Computer Science 136
Data Structures
Lecture #7 (September 24, 2021)

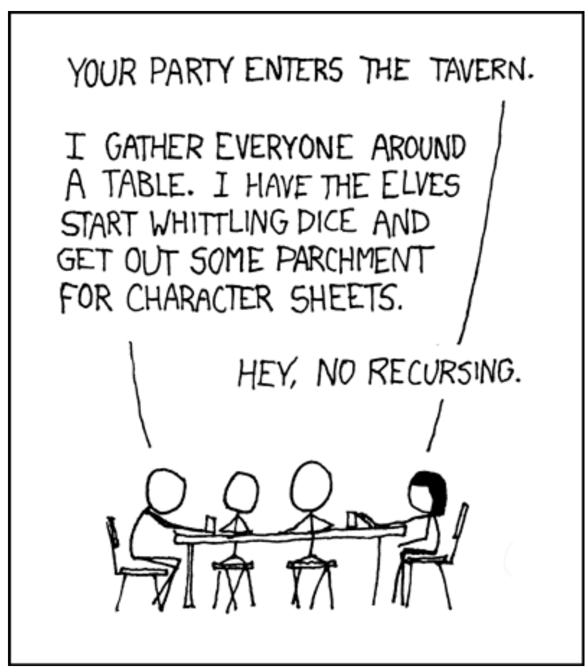
1. Announcements:
   (a) Labs due on Tuesday; the cycle begins!
   (b) Questions?
2. Complexity (left over from Wednesday!):
   (a) Formal definition of what it means for $f(x)$ to be $O(g(x))$. A definition worth remembering. Write it here:

      i. $f(x)$ is bounded above by some constant times $g(x)$.
      ii. $f(x)$ need not ever be equal to $c \cdot g(x)$. The bound need not be *tight*.
      iii. It only needs to be bounded above to the right of some x_0.
      iv. Really, we're only concerned about the magnitude of $f(x)$. In practice, $f(x) \geq 0$ since f is often a measure of time or space utilization.

   (b) Typical approach to "simplifying" a complex function to it's big-O equivalent:
      i. If a function is a sum of terms, keep the term that grows the fastest. For example, $\frac{1}{2}n^2 - \frac{n}{2}$, you keep just the $\frac{1}{2}n^2$ term; in the long run, the polynomial's trend is governed by this term.
      ii. Thus, the function has a single term. If this term has a coefficient that is not 1, cross out the coefficient. Thus, $\frac{1}{2}n^2$ is $O(n^2)$—a quadratic—and $f = 1000$ is written $O(1)$—a constant function.
      iii. Logarithms of all bases are related by a constant. Thus, it does not matter if $f(n) = \log_2 n$ or $f(n) = \ln n$: they're both written $O(\log n)$.

   (c) E.g.: `Vector`s that extend to size n by 1 requires $\frac{n(n-1)}{2}$ copies of old data to new, or $O(n^2)$ time.
   When you double the `Vector`*'s length, the time to expand increases by 4.*
   (d) E.g.: `Vector`s that extend to size n by doubling takes copies old data new new in $2^{\log_2 n} - 1 = n - 1$ steps, or $O(n)$ time.
   When you double the `Vector`*'s length, the time to expand increases by 2.*

3. Recursion: big ideas, little code.
   (a) Basic idea: Reduce hard problem to simpler problem plus a little work.
      i. Base case. The simplest problem(s) you can solve. Think zero.
      ii. Progress. If not a base case, a little bit of work necessary to reduce problem to a simpler problem.
      iii. Recursion. A call to (perhaps another) method that solves the subproblem.
   (b) Counting down to zero from n:
      i. Base case: if n == 0: count 0, we're done.
      ii. Progress: Count n. Now only n-1 to zero is left.
      iii. Recursion: Count down from n-1.
   (c) Reversing a string: `reverse("bard")` is `"drab"` and `reverse("grub")` is `"burg"`. (This, of course, is helpful for finding *palindromes*, but just as helpful in finding even more exotic *semordnilaps*.)
   (d) The `hello, world` of recursion: Towers of Hanoi.
   (e) Print all substrings of a string (!), `printSubstrings(s)`.
4. Challenge: Can you write down the n distinct values $x_i \in \{1...n\}$ in an order such that that makes $x_i + x_{i+1}$ a perfect square for $0 \leq i < n$?



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