CSCI 334:
Principles of Programming Languages

Lecture 21: Logic Programming

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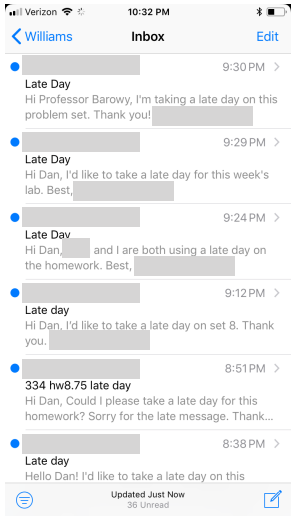
Announcements

Be a TA!

Announcements



Logic Programming

- Logic programming began as a collaboration between AI researchers (e.g., John McCarthy) and logicians (e.g., John Alan Robinson) to solve problems in artificial intelligence.
- Cordell Green built the first "question and answer" system using Robinson's "unification algorithm," demonstrating that it was practical to prove theorems automatically.

Prolog

- Alain Colmerauer and Phillippe Roussel at Aix-Marseille University invented Prolog in 1972.
- They were inspired by a particular formulation of logic, called "Horn clauses," popularized by the logician Robert Kowalski.
- Horn clauses have a "procedural interpretation," meaning that they suggest a simple procedure for solving them, called "resolution."
- John Alan Robinson's unification algorithm is an efficient algorithm for doing resolution, and this is essentially the algorithm used by Prolog.

Declarative Programming

- Declarative programming is a very different style of programming than you have seen to this point.
- Mostly, you have seen imperative programs.
- In imperative-style programming, the programmer instructs the computer how to compute the desired result.
- In declarative-style programming, the computer already knows how to compute results.
- Instead, the programmer asks the computer what to compute.

Declarative Programming

- Most of you have probably been CS majors for long enough that we have sufficiently damaged your brain such that you do not recognize the difference between these two concepts.
- In fact, imperative-style programming is a very unnatural way of communicating desires.
- Declarative: "Make me a PB&J sandwich."
- Imperative: https://youtu.be/cDA3_5982h8

Prolog

- The goal of AI is to enable a computer to answer declarative queries.
- I.e., it already knows how to answer you.
- Prolog was an attempt to solve this problem.
- Since this was early work, the input language was somewhat primitive: predicate logic.
- As you will see, formulating queries in pure logic is not the easiest thing to do.
- However, for certain classes of logic, there are known efficient, deterministic algorithms for solving every possible query.

Horn Clause

- Horn clauses are composed of two simple pieces:
  - facts
  - rules (clauses)
- Rules are composed of facts
- Complex facts may also be composed using conjunction.
- We will explore these concepts using Prolog syntax.
- Note that Horn clauses can be "satisfied" in polynomial time.
- In fact, Horn logic is the most expressive form of logic known to be satisfiable in polynomial time.

Facts (Prolog syntax)

- Here are some facts:

```
raining.
cloudy.
thursday.
```

- Facts are assumed to be true.
- Facts of this form are sometimes called "atoms", since they are indivisible.
- The meaning of these facts is up to the programmer.
- Facts can also be compound:

```
raining,cloudy.
cloudy,thursday.
```

- "`,`" denotes "logical and".
- Note that, in Prolog, facts are always lowercase and must begin with a letter.

Rules (Prolog syntax)

- Here are some rules:

```
sleep_deprived :- thursday.
unhappy :- raining,cloudy.
```

- The interpretation of a rule `X :- Y` is: if `Y` is true, then `X` is true
- In other words, `Y` is the antecedent and `X` is the consequent.
- So, we might interpret the above as:
  "students are sleep deprived if it is Thursday"
  "I am unhappy if it is raining and cloudy."

Variables (Prolog syntax)

- Note that I just used a generalization of rules without definition:

```
X :- Y
```

- Prolog explicitly allows generalizations of facts like this.
- We call these generalizations "variables", because their precise values (i.e., facts) may not be known to us.
- In the "execution" of a Prolog program, we seek to "instantiate" variables with facts.
- In Prolog, variables are always written starting with an uppercase letter.
- We will come back to variables shortly.

Complex facts (Prolog syntax)

- Prolog allows one additional form:

```
musician(mia).
musician(john).
friends_with(mia,john).
```

- Statements of this form are called "complex facts."
- Again, the interpretation is up to you.
- E.g.,

  "Mia is a musician."

  "John is a musician."

  "Mia is friends with John."
- Note that we do not automatically assume that

  "John is friends with Mia"!

Queries

- Taken together, facts and rules form a "knowledge base."

```
raining.
cloudy.
thursday.
sleep_deprived :- thursday.
unhappy :- raining,cloudy.
```

- A query asks the knowledge base a question. E.g.,

```
?- sleep_deprived.
true
?- unhappy.
true
```

Resolution

- "Resolution" is the name of the procedure that Prolog uses to "satisfy" a query.

```
raining.
cloudy.
thursday.
sleep_deprived :- thursday.
unhappy :- raining,cloudy.
```

- Essentially, we seek to reduce a query expression to the expression true by substitution.
- Remember that facts are assumed to be true.

Resolution

```
1. raining.
2. cloudy.
3. thursday.
4. sleep_deprived :- thursday.
5. unhappy :- raining,cloudy.
```

```
?- sleep_deprived.
```

- For a given query, we first seek either a fact that immediately makes the query true, or we seek a rule whose consequent is the query.
- When a rule is reduced to the form `X :- true`, then X is true.

```
6. sleep_deprived :- thursday (by KB4)
7. sleep_deprived :- true (by KB3)
8. true (by deduction)
```

Resolution

- Given the following knowledge base,

```
1. a :- b,c.
2. b :- d,e.
3. b :- g,e.
4. c :- e.
5. d.
6. e.
7. f :- a,g.
```

- let's try to satisfy the following query using resolution:

```
?- a.
```

Resolution

- Note that we get a slightly different outcome if the same set of facts are written in a slightly different order:

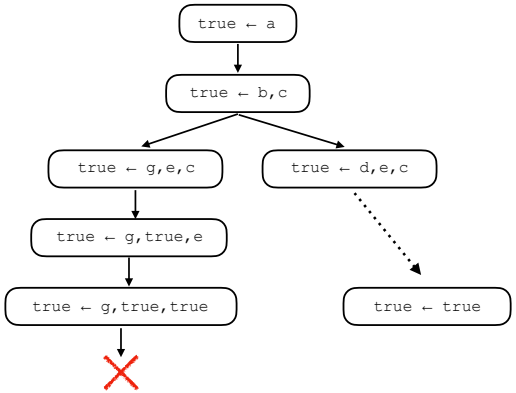
```
1. a :- b,c.
2. b :- g,e.
3. b :- d,e.
4. c :- e.
5. d.
6. e.
7. f :- a,g.
```

- Again, let's try to satisfy the following query using resolution:

```
?- a.
```

Proof Search

- Nonetheless, Prolog is not generally sensitive to the order of the facts in a database. How does this work?
- The answer is that resolution is actually a form of backtracking search.



Resolution with Variables

- Resolution with variables can be very computationally expensive.
- Unification allows resolution with variables to be completed in polynomial time.
- The basic insight is to "instantiate" variables "on demand" instead of enumerating all possible variable instantiations into facts.
- Hindley-Milner is essentially just unification.

```
1. musician(mia).
2. musician(john).
3. friends_with(X,Y) :- musician(X),musician(Y).
```

- Let's resolve the following query:

```
?- friends_with(mia,john).
```

Resolution with Variables

- When asking a query that utilizes variables, Prolog will both search for a satisfying assignment and it will return that assignment.
- There may be more than one possible assignment.
- If so, use the ";" command to ask for another solution.
- Let's resolve the following query:

```
?- friends_with(mia,Who).
```

- We may even ask:

```
?- friends_with(Who1,Who2).
```

Exercise

- Construct the a knowledge base containing the following facts:
  - "Giants eat people."
  - "Giants eat bunnies."
  - "Bunnies eat grass."
  - "People eat bunnies."
  - "People eat people."
  - "Those who are eaten by others hate those others."
  - "Monsters love those who hate themselves."
- Then supply a query that can answer:
  - "Who do monsters love?"