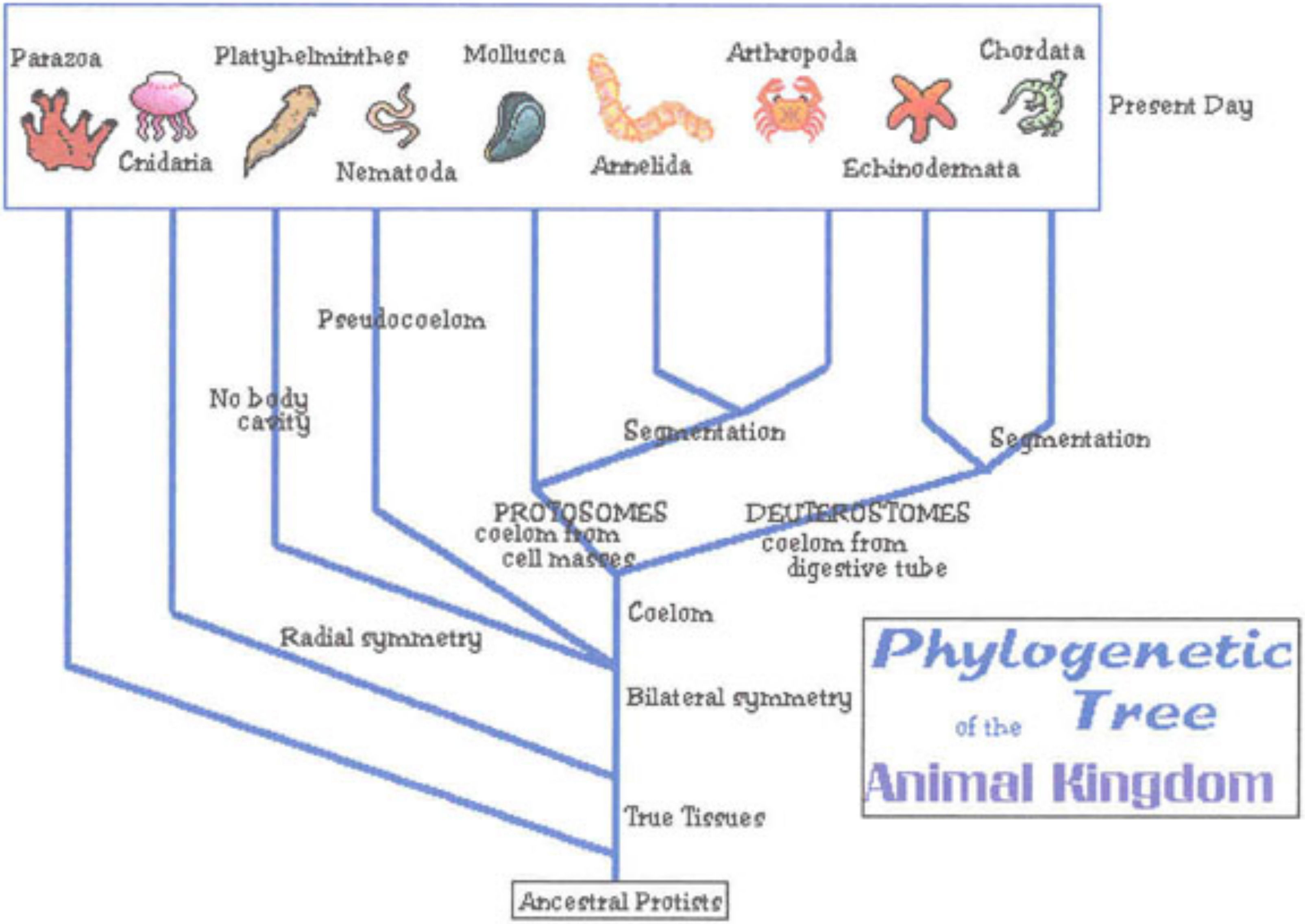


Tree Features

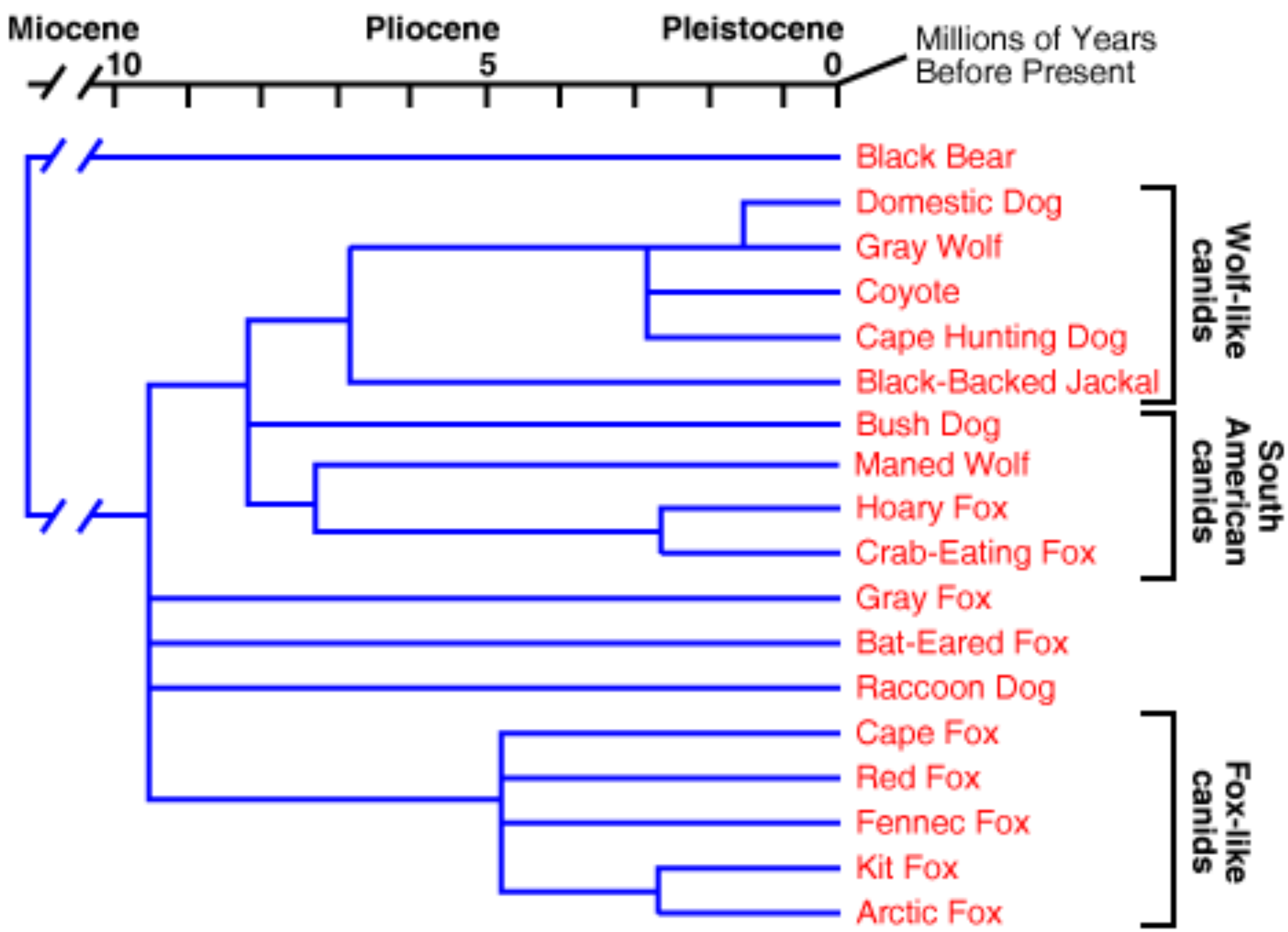
- Trees express hierarchical relationships
  - Directed: root to leaf
- Root at the top
- Leaf at the bottom
- Interior nodes in middle
- Parent, children, ancestors, descendants, siblings
- Degree (of node): number of children of node
- Degree (of tree): maximum degree (across all nodes)
- Depth of node: number of *edges* from root to node
- Height: maximum depth (across all nodes)

Other Trees

- Phylogenetic tree
- Directories of files
- Game trees
  - Build a tree
  - Search it for moves with high likelihood of winning
- Expression trees



Parazoa
Cnidaria
Platyhelminthes
Nematoda
Mollusca
Annelida
Arthropoda
Echinodermata
Chordata
Present Day
Pseudocoelom
No body cavity
Segmentation
Segmentation
PROTOSOMES
coelom from cell masses
DEUTEROSTOMES
coelom from digestive tube
Coelom
Radial symmetry
Bilateral symmetry
True Tissues
Ancestral Protists
Phylogenetic Tree of the Animal Kingdom


Miocene
Pliocene
Pleistocene
10
5
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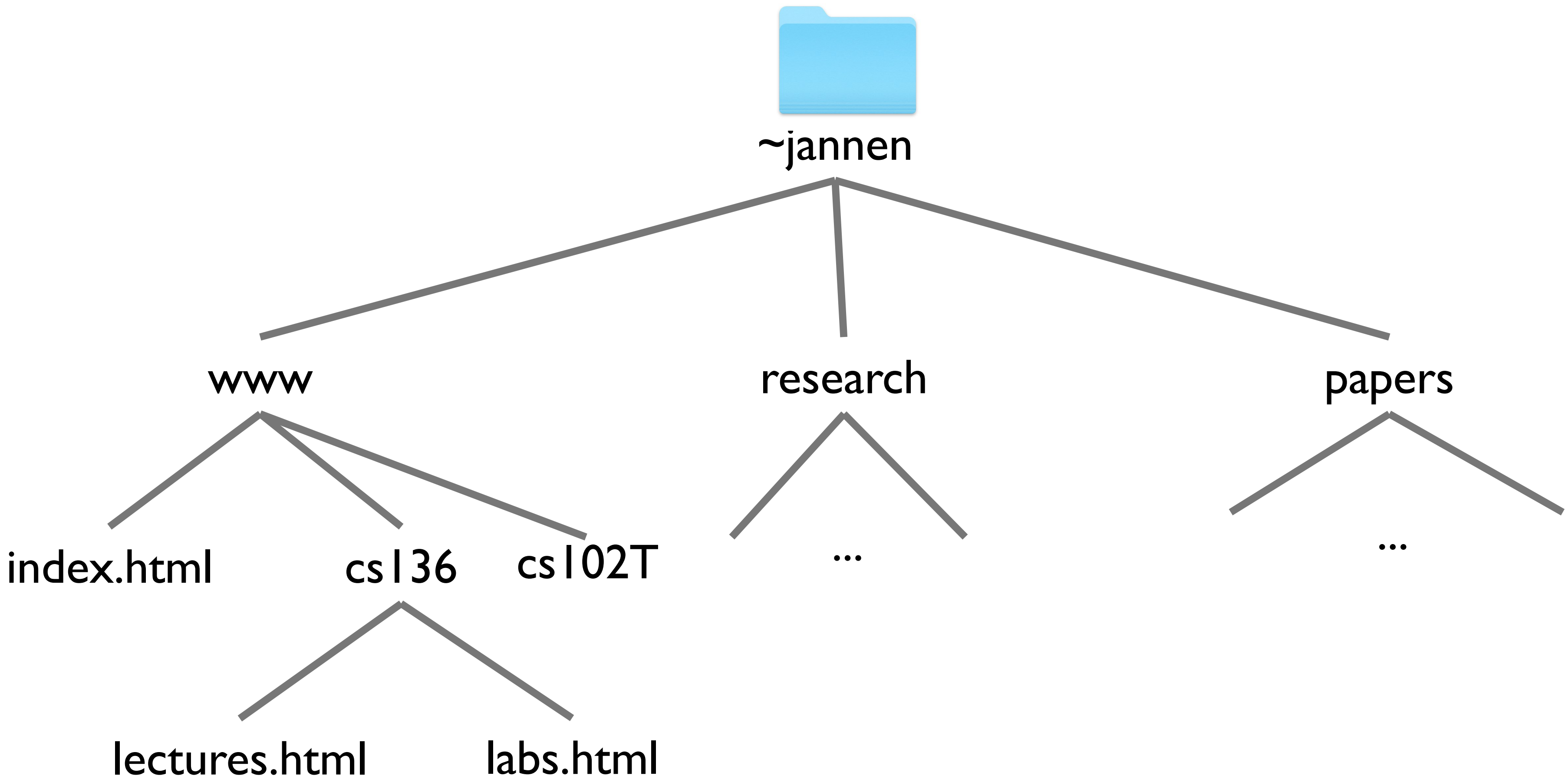
Millions of Years
Before Present
Black Bear
Domestic Dog
Gray Wolf
Coyote
Cape Hunting Dog
Black-Backed Jackal
Bush Dog
Maned Wolf
Hoary Fox
Crab-Eating Fox
Gray Fox
Bat-Eared Fox
Raccoon Dog
Cape Fox
Red Fox
Fennec Fox
Kit Fox
Arctic Fox
Wolf-like canids
South American canids
Fox-like canids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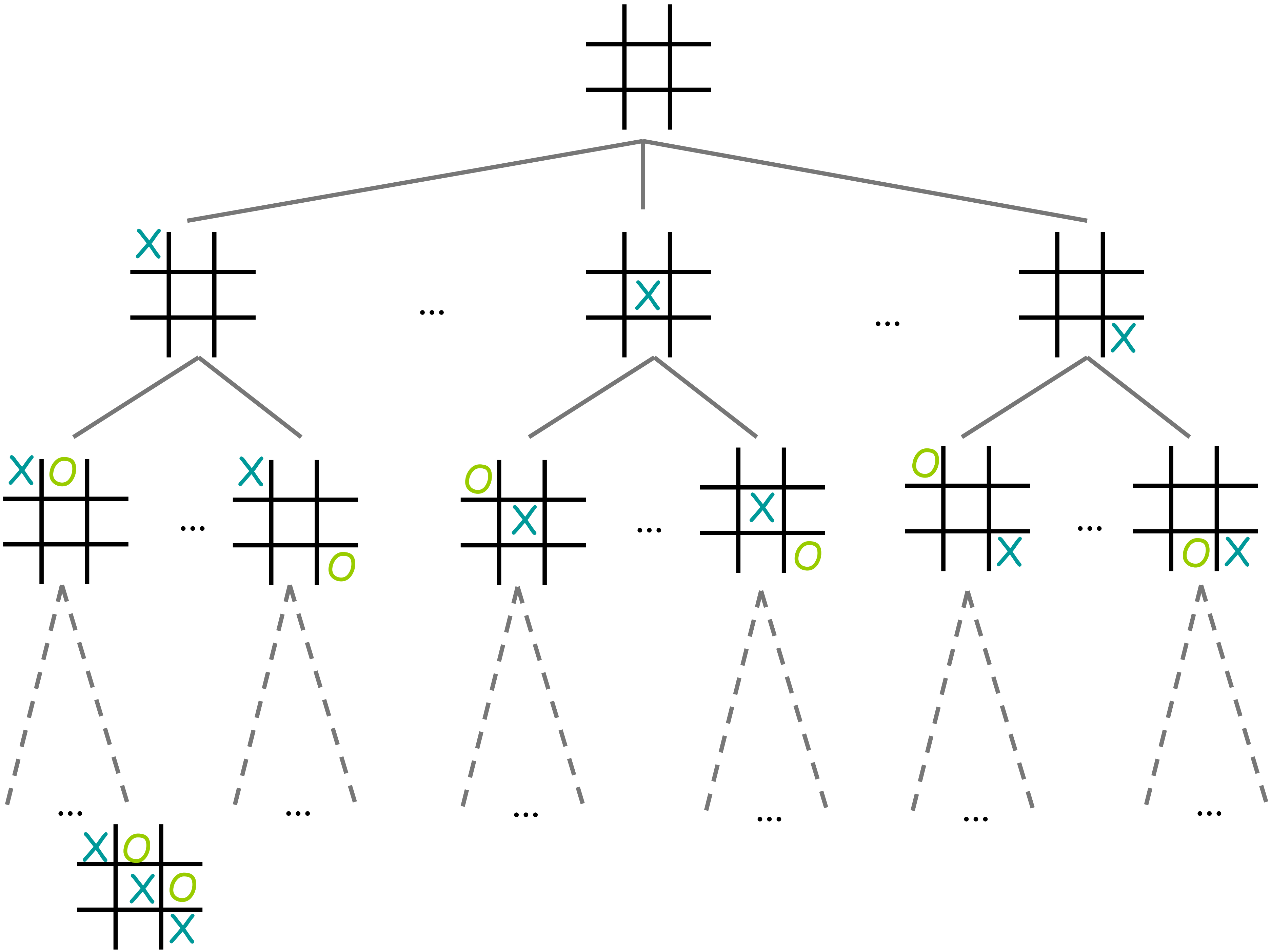
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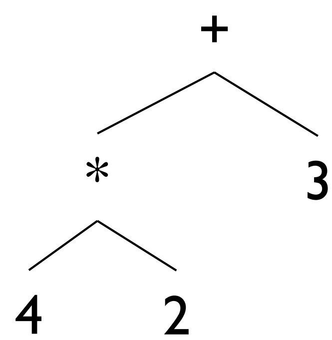
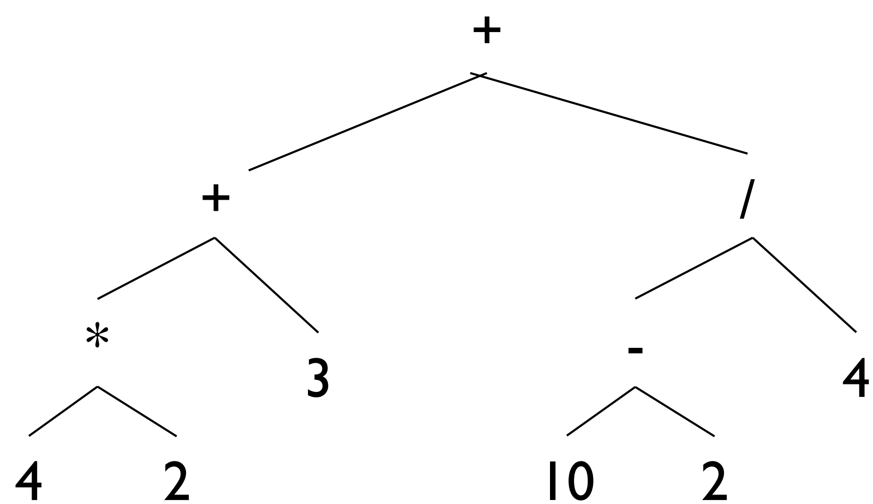
Expression Trees

4 * 2 + 3



(4 * 2 + 3) + ((10 – 2)/ 4)



Introducing Binary Trees

- Degree of each node <= 2
- Recursively defined. A tree can either be:
  - Empty
  - Root with left and right subtrees
- SLL: Recursive nature was captured by nodes (`Node<E>`) on inside
- Binary Tree: No "inner" node class; single `BinaryTree` class does it all
- (Not part of the `structure` hierarchy)

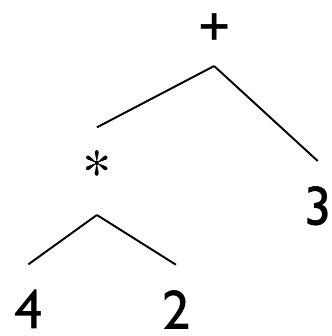
Binary Trees for (Math) Expressions

- General strategy
  - Make a binary tree (BT) for each leaf node
  - Move from bottom to top, creating BTs
  - Eventually reach the root
  - Call "evaluate" on final BT

- Example
  - How do we make a binary expression tree for: (4*2)+3
    - Leaves are numbers
    - Non-leaf nodes are operators
      - We will apply each operator to its children (ex: left + right)

Example: Expression Trees

4 * 2 + 3



Build using constructor

```
new BinaryTree<E>(value, leftSubTree, rightSubTree)
```

```
BinaryTree<String> fourTimesTwo =
                    new BinaryTree<String>(“*”,
                    new BinaryTree<String>(“4”),
                    new BinaryTree<String>(“2”));
BinaryTree<String> fourTimesTwoPlusThree =
                    new BinaryTree<String>(“+”,
                    fourTimesTwo,
                    new BinaryTree<String>(“3”));
```

Evaluating Expression Trees

- Starting at the root,
  - Evaluate left subree
  - Evaluate right subtree
  - Perform operation (+, -, *, /) with left and right

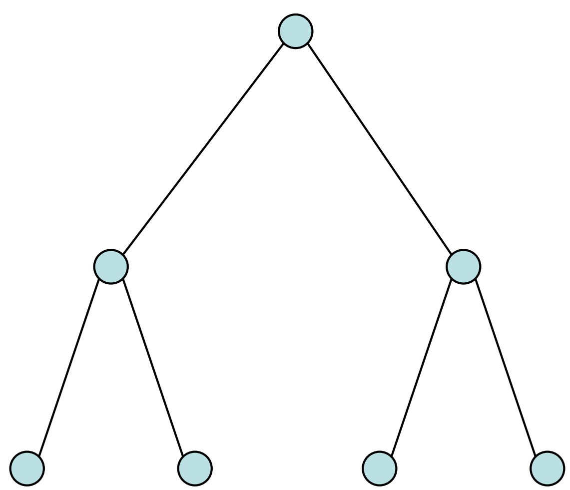
```
int evaluate(BinaryTree<String> expr) {
     if (expr.height() == 0) {
          return Integer.parseInt(expr.value());
     } else {
          int left = evaluate(expr.left());
          int right = evaluate(expr.right());
          String op = expr.value();
          switch (op) {
               case “+” : return left + right;
               case “-” : return left - right;
               case “*” : return left * right;
               case “/” : return left / right;
          }
          Assert.fail(“Bad op”);
          return -1;
     }
}
```

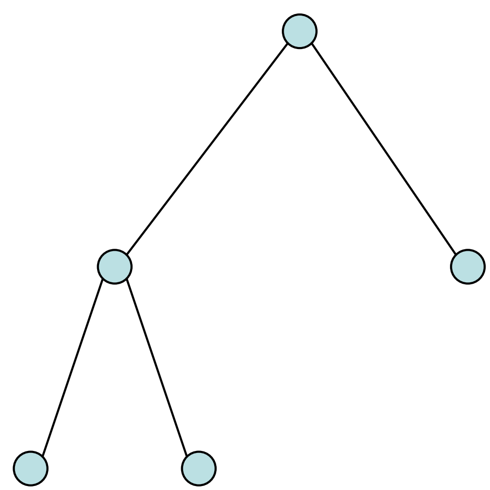
More Tree Terminology

- Some of the terminology is non-standard
- We will try to be consistent in this class, but…
  - We want to be able to communicate to our friends outside of Williams CS too!
- I *hate* jargon, but having a language for our data structures gives us the ability to express ideas and describe algorithms

Full vs. Complete (non-standard!)

- Full tree – A full binary tree of height h has *leaves only* on level h, and each internal node has exactly 2 children.
- Complete tree – A *complete* binary tree of height h is *full* to height h-1 and has all leaves at level h in leftmost locations.





All full trees are complete, but not all complete trees are full!