

Please show your work to receive partial credit. Clearly indicate your final answer.

1. **Two's Complement:** For this question, please consider the following 12-bit **two's complement** representation of an integer \mathbf{x} :

$$\mathbf{x} = 1001\ 1100\ 1011$$

What is the decimal value of \mathbf{x} ?

Since this is a 12 bit two's complement number, the leftmost bit will indicate the sign. The '1' implies this is a negative number. We can calculate the value of this number in one of two different ways.

a.) *Add all the bits together with the first bit's value being negatively weighted:*

$$1 \times (-2)^{11} + 1 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^3 + 1 \times 2^1 + 1 \times 2^0 = \\ -2048 + 256 + 128 + 64 + 8 + 2 + 1 = -1589$$

b.) *Figure out what positive number this represents by flipping all the bits and adding 1 before converting that unsigned number to its decimal representation and then insert a negative sign.*

$$\mathbf{x} = 1001\ 1100\ 1011 \\ -\mathbf{x} = 0110\ 0011\ 0101$$

$$1 \times 2^{10} + 1 \times 2^9 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^2 + 1 \times 2^0 = 1024 + 512 + 32 + 16 + 4 + 1 = 1589$$

What is the decimal value of \mathbf{x} if we instead interpret it as a 12-bit unsigned number?

We just use the equation for converting an unsigned binary number to decimal.

$$1 \times 2^{11} + 1 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^3 + 1 \times 2^1 + 1 \times 2^0 = \\ 2048 + 256 + 128 + 64 + 8 + 2 + 1 = 2507$$

2. **Truncating:** Consider the primitive C types `short` and `char` on our lab's x86_64 systems. The type `short` is used to represent 16-bit two's complement numbers, and the type `char` is used to represent 8-bit two's complement numbers. What output is printed after executing the following C code snippet?

```
short s = 0x58b;  
char c = (char) s;  
printf("%d", c);
```

Hint: The `printf` format string character `%d` is used to print the decimal value of an integer type, including the primitive type `char`.

Each hexadecimal digit is equivalent to 4 binary digits. Thus the binary representation is:

0101 1000 1011

When we cast a short to a char, we drop the leading byte of the short due to truncation. That means we are left with:

1000 1011

chars are considered signed, so we would interpret this byte as a twos complement representation. We would calculate its value in the following manner:

$$1 \times (-2)^7 + 1 \times 2^3 + 1 \times 2^1 + 1 \times 2^0 = -128 + 8 + 2 + 1 = -117$$

3. **Bit operations:** What does this C expression evaluate to? Please express your answer in hexadecimal.

```
0xabbedeed & (0xff << 16);
```

If we keep in mind that every hexadecimal digit corresponds to 4 binary digits, we see that 0xff takes up 8 binary digits. We are then shifting that value 16 binary digit positions left which is equivalent to 16/4 = 4 hexadecimal digits left. Thus, we get

$$(0xff \ll 16) = 0xff0000$$

When we do a bitwise and of this value with 0xabbedeed, only the bits in 0xabbedeed that align with non-zeros will be in our result:

```
  0xabbedeed  
& 0x00ff0000  
-----  
  0x00ba0000
```