CSCI 136
Data Structures &
Advanced Programming

Lecture 26

Fall 2018

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Administrative Details

- Lab 9: Super Lexicon is online
  - Partners are permitted this week!

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Last Time

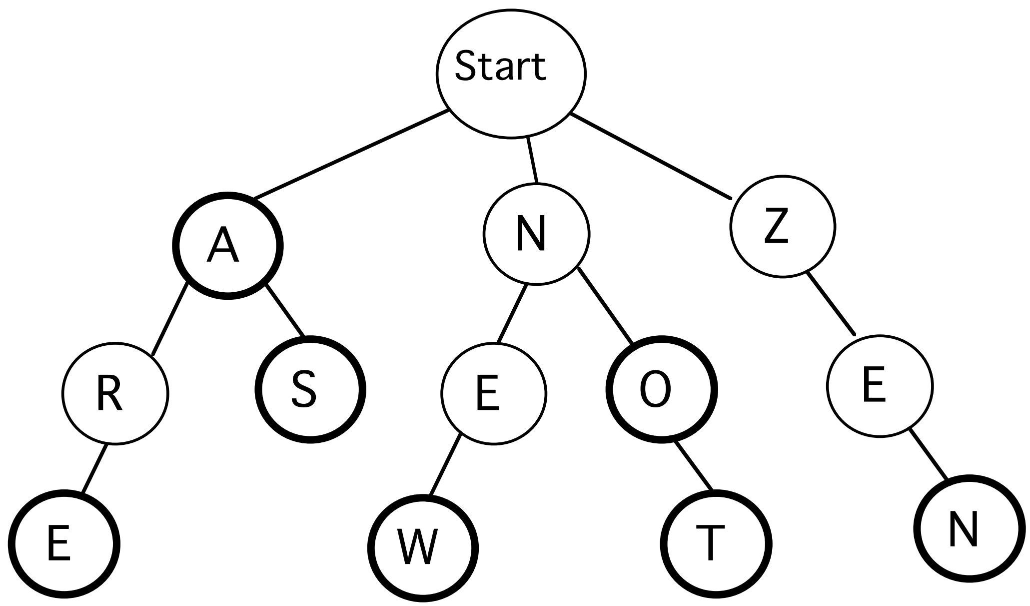
- *Efficient* Binary search trees (Ch 14)
  - AVL Trees
    - Height is O(log n), so all operations are O(log n)
  - Red-Black Trees
    - Different height-balancing idea: height is O(log n)
    - All operations are O(log n)

Today's Outline

- Lab 9: Super Lexicon
- Introduction To Graphs
  - Basic Definitions and Properties
  - Applications and Problems

Lab 9 : Lexicon

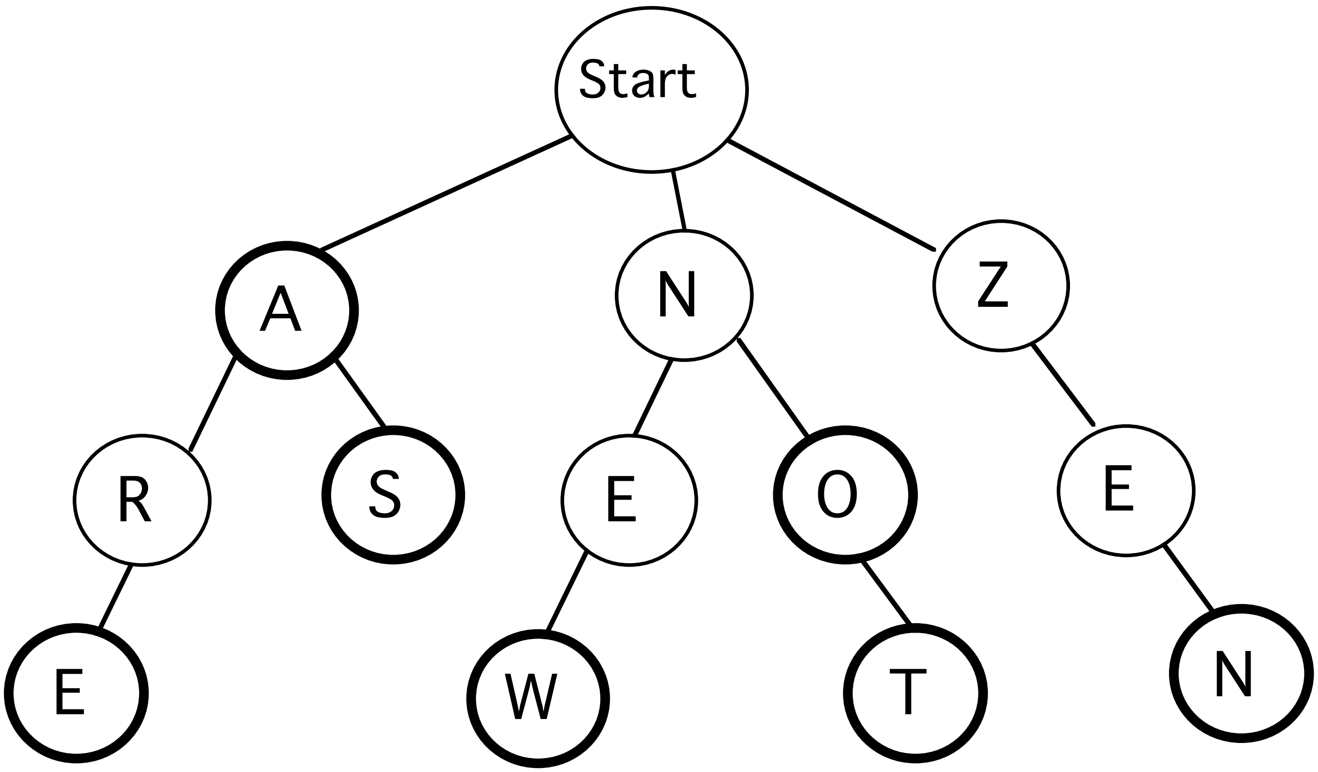
- Goal: Build a data structure that can efficiently store and search a large set of words
- A special kind of tree called a *trie*



Lab 9 : Tries

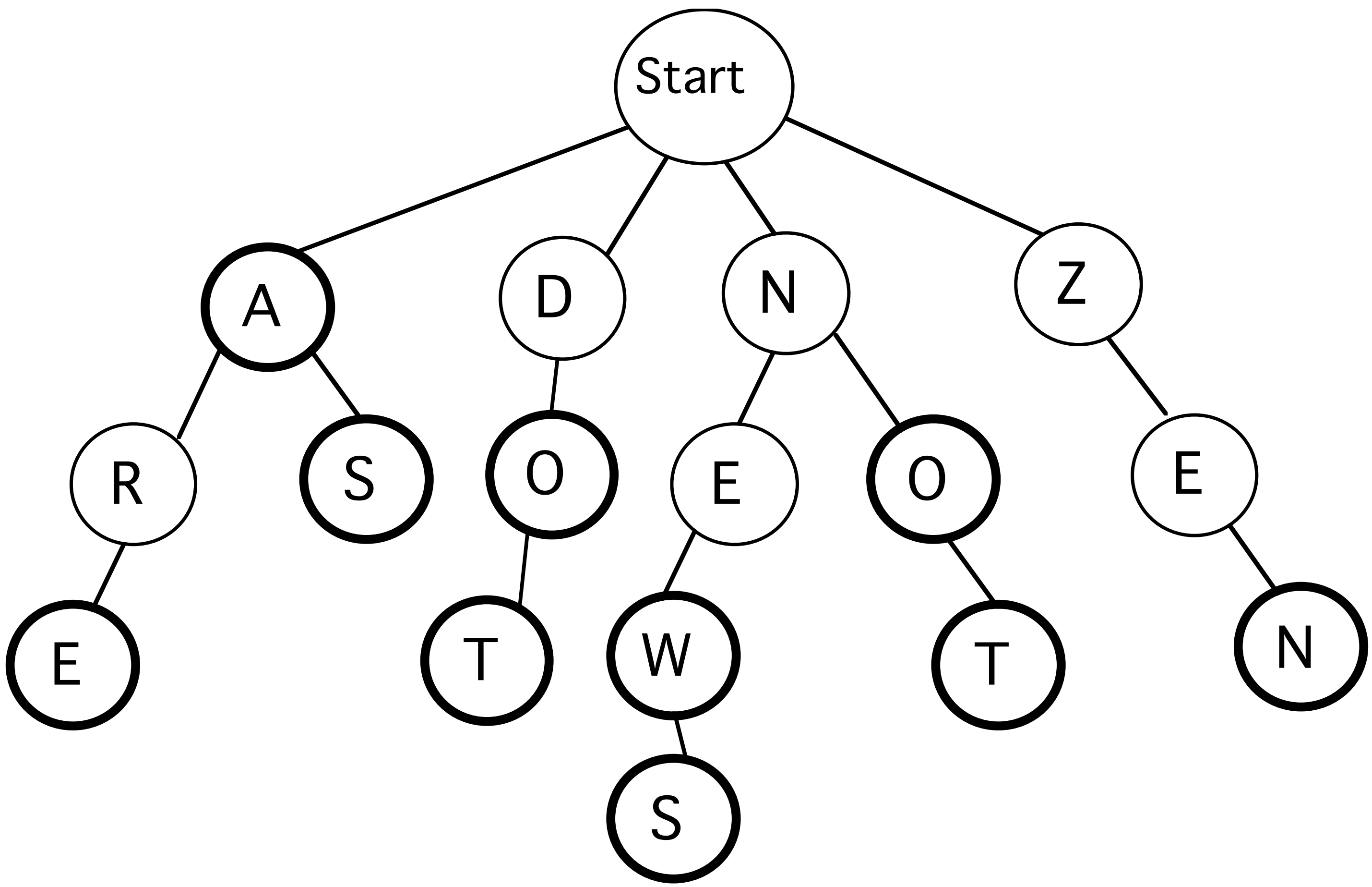
- A trie is a tree that stores words where
  - Each node holds a letter
  - Some nodes are "word" nodes (dark circles)
  - Any path from the root to a word node describes one of the stored words
  - All paths from the root form prefixes of stored words (a word is considered a prefix of itself)

Tries



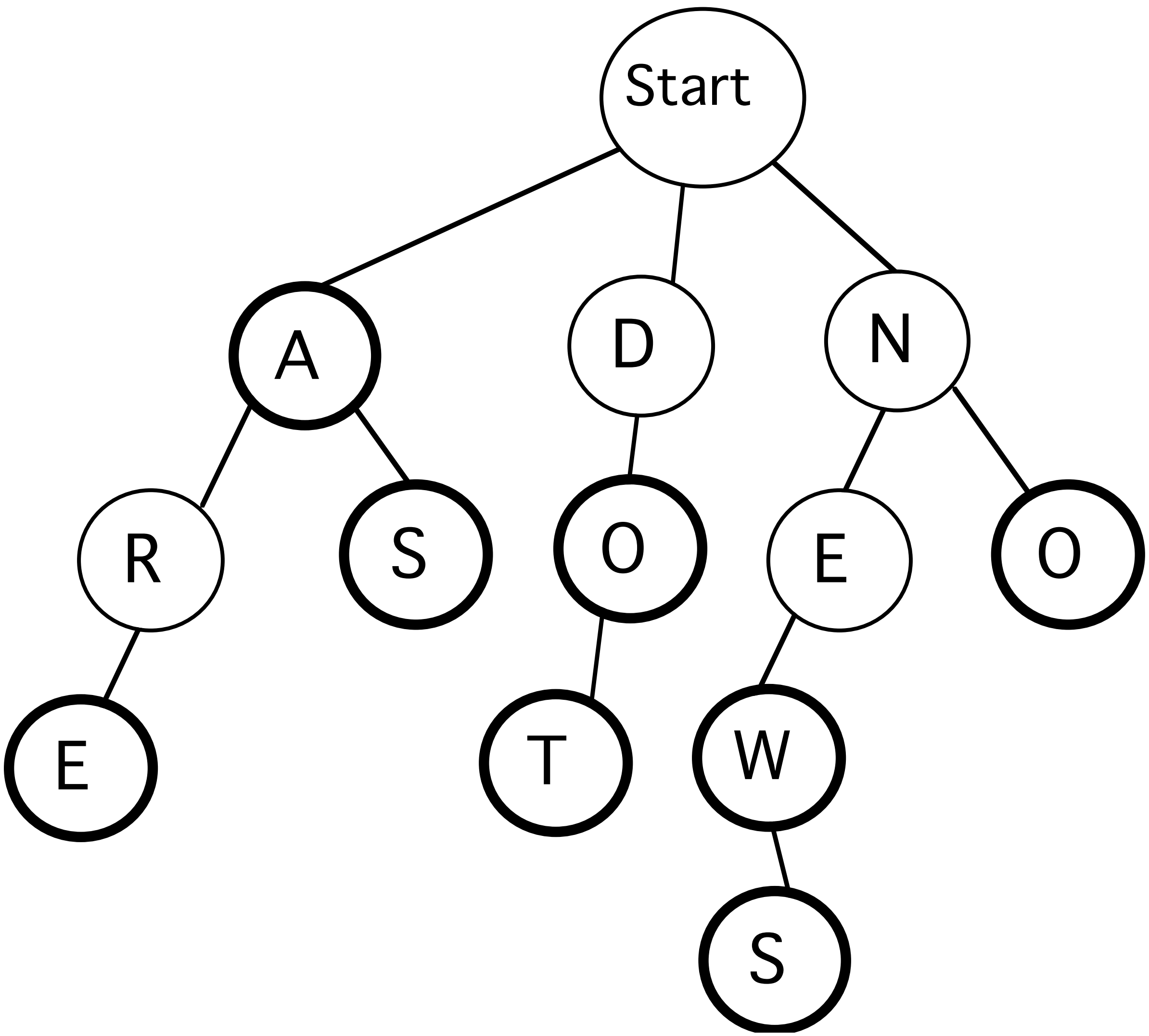
Now add “dot” and “news”

Tries



Now remove “not” and “zen”

Tries



Lab 9 : Lexicon

An inteface that provides the methods

```
public interface Lexicon {
    public boolean addWord(String word);
    public int addWordsFromFile(String filename);
    public boolean removeWord(String word);
    public int numWords();
    public boolean containsWord(String word);
    public boolean containsPrefix(String prefix);
    public Iterator<String> iterator();
    public Set<String> suggestCorrections(String
            target, int maxDistance);
    public Set<String> matchRegex(String pattern);
}
```

Lab 9

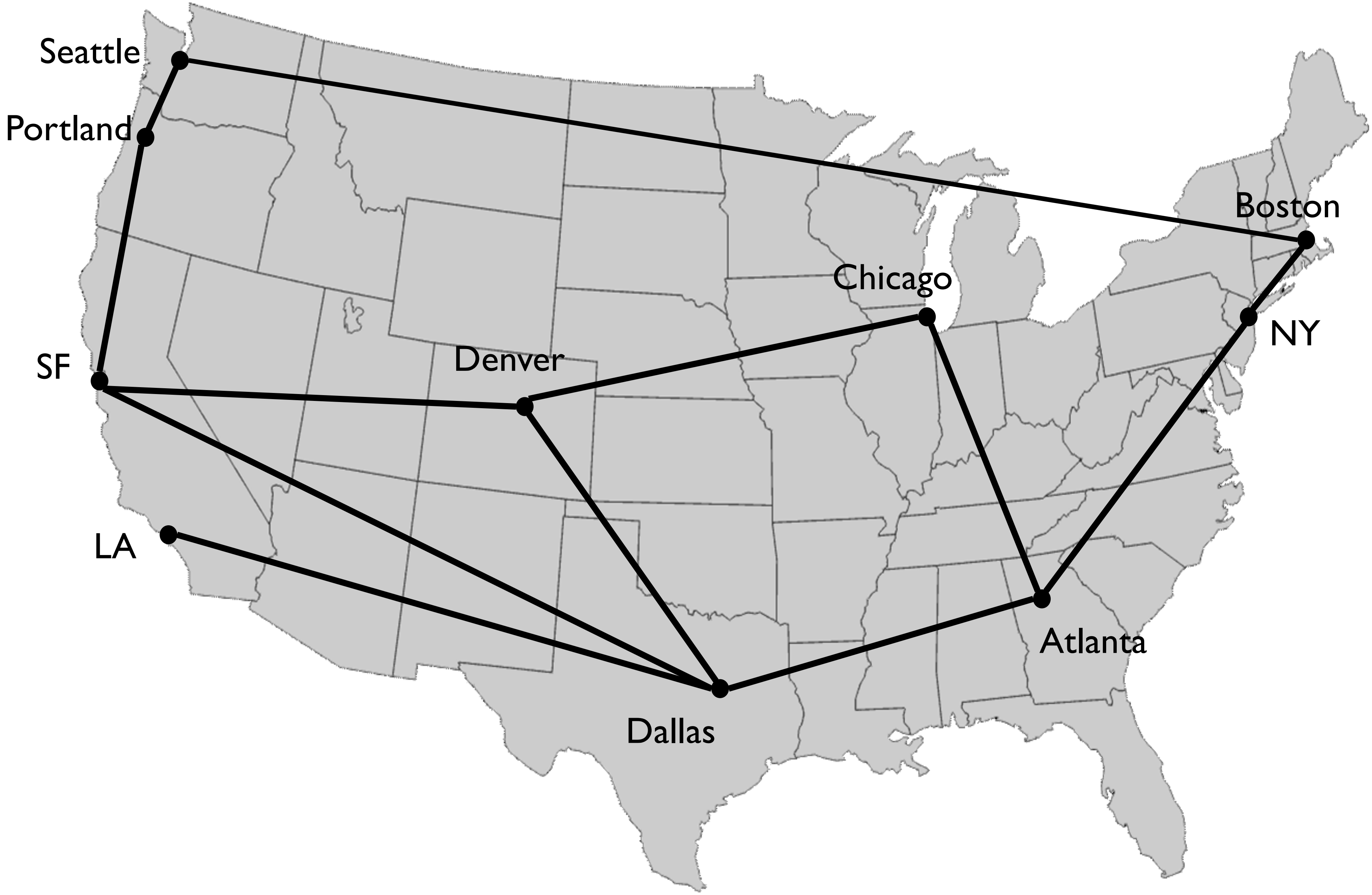
- Implement a program that creates, updates, and searches a Lexicon
  - Based on a LexiconTrie class
    - Each node of the Trie is a LexiconNode
    - Analogous to a SLL consisting of SLLNodes
  - LexiconTrie implements the Lexicon Interface
  - Supports
    - adding/removing words
    - searching for words and prefixes
    - reading words from files
    - Iterating over all words

Graphs Describe the World[1]

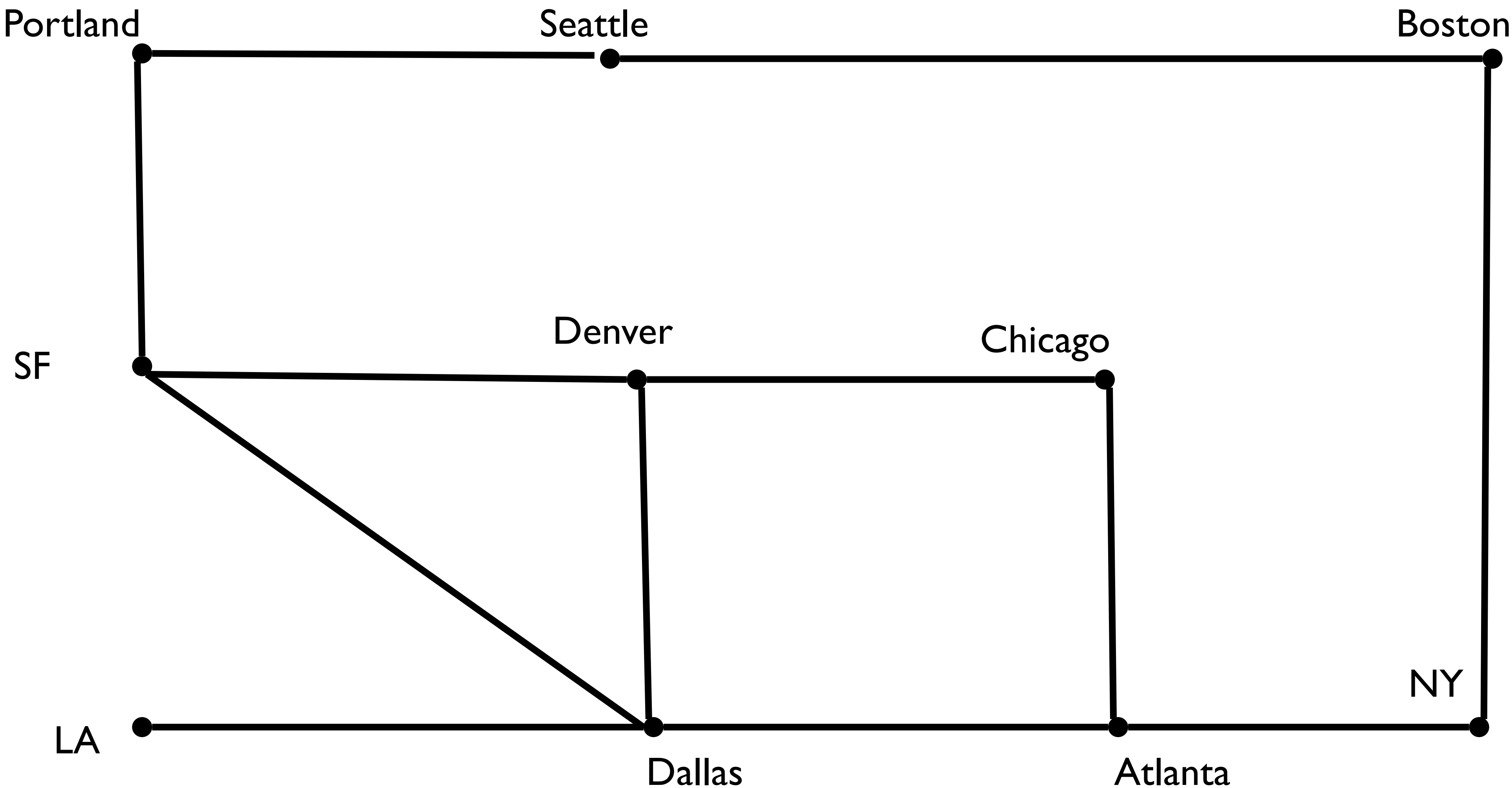
- Transportation Networks
- Communication Networks
- Molecular structures
- Dependency structures
- Scheduling
- Matching
- Graphics Modeling
- ....

[1]But don't tell Tom Garrity---he'll just be sad....

Nodes = subway stops; Edges = track between stops

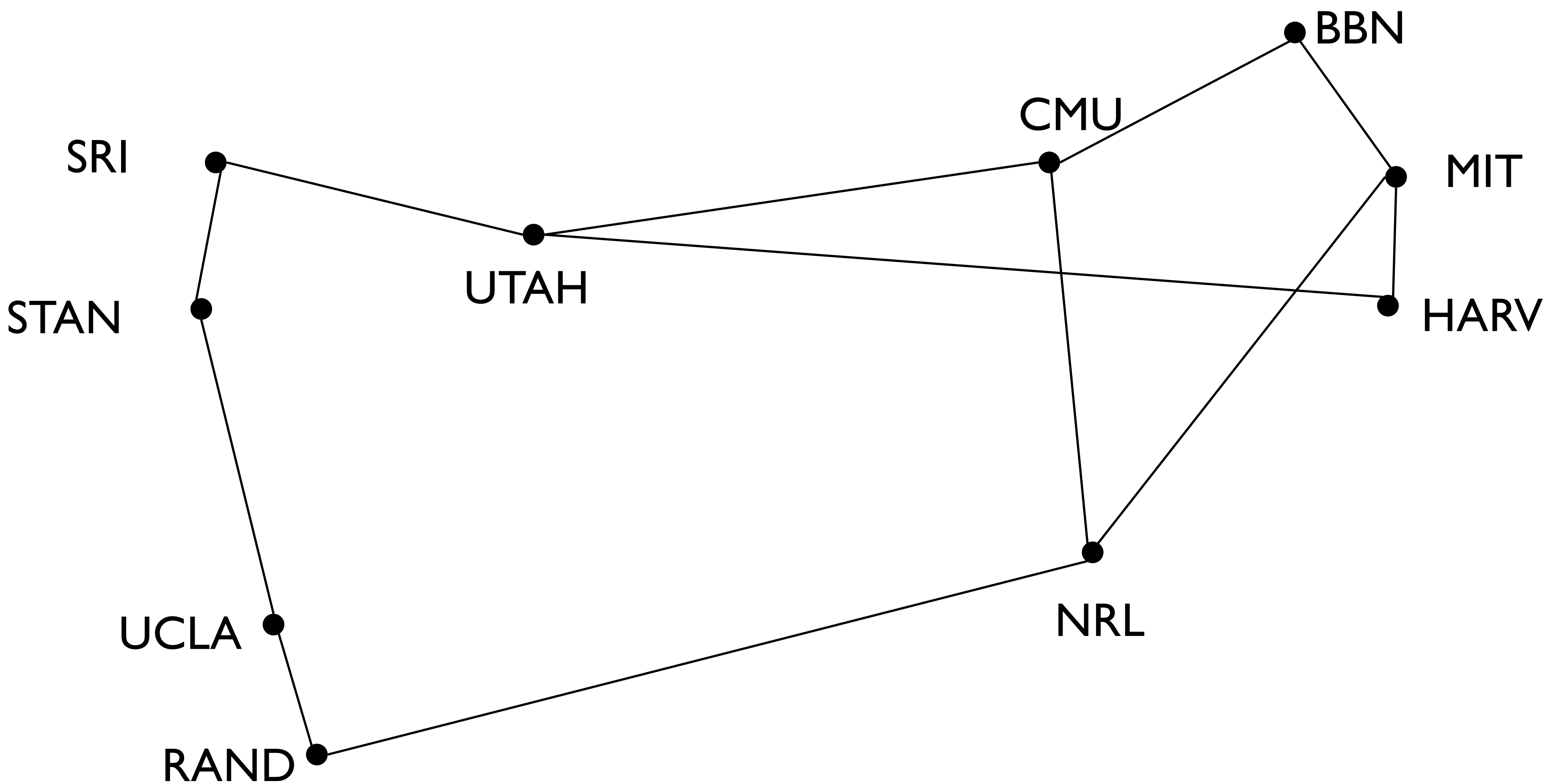


Nodes = cities; Edges = rail lines connecting cities

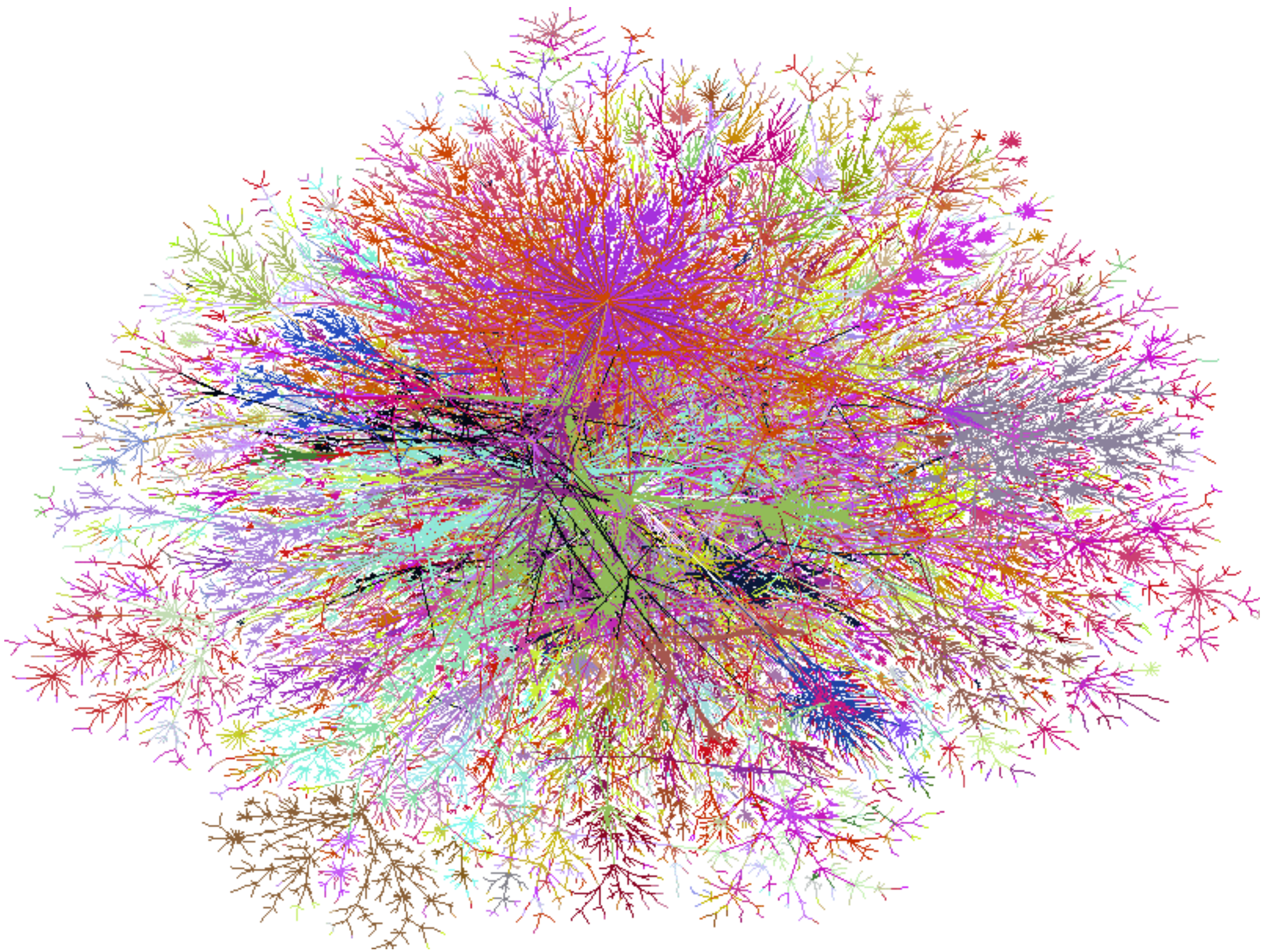


Note: Connections in graph matter, not precise locations of nodes

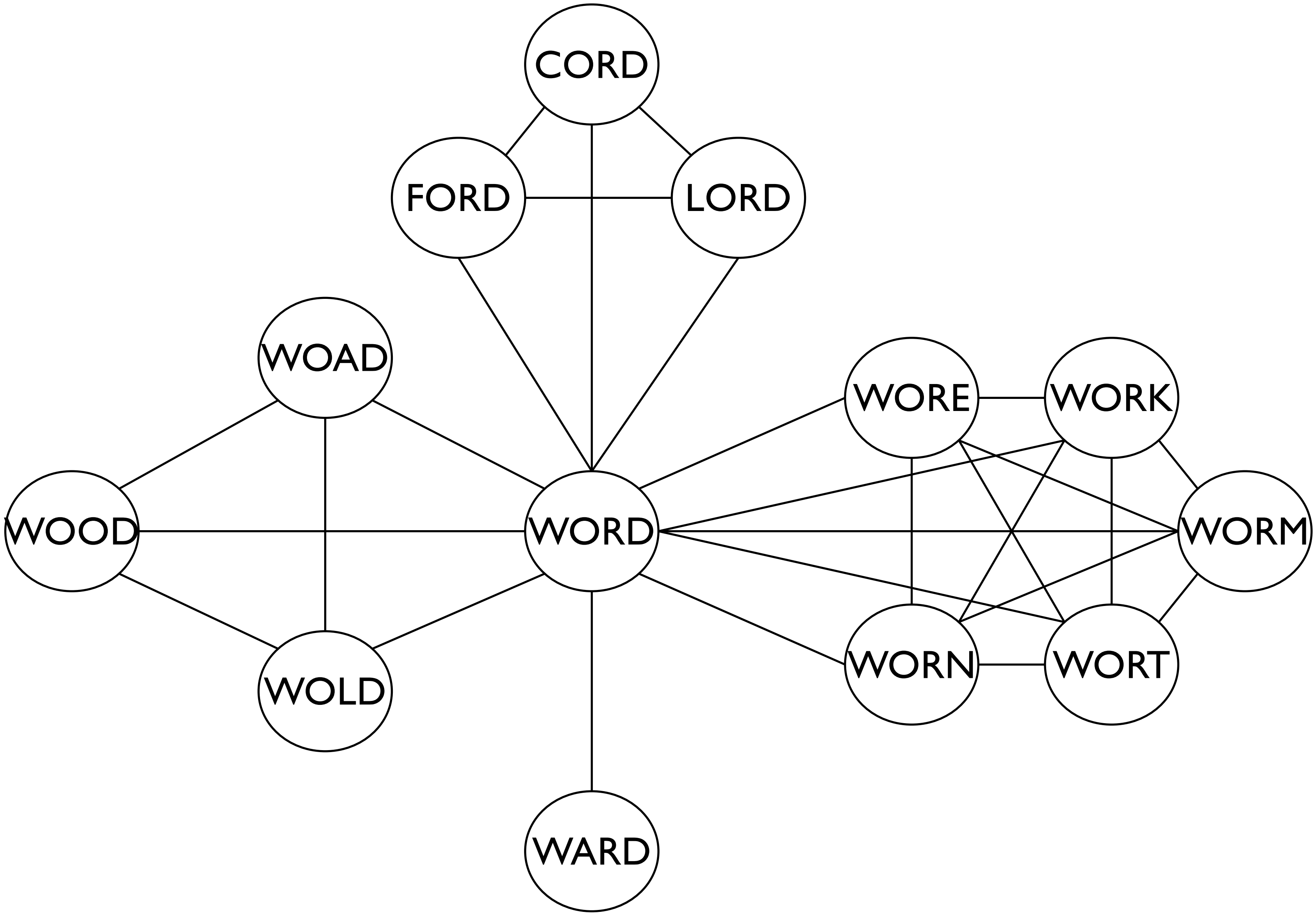
Internet (~1972)



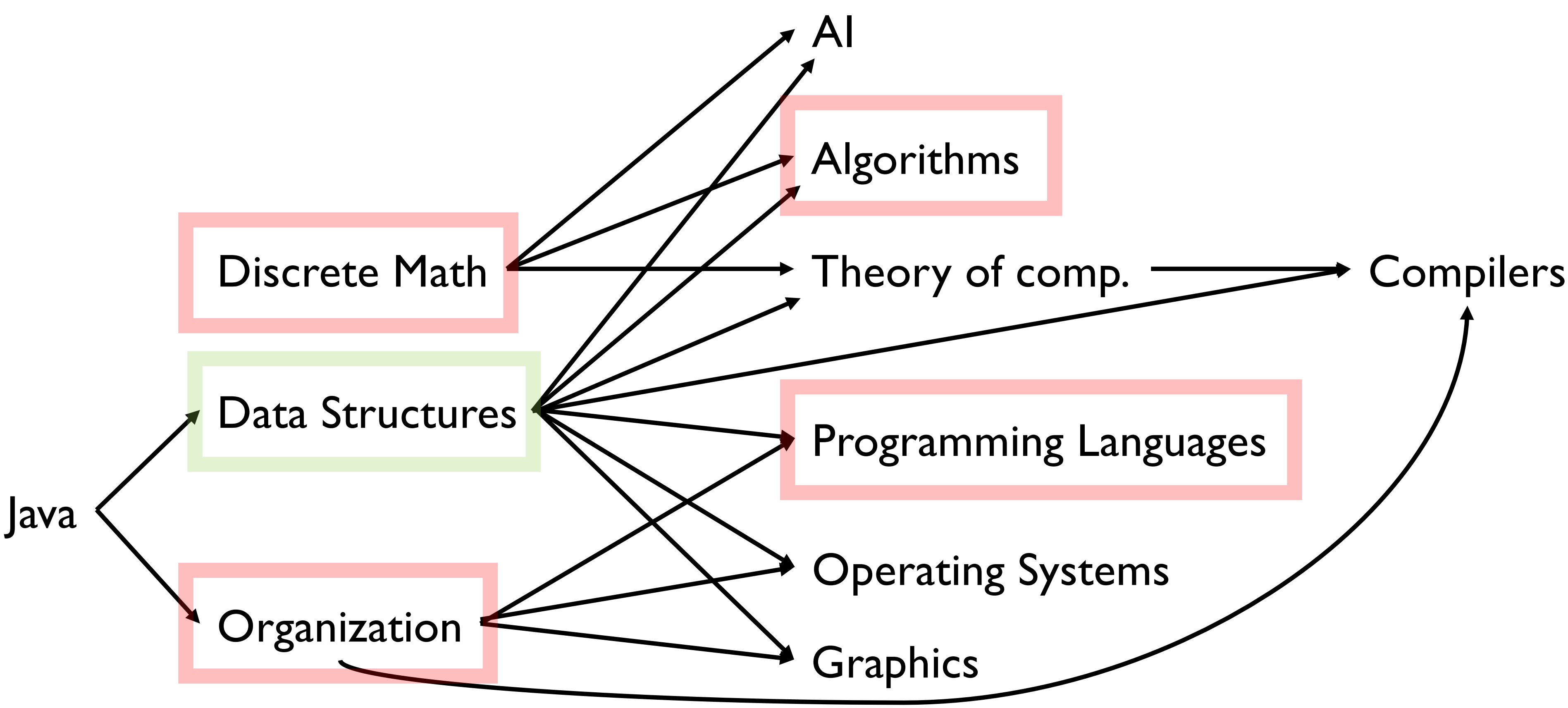
Internet (~1998)



Word Game

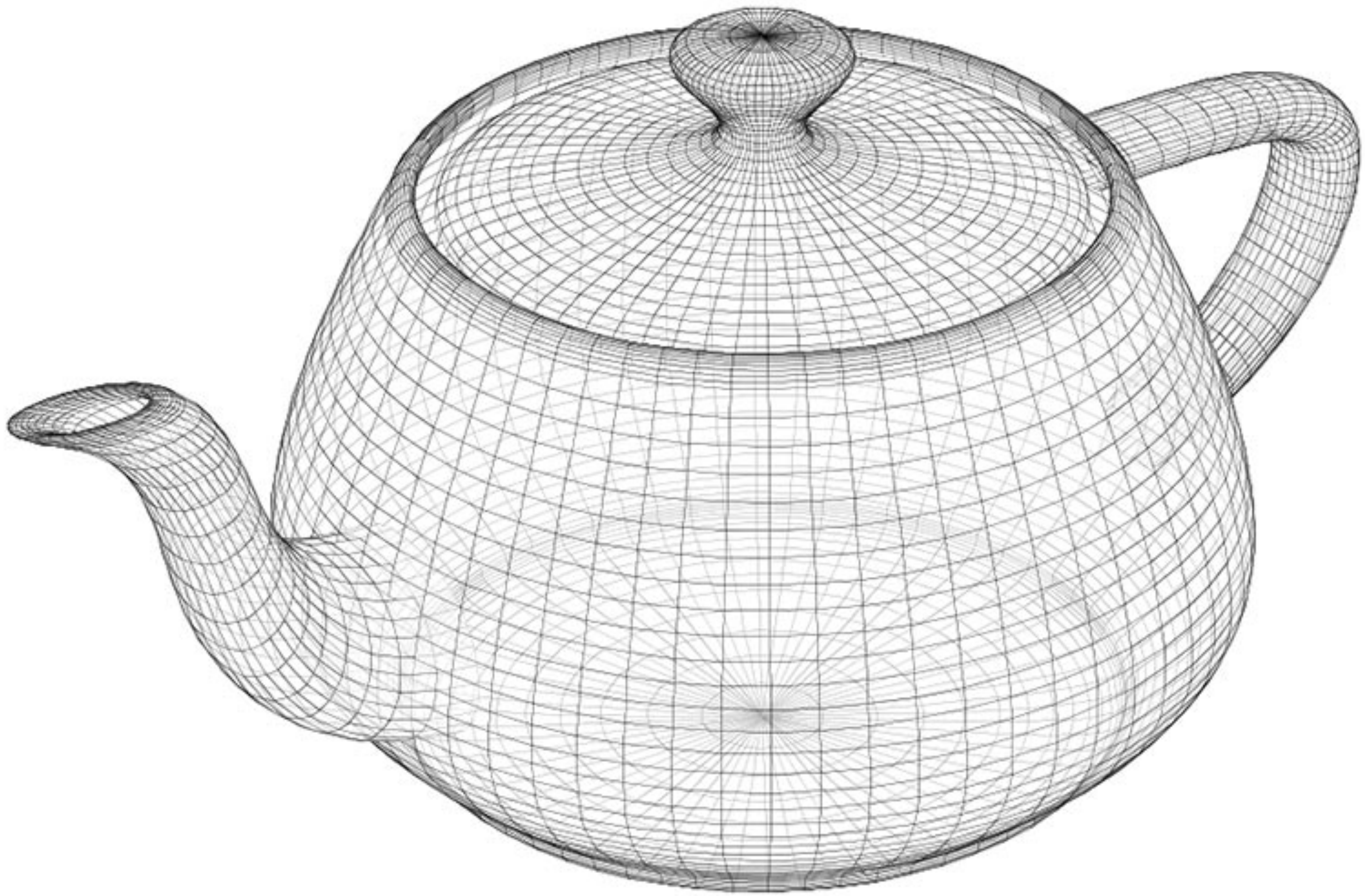

CORD
FORD
LORD
WOAD
WORE
WORK
WOOD
WORD
WORM
WOLD
WORN
WORT
WARD

CS Pre-requisite Structure (subset)

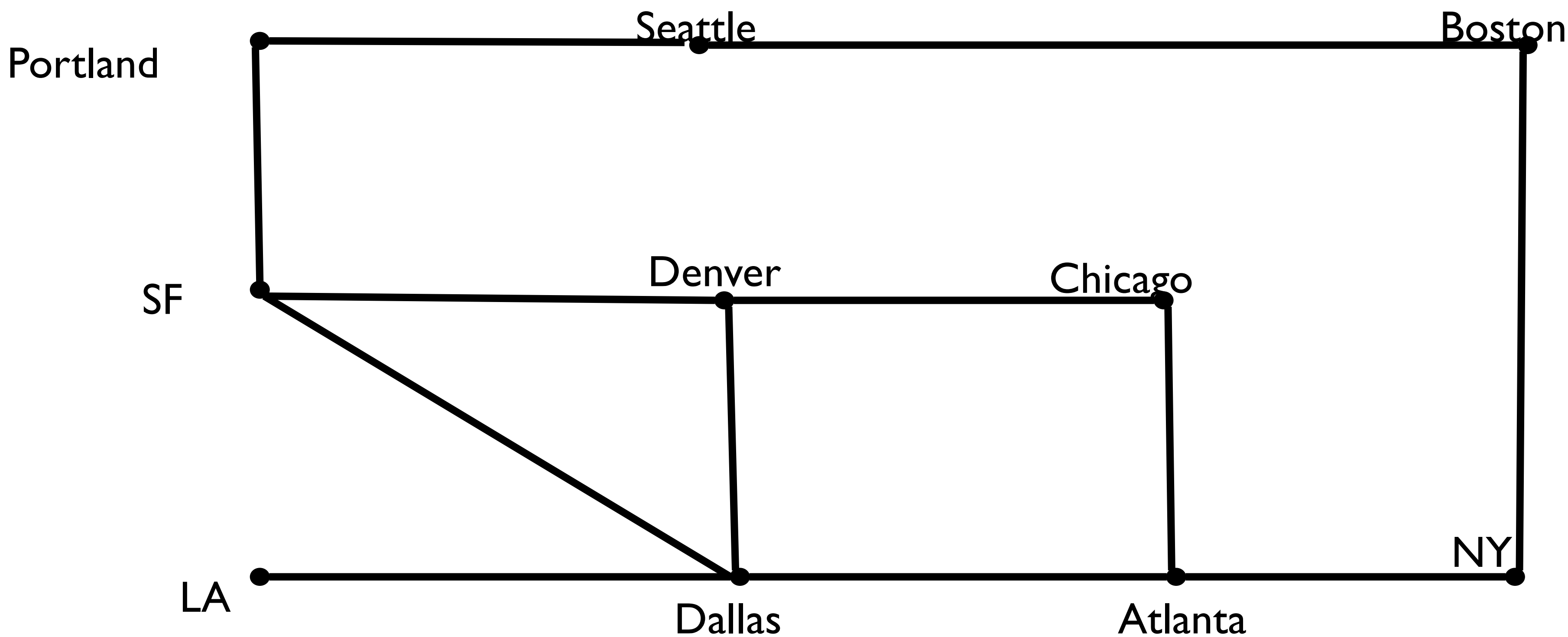


Nodes = courses; Edges = prerequisites ***

Wire-Frame Models



Basic Definitions & Concepts



Def'n: An *undirected graph* G = (V,E) consists of two sets

•V : the *vertices* of G, and E : the *edges* of G

•Each edge e in E is defined by a set of two vertices: its *incident vertices*. We write e = {u,v} and say that u and v are *adjacent.*

Walking Along a Graph

- A *walk from u to v* in a graph G = (V,E) is an *alternating* sequence of vertices and edges

  $$u = v_0, e_1, v_1, e_2, v_2, \ldots, v_{k-1}, e_k, v_k = v$$

  such that each $e_i = \{v_i, v_{i+1}\}$ for $i = 1, \ldots, k$
- Note a walk starts and ends on a vertex
- If no *edge* appears more than once then the walk is called a *path*
- If no *vertex* appears more than once then the walk is a *simple path*

Walking In Circles

- A *closed walk* in a graph G = (V,E) is a walk

  $$v_0, e_1, v_1, e_2, v_2, \ldots, v_{k-1}, e_k, v_k$$

  such that each $v_0 = v_k$
- A *circuit* is a path where $v_0 = v_k$
  - No repeated edges
- A *cycle* is a *simple* path where $v_0 = v_k$
  - No repeated vertices (uhm, except for v_0!)
- The length of any of these is the number of *edges* in the sequence

Little Tiny Theorems

- If there is a walk from u to v, then there is a walk from v to u.
- If there is a walk from u to v, then there is a path from u to v (and from v to u)
- If there is a path from u to v, then there is a simple path from u to v (and v to u)
- Every circuit through v contains a cycle through v
- Not every closed walk through v contains a cycle through v! [Try to find an example!]

Another Useful Graph Fact

- Degree of a vertex *v*
  - Number of edges incident to *v*
  - Denoted by *deg(v)*
- Thm: For any graph *G = (V,E)*

$$\sum_{v \in V} \deg(v) = 2|E|$$

where |*E*| is the number of edges in *G*

- Proof Hint: Induction on |*E*|: How does removing an edge change the equation?
  - Or: Count pairs *(v,e)* where *v* is incident with e

Reachability and Connectedness

- Def'n: A vertex v in G is *reachable* from a vertex u in G if there is a path from u to v
- v is reachable from u *iff* u is reachable from v
- Def'n: An undirected graph G is *connected* if for every pair of vertices u, v in G, v is reachable from u (and vice versa)
- The set of all vertices reachable from v, along with all edges of G connecting any two of them, is called the *connected component of v*