CSCI 136
Data Structures &
Advanced Programming

Lecture 18

Fall 2018

Instructor: Bills

Administrative Details

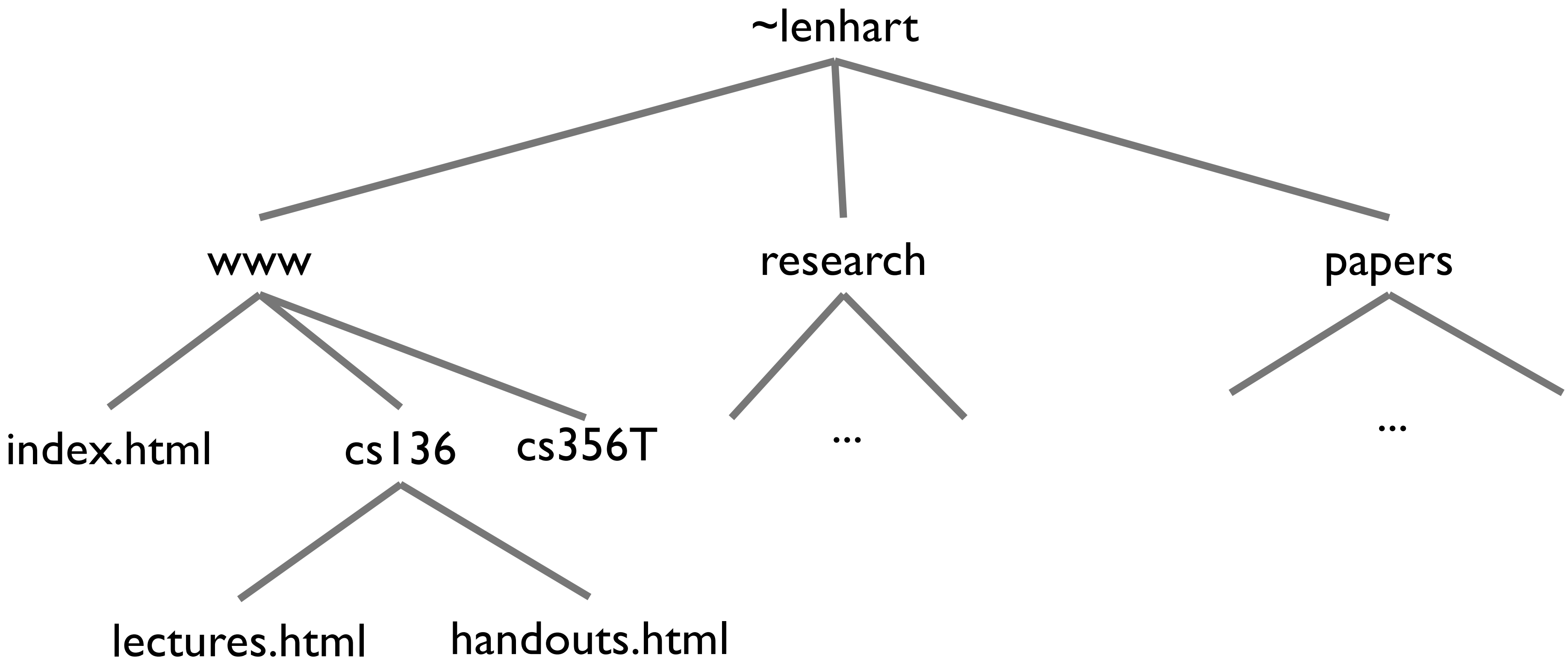
- Lab 7 Today: PostScript
  - No partners this week
  - Review before lab; come to lab with design doc
  - Check out the javadoc pages for the 3 provided classes
    - Token – A wrapper for semantic PS elements,
    - Reader – An iterator to produce a stream of Tokens from standard input or a List of Tokens,
    - SymbolTable – A dictionary with String keys and Token values: For user-defined names

Last Time:

- Ordered Structures
- Trees
  - Structure, Terminology, Examples

Today

- Trees
  - Implementation
  - Recursion/Induction on Trees
  - Applications
  - Traversals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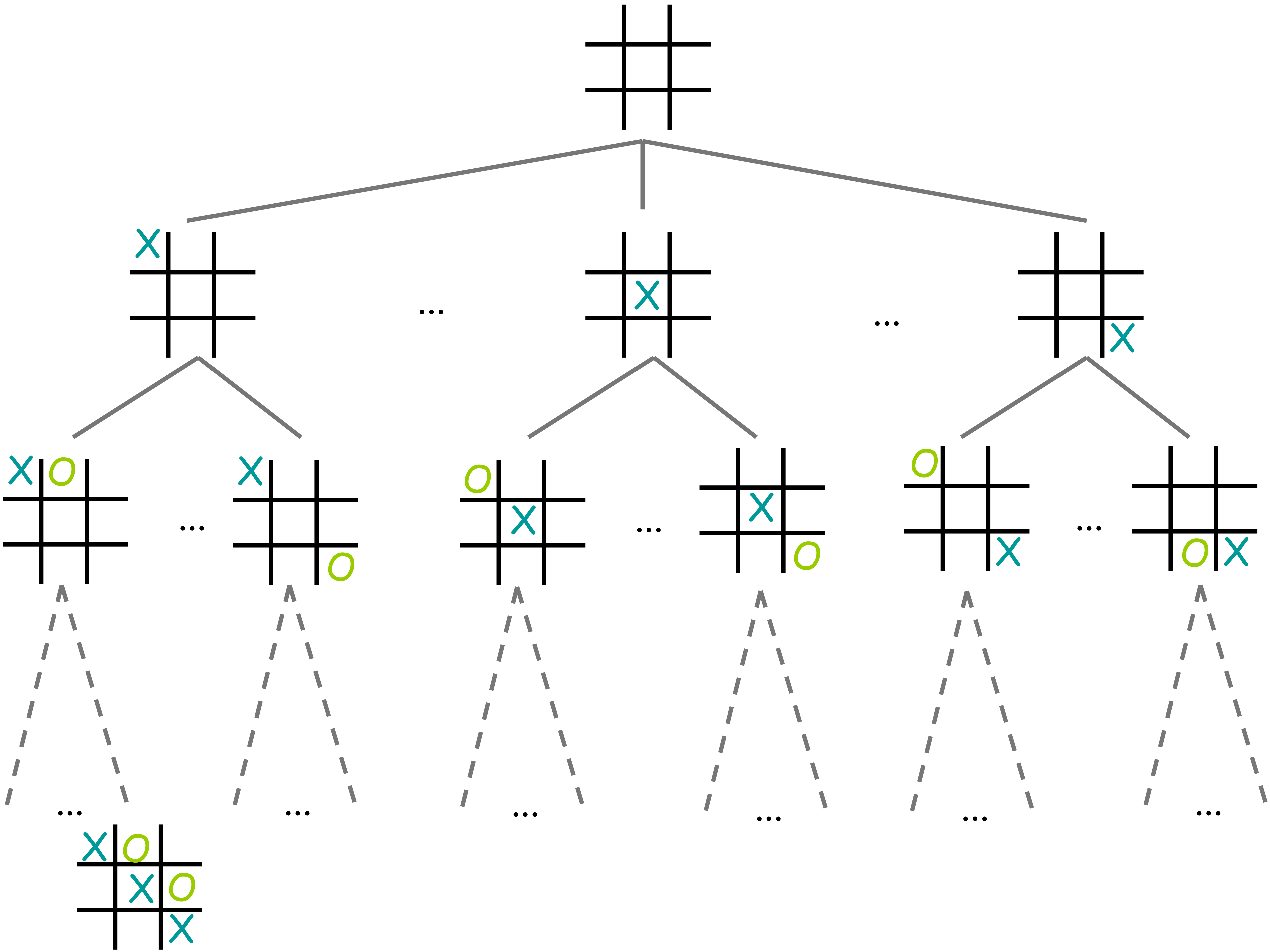
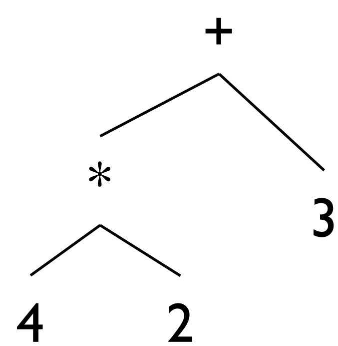


Tree Features

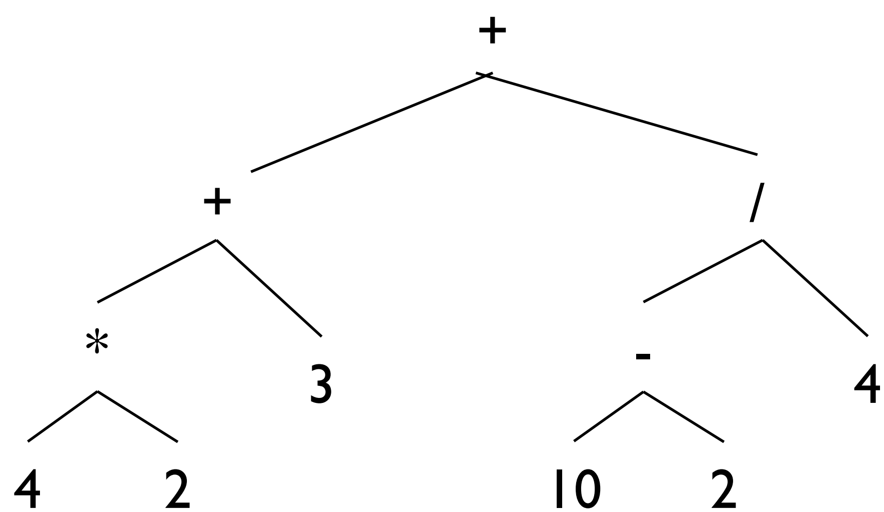
- Hierarchical relationship
- Root at the top
- Leaf at the bottom
- Interior nodes in middle
- Parents, children, ancestors, descendants, siblings
- Degree (of node): number of children of node
- Degree (of tree): maximum degree (across all nodes)
- Depth of node: number of *edges* from root to node
- Height of tree: maximum depth (across all nodes)

Expression Trees

4 * 2 + 3



(4 * 2 + 3) + ((10 – 2)/ 4)

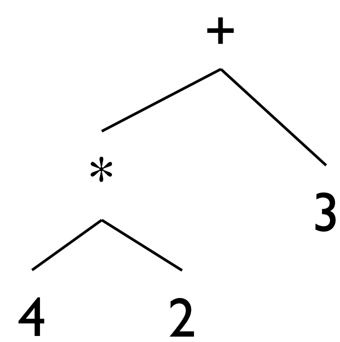


Introducing Binary Trees

- Degree of each node at most 2
- Recursive nature of tree
  - Empty
  - Root with left and right subtrees
- SLL: Recursive nature was captured by hidden node (Node<E>) class
- Binary Tree: No "inner" node class
  - Single BinaryTree class does it all
  - Is it a tree or a node?
    - It's a node that's a root of a tree!
  - And it's not part of Structure hierarchy!

Expression Trees

4 * 2 + 3



Build using constructor

```
new BinaryTree<E>(value, leftSubTree, rightSubTree)
```

```
BinaryTree<String> fourTimesTwo =  new BinaryTree<String>
        (“*”,new BinaryTree<String>(“4”),new BinaryTree<String>(“2”));

BinaryTree<String> fourTimesTwoPlusThree = new BinaryTree<String>
        (“+”, fourTimesTwo,  new BinaryTree<String>(“3”));
```

Expression Trees

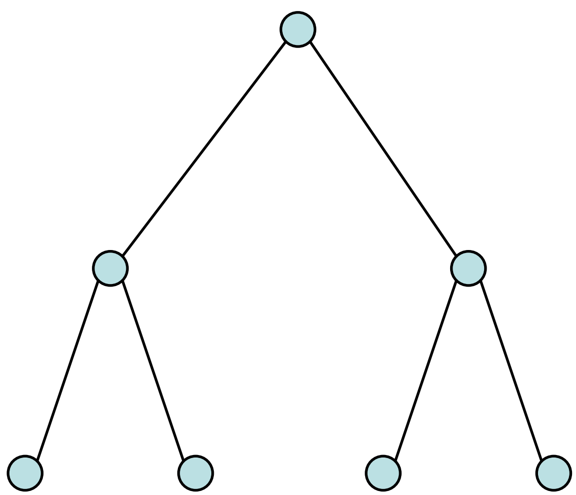
- General strategy
  - Make a binary tree (BT) for each leaf node
  - Move from bottom to top, creating BTs
  - Eventually reach the root
  - Call "evaluate" on final BT
- Example
  - How do we make a binary expression tree for (((4+3)*(10-5))/2)
    - Postfix notation: 4 3 + 10 5 - * 2 /

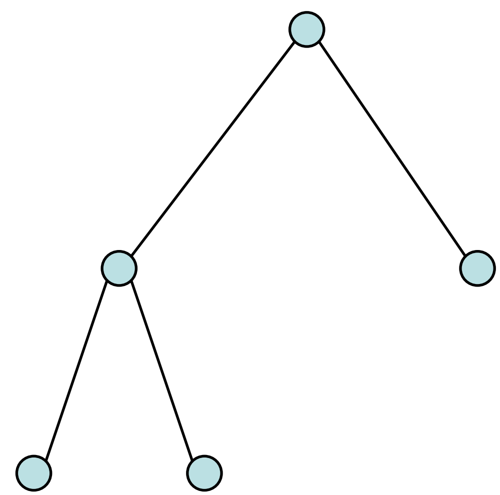
```
int evaluate(BinaryTree<String> expr) {

    if (expr.height() == 0)
      return Integer.parseInt(expr.value());

    else {
      int left = evaluate(expr.left());
      int right = evaluate(expr.right());
      String op = expr.value();
      switch (op) {

      case "+" : return left + right;
      case "-" : return left - right;
      case "*" : return left * right;
      case "/" : return left / right;
      }

      Assert.fail("Bad op");
      return -1;
   }
}
```

Full vs. Complete (non-standard!)

- Full tree – A full binary tree of height h has *leaves only* on level h, and each internal node has exactly 2 children.

- Complete tree – A *complete* binary tree of height h is *full* to height h-1 and has all leaves at level h in leftmost locations.

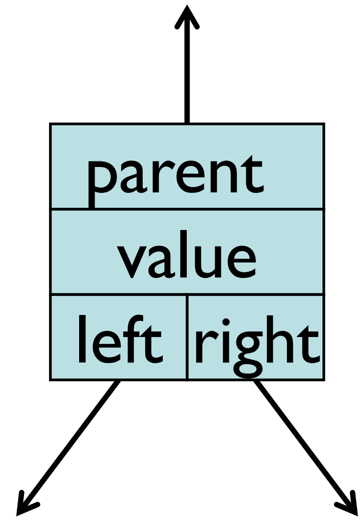


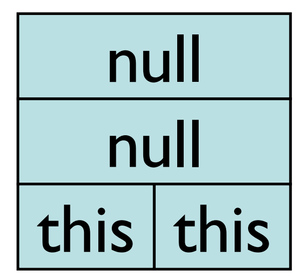


All full trees are complete, but not all complete trees are full!

Implementing BinaryTree

- BinaryTree<E> class
  - Instance variables
    - BinaryTree: parent, left, right
    - E: value
- left and right are never null
  - If no child, they point to an "empty" tree
    - Empty tree T has value null, parent null, left = right = T
  - Only empty tree nodes have null value

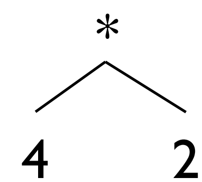


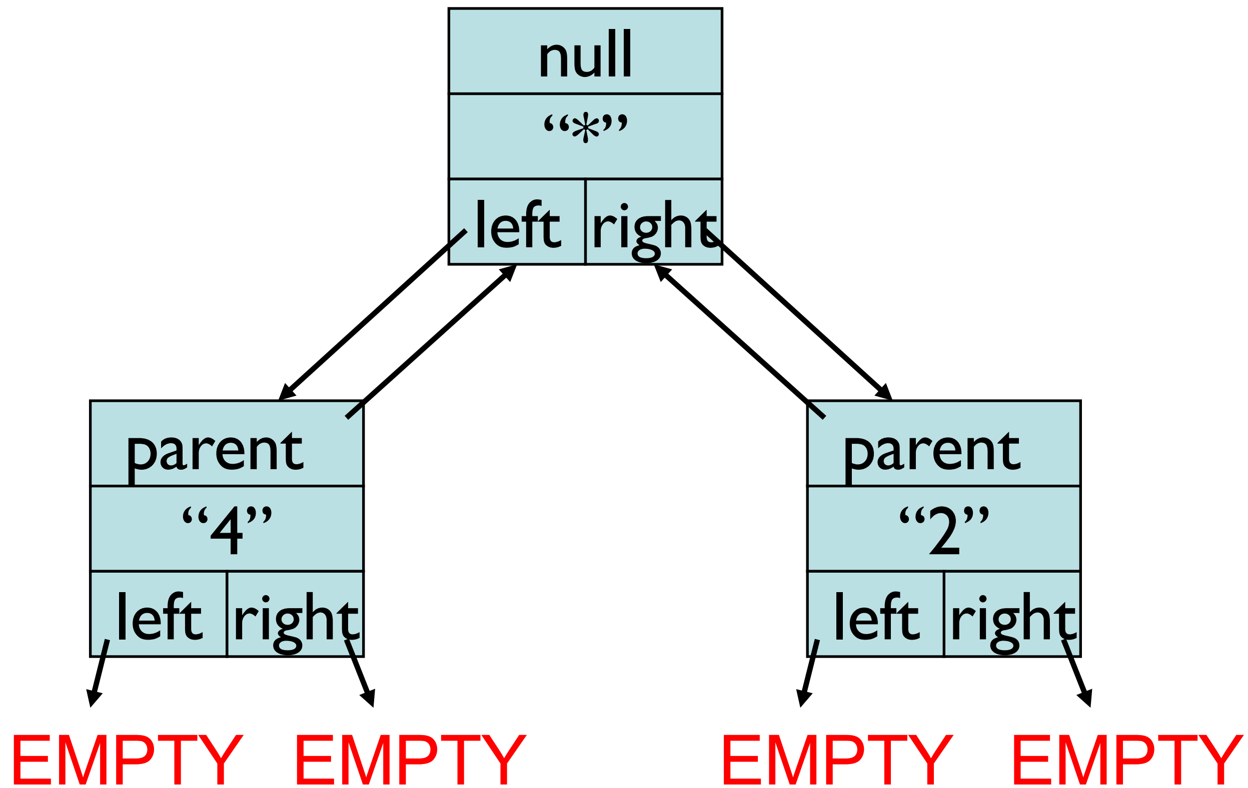


EMPTY BT

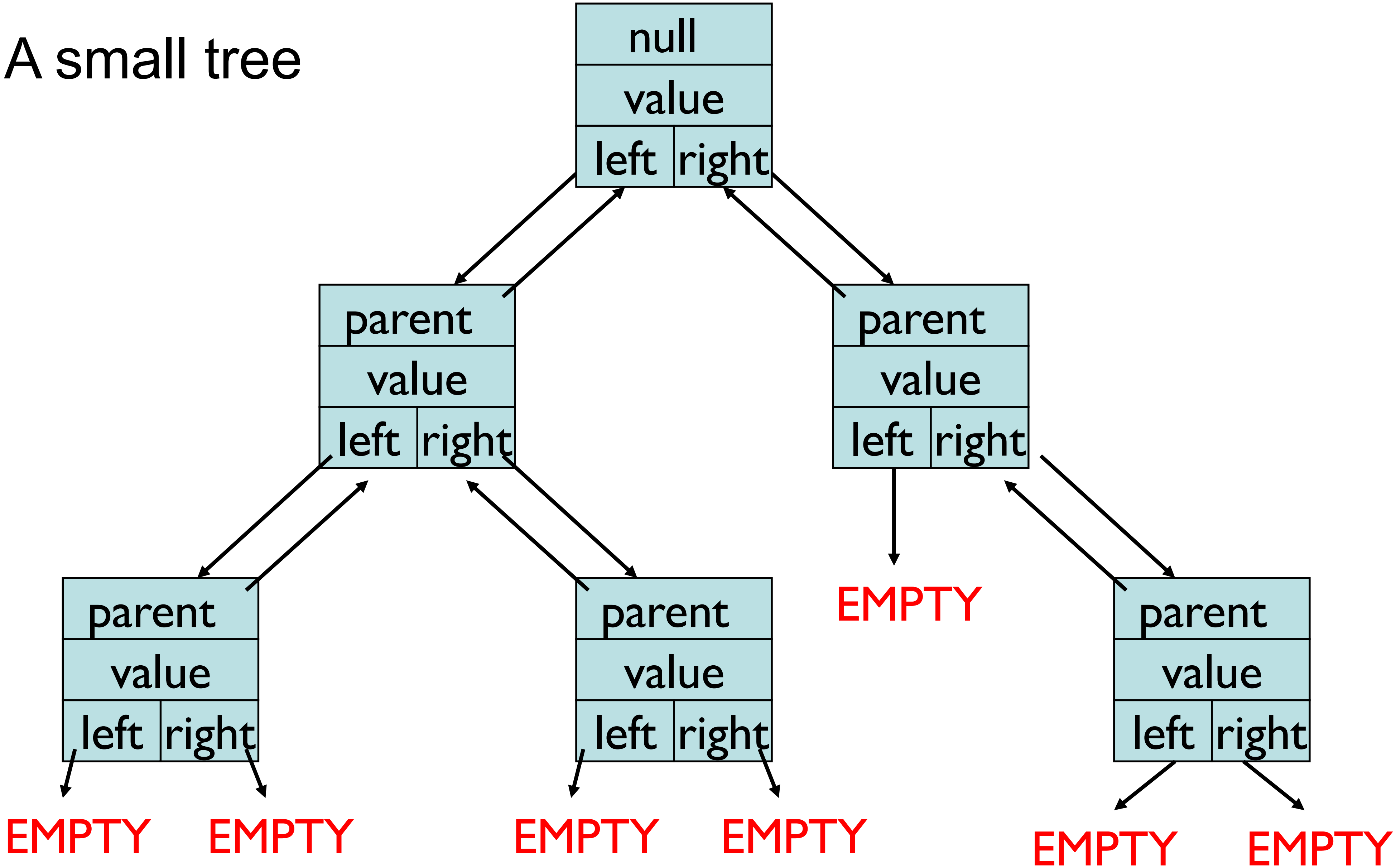
Implementing BinaryTree

- BinaryTree class
  - Instance variables
    - BT parent, BT left, BT right, E value





A small tree



EMPTY != null!

Implementing BinaryTree

- Many (!) methods: See BinaryTree javadoc page
- All "left" methods have equivalent "right" methods
  - public BinaryTree()
    - // generates an empty node (EMPTY)
    - // parent and value are null, left=right=this
  - public BinaryTree(E value)
    - // generates a tree with a non-null value and two empty (EMPTY) subtrees
  - public BinaryTree(E value, BinaryTree<E> left, BinaryTree<E> right)
    - // returns a tree with a non-null value and two subtrees
  - public void setLeft(BinaryTree<E> newLeft)
    - // sets left subtree to newLeft
    - // re-parents newLeft by calling newLeft.setParent(this)
  - protected void setParent(BinaryTree<E> newParent)
    - // sets parent subtree to newParent
    - // called from setLeft and setRight to keep all "links" consistent

Implementing BinaryTree

- Methods:
  - public BinaryTree<E> left()
    - // returns left subtree
  - public BinaryTree<E> parent()
    - // post: returns reference to parent node, or null
  - public boolean isLeftChild()
    - // returns true if this is a left child of parent
  - public E value()
    - // returns value associated with this node
  - public void setValue(E value)
    - // sets the value associated with this node
  - public int size()
    - // returns number of (non-empty) nodes in tree
  - public int height()
    - // returns height of tree rooted at this node
  - But where's "remove" or "add"?!?!

BT Questions/Proofs

- Prove
  - The number of nodes at depth n is at most 2^n.
  - The number of nodes in tree of height n is at most $2^{(n+1)}-1$.
  - A tree with n nodes has exactly n-1 edges
  - The size() method works correctly
  - The height() method works correctly
  - The isFull() method works correctly