Lecture 22

Binary Search Trees I

- Binary Search (review)
  - Example
  - Log Reminders
  - Algorithm and Limitations
- Binary Search Trees
  - Design
  - Subtree Conditions
  - Operations (Part I)

Binary Search
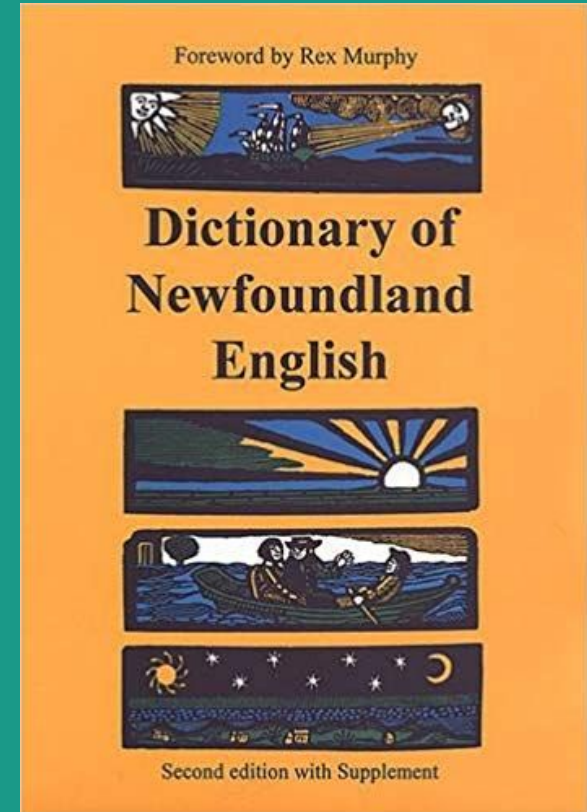
review

Example

Searching an English Dictionary

Suppose that we want to look up a word in an English Dictionary.

- How would you search to find the word?
- How many page flips would it take?

Prediction for the Dictionary of Newfoundland English:

- 10 page flips for book.
- 16 for Kindle locations.

847 pages
49578 locations
Includes Preface and Supplements

toutin?

and associated behaviour. In winter the fish migrate offshore into northern waters and come inshore, following the caplin, in summer.

northern dancers: aurora borealis or northern lights; MERRY DANCERS.

C 65–2 The Northern Lights (also Northern Dancers and Merry Dancers) are really going it tonight. We're going to have a nice day tomorrow, Q 73–9 Look at the northern dancers shining tonight.

northern fishery: cod-fishery prosecuted by eastern Newfoundland fishermen in waters of the north-east coast and Labrador; LABRADOR FISHERY.

[1736–9] 1954 INNIS 148 Fogo & Twillingate, the Northern Fisheries. 1819 ANSPACH 443 What is called the *northern-fishery,* is carried on on the northern coasts of the island and the adjoining parts of Labrador by planters from Conception Bay, Trinity Bay and Saint John's, who go there early in the season in large schooners carrying several skiffs, a numerous crew, and provisions for the

whole fishing season. 1832 MCGREGOR i, 232–3 The northern fishery, now enjoyed by France, was carried on by the planters, by proceeding in schooners, with necessary stores and skiffs, to the northern harbours of Newfoundland, much in the same way as the fishery is at present conducted at Labrador, and the schooners sent back with the fish to the respective merchants.

northern indian: Montagnais or Naskapi indian of Labrador (1839 TUCKER 126).

northern man: inhabitant of the north-east coast of Newfoundland.

1894 HOWLEY *MS Reminiscences* 30 Harbour Main District in particular supplies the bulk of the labourers [on the railway] and they are the only ones who appear to stick to the work. . . The Northern men hate pick and shovel work. 1902 MURPHY 72 "Fanny's Harbour Bawn": I did address this young man, and unto him did say, / Are you from Bonavista or are you from the bay? /1 think

tatter n Phr *every tatter:* every stitch of canvas or sail.

1936 SMITH 12 The next command was to loosen the two top gallant sails, and it wasn't long before Skipper Job. . .had every tatter on her.

tattle-tongue n Cp *NID* tattletale. Child who tells tales on another.

1971 NOSEWORTHY 254 ~ A children's nickname for one who tattles. P 11–79 ~ a person who cannot keep things to himself.

tatty v To jump from one floating ice-pan to another (P 148–67); cp TABBY v.

taunt See TANT.

taut a Of an ice-floe, tight against (the land); solidly frozen; cp SLACK v.

1887 BOND 59 The ice will be taut on the land.

taut* n See TAWT.

tavern n Long-stemmed tobacco pipe with large bowl (Q 67–8).

T 70/2–64[1] They'd get [pipes] that length—taverns, they used to call 'em. You'd have to reach your arms out to the bowl. That was the thing to smoke out o'!

tawt n also **taut, thawt, thort** [ðɑːt, tɔːt, tɔːt] *OED* thought[2], thaught now dial (1622–1886), thoft north dial (1000–); ELMER 106–8 [thoughts] s and e coasts of England, [tawts] two ports in Wales. See *OED* thwart sb[2] (1736–) for the unclear relations between that standard term and thought[2].

1 A board across an undecked boat on which rower sits, often with specifying word *after, forward, midship;* thwart.

1937 DEVINE 51 Thorts. Seats of rowers on a boat; thwarts. P 133–58 Taut: thwart. T 14/6–64 And the aft-tawt will be about two inches shorter than the mid-ship-tawt because the after-tawt goes on the

37184

24791

49578

union organization...by the Newfoundland Board of Labour, which bases its position on a federal judicial opinion. 1986 SIDER 143 Factories hire the labor for their boats as 'co-adventurers'–which means that they are paid with a share of the catch, rather than a wage. 1987 FIZZARD 218 Previously [to 1974] the amount paid to the [trawler] fishermen was dependent on the amount of fish brought in by their vessel: more fish meant more wages; less fish meant lower wages. Called the 'co-adventurer' system, it required the individual fisherman to share the risk of the voyage with the owner.

coaker n

1985 ASHTON 204–5 "The Gambo Way": We called into Fair Islands, took a pilot for Hare Bay. /The old Coaker gave no trouble, she hurled us up that day.

coast n

1 1977 WHITELEY 29 [When he] came to the coast in 1856 James Buckle had his establishment on 'John's Island.' 1984 POWELL 72 Very often when the Newfoundlanders came on the coast in the spring they would say, 'What were you doing all winter, sleeping?' 1988 *Evening Telegram* 31 May, p. 11 Things weren't much different along the coast in 1940 than they were in the first war. That ended in November, 1918 but we didn't know it was over till January of 1919 and then only because two veterans from up around Mud Lake on Hamilton Inlet were walking home.

coastal a Phr *coastal boat.*

1984 POWELL 119 These were two things we just had to have, but this was the last trip of the coastal boat for the season. 1987 KING (ed) 112 "The Southern Shore": Like wild sea birds/these people come/across my imagination/went up the south coast/by coastal boat one summer.

coasting vbl n

~ s. Trees which have been barked, and left standing. **1836** [wix]² 50 Even the sight of a 'whiting' in the woods, that is, of a tree stripped of its bark for the uses of the fishery. . .which tells of the place's having been visited, though in the preceding summer, or a year or two before, by the foot of man. . . [cl900] **1968**RLS8, p. 26 Witing, white end: a tree from which the rind has been stripped such as fir or spruce. **1903** *Nfld Qtly* Dec, p. 56 He went to the wood-pile and selected a nice handy looking whiting and cut it up into junks about two feet long, just sufficient to lay nicely across the dog-irons. **1937** DEVINE 57 ~ s. Standing fir or spruce trees stripped of the lower part of their rind for use in covering dry fish. When later cut down for firewood or lumber, they have become quite dry throughout. Also *white ends*. P 245–61 White-end. A spruce tree, with limbs and bark removed, then dried and cut for fuel. T 12–64 'Dry as a whiten.' A whiten is the rampike or very dry stick used for making splits. T 141/67–65² An' then there was rindin'–you had to go up an' rind your white-ends, rind your rind, sir. That was only thing then to cover fish with. C 68–4 They would find some whitings (trees that were cut and placed in piles). M 69–17 The rinds were taken from standing trees, and as a result that part of the tree from which the bark was removed became dry and was then called white ends, and was cut and used as kindling.

whittle n *EDD* ~ sb² Do D So Co. Heavy flannel or quilted material wrapped around a baby.

T 911–71 We had our children bound up till dey git two or t'ree months old, in whittles dey used to call it you see. **1972** MURRAY 89 Then, winter and summer, children were bound up in 'whittles,' night and day, for two or three months at least, though after this period they were given more freedom of movement during the day. Putting a child in 'whittles' or 'whetals' was also termed 'dressing

truckman: owner and operator of a 'truck cart.'

T 731–70 A truckman meant a man that had a horse and long cart and stood in the coves until he was hired. 1976 CASHIN 84 His avocation in life was that of what is termed nowadays a truckman. He had left his horse and cart in front of the old mechanics' hall on Water Street, imbibed a little too freely, then he came aboard to see a friend of his off to the sealing voyage. . .

truckle v *EDD* ~ v 1 s w cties. To trundle or roll (a wheeled toy); CHUCKLE, RUCKLE.

C 70–25 When the car or truck was pushed along. . .it was called *truckling* the car. In fact the name of the game was 'truckling cars.'

truckle n *OED* ~ sb 4 'low-wheeled car' Ir (1689-); *EDD* sb^1 Ir obs. A small cart with four wheels; TRUCKLY-MUCK.

1937 DEVINE 54 ~ A home made hand cart with small solid wheels. M 68–24 He was on a home-made cart (truckels) with four wheels drawn by a very obedi ent dog.

truckly-muck n also **chuckle-de-muck*, truck-lemuck, truckly.** Cp *EDD* truckly-mux D; truck-amuck sb 1 'trolly,' 2 'farm-cart' D for sense 1.

1 A small cart with four wheels; TRUCKLE.

1895 *J A Folklore* viii, 39 ~ a small two-handed car for dogs, with a handle for a man to keep it straight. [cl900] 1978 *RLS* 8, p. 26 Trucklemuck. A small two wheeled hand car with a guiding stick in front, usually used with dogs to haul while the owner also hauls with a rope over one shoulder & his other hand holding the handle or tiller. 1937 DEVINE 54 Truckley-muck. A home made hand cart with small solid wheels. P 267–58 ~ Four-wheeled cart used on the [fishing] stage.

remarkable hill, called the Chapel Tolt, behind, and the Long Hill Deer country, on my left. 1842 JUKES ii, 221 The principal of these [elevations] are called. . . Little Gut Outlook, the Tolt, and the Monument. 1868 HOWLEY *MS Reminiscences* 34 In the far distance some high hills or tolts are visible rising like gigantic hay pooks above the tree tops. 1895 / *A Folklore* viii, 39 ~ a solitary hill, usually somewhat conical, rising by itself above the surrounding country. [cl900] 1978 *RLS* 8, p. 23 ~ A rounded conical isolated hill standing above the general level. Like a hay pook. Seems to correspond nearly with the S.A. 'kopj' but smaller. 1953 *Nfld & Lab Pilot* ii, 2–3 On the eastern coast hummocks of granite protrude and are known locally as Tolts. 1965 *Evening Telegram* 31 Dec He reveals only that he climbs to the top of a 1,822-foot toit on the Gaff Topsails. 1971 NOSEWORTHY 256 ~ A hill standing alone in flat country. (*'Hill* —highest, *tolt* —next highest, then *knap.'*) P 245–79 She lives in on the

Tolt Road [near St John's].

tom n
Comb **tom cod**: see TOM COD.
torn Conner, tommy ~ : blue perch *(Tautogolabrus adspersus),* a bottom-feeding fish of inshore waters, esp common around wharves and fishing stages; CONNER.
M 70–27 The tommy Conner is the local name for a Conner. C 71–93 Tom conners [are] small fishes of a greyish colour usually found around wharves where cod are landed.
torn fox: eastern fox sparrow *(Passerella iliaca iliaca);* FOXY TOM.
1959 MCATEE 66 ~ Fox sparrow (From its colour; the most widespread subspecies has a good deal of chestnut-rufous spotting, striping, and washing. Nfld. 'Labr.'.N.S.)
tom-tee: tom-tit; black-capped chickadee; PIG-A-WEE *(Parus atricapillus bartletti)* (1959 MCATEE

37959

37185 —— 38733

(1958-). See also COD TRAP. For fur-trade terms, see TRAP [2].

1 A type of fixed fishing-gear used in inshore waters, box-shaped, with a length of net stretching from shore to entrance through which migrating cod (and salmon) enter and are trapped.

1863 HIND [i], 297 On the day of our visit he took 9,000 cod fish out of his traps. . . A shoal of fish coming in either direction in thirty to forty feet of water, the depth of the net, find their course intercepted; some of the fish pass round the seaward side of the net and escape; the others or some of them, coming landwards, enter the first compartment, swim round its side, and a portion pass into the second compartment, swim round its side, and, always pursuing a straight course, ultimately enter the third compartment, and finally the pound or fourth compartment. The fish, when swimming round the sides of the net, are observed to pass by the narrow doors, keeping always 'straight ahead'; so that, if the doors are always *flush* with the sides of the net, the fish swim straight on and do not turn out of their course to pass through them, and consequently remain in the pound when once there. It is needless to say that the net *is floored* with net, and really forms a gigantic bag with square sides and narrow perpendicular inlets. [cl875] 1975 WHITELEY 153 [He] used the first trap in Newfoundland made by American Net & Twine Co Boston for Job Bros & Co from plan furnished by Capt James Joy— dimensions taken from my trap at Bonne Esperance —the following year the same firm had traps made for Indian Harbor; these were the first traps used on the Labrador. 1884 DEMING 90 The local title of 'trap' is applied to a contrivance much like our fish-pounds, but made entirely of nets. 1889 HOWLEY *MS Reminiscences* 3 Reports of wreckage, especially the destruction of traps and other fishing gear, were heard of all along. [1905] 1912 *Nfld Law*

beef; anything hard to chew (P 148–63).

toutin n also **toutan, touten, touton, towtent** ['tæuʧn̩, 'taɒfɔn̩, 'tæutənz, 'tæuʧnz, 'teuʧnz]. Cp *DAE* dough cake 'flapjack or a kind of cake made with bread dough' (1839-) and doughnut (1805-).

1 A piece of bread dough fried in fat; DAMPER DOG.

1891 *Holly Branch* 12 Skipper Tom and crew. . .remained longer at home whenever they came with a little 'gob,' stopped on the 'jigging ground' and fry 'toutins' till the smoke of the fat. . .had almost set him mad. 1915 *Nfld Qtly* Oct, pp. 32–4 The old captain could not eat hard bread, so we would have toutons for breakfast, sweet pudding for dinner and toutons again for supper. 1937 DEVINE 53 Toutan—a fried cake with bits of fat pork in it. 1966 HORWOOD 166 . . .*toutins,* which are pieces of bread dough fried golden-brown in fat. T 94/5–64 Toutins is rose dough. You knead it down, and cut off little pieces an' you poke your finger through 'em and you have

something, bacon or fat-back, and lay 'em on. P 130–67 It's been a long time since we had toutins for breakfast. It was usually served for breakfast because with slow rising yeast, bread was mixed in the evening, allowed to rise during the night. P 127–77 ~ A dough pancake, made by wrapping a piece of dough around a piece of baloney and frying. 1981 *Evening Telegram* 15 Aug, p. 211 was [in the kitchen] waiting for the toutons to brown.

2 A bun made with flour, molasses and bits of pork (P 267–58); cp BANGBELLY , combs, with MOLASSES, PORK.

1896 *J A Folklore* ix, 36 Towtents—pork cakes made of pork chopped fine and mixed with flour. T 30/3–64[1] Toutins. . .dough mixed up with pork cut up in little small pieces and baked. T 96–64[2] Take the flour an' put the pork in and the bakin' powders an' then roll them an' bake. That's the toutins. M 69–17 The toutin was the traditional food of woodsmen, for if they contained enough pork, they

38153

37960 ⁻ 38346

Toutin

Yummy?!

toutin n also **toutan, touten, touton, towtent** [ˈtæuʈʂn̩, ˈtaʊfʂn̩, ˈtæutənz, ˈtæuʈʂn̩z, ˈtɐuʈʂn̩z]. Cp *DAE* dough cake 'flapjack or a kind of cake made with bread dough' (1839-) and doughnut (1805-).

   1 A piece of bread dough fried in fat; DAMPER DOG.

Summary of Guesses

Range of Possibilities			Guesses		
left position+1	**right** position-1	**length** right - left + 1	**position** left + ⌊length/2⌋	**word**	**result**
1	49578	49578	24790	*northern dancers*	too small
24791	49578	24788	37184	*tatty*	too small
37185	49578	12394	43382	Supplemental Words *coaker*	too big
37185	43381	6197	40283	*whittle*	too big
37185	40282	3098	38734	*truckman*	too big
37185	38733	1549	37959	*tom cod*	too small
37960	38733	774	38347	*trap*	too big
37960	38346	387	38153	*toutin*	correct!

Log Reminders

Reminders on log(n)

Remember that log(n) is the number of times we can divide an integer n ≥ 0 by 2 until reaching 1.

In other words, it is the length of the following sequence.

$$n/2, \ n/4, \ n/8, \ …, 1$$

A common goal in computer science is to reduce factors of n to factors of $\log_2(n)$.

- Linear searching in a sorted array is O(n)-time.
  Binary searching in a sorted array is O($\log_2(n)$)-time.
- Sorting an array is O(n^2)-time using obvious algorithms (e.g. selection or insertion sort).
  Sorting an array is O(n · $\log_2(n)$)-time using better algorithms (e.g. merge or quick sort).

We use $\log_2(n)$ so often that we often denote it by log(n) or lg(n) or log n.

Also, remember that O(log(n)) is much smaller than O(n).

- 2^{10} = 1024 so log(1024) = 10. Or roughly, log(1,000) < 10.
- 2^{20} = 1048576 so log(1048576) = 20. Or roughly, log(1,000,000) < 20.
  This can be computed using log(x · y) = log(x) + log(y).
  That is, log(1000000) = log(1000 · 1000) = log(1000) + log(1000) < 10 + 10 = 20.

Original Log Commercial | The Ren & Stimpy Show | NickRewind

Algorithm and Limitations

Binary Search Algorithm

Given a sorted array of length n, we wish to determine if a target value t appears within the array.

The *binary search algorithm* compares t to the value m in the middle of the array.

- If t < m, then m can only be located in the first half of the array.
- If t = m, then t has been found.
- If t > m, then m can only be located in the second half of the array.

Searching for -1 in the sorted array.

-10	-8	-5	-5	-2	0	1	3	3	7	11	13	20	22	22	25

-10	-8	-5	-5	-2	0	1	3

0	1	3

0

The algorithm takes O(log(n))-time as opposed to linear search which takes O(n)-time.

Notice that the algorithm also identifies when an target value is not in the array.

Limitations of Binary Search

Binary search assumes that its values are sorted and are accessible in O(1)-time via indexes. Therefore, it is effective in an array and not a standard linked list.

Array location

i-th element position
calculated as
SizeOf(element)*i

Linked List First Node
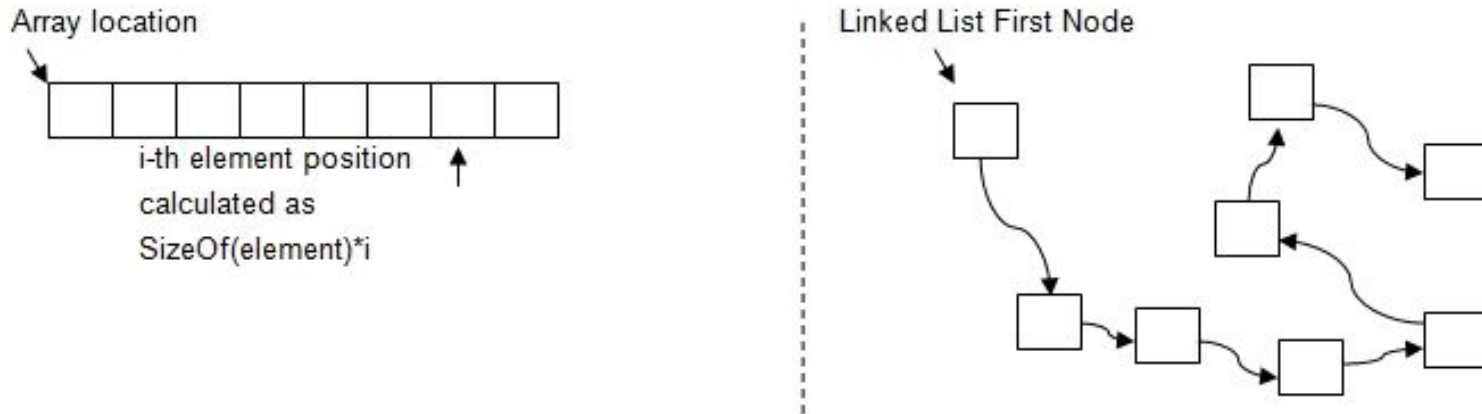
Without further thought, it is best suited for *static data* and does not work well with *dynamic data*. More specifically, deleting or inserting a value in a sorted array of length n takes O(n)-time.

This is fine for something like an English dictionary, which is not updated frequently, but in computer science we use dictionaries for a variety of purposes.

Binary Search Trees (BST)

Design

Goals

Previously we discussed how the binary search algorithm is not always suitable for dynamic data. More specifically, if we maintain a sorted array, then it takes O(n)-time to insert and delete values, where n is the number of values currently stored in the array.

Let's try to achieve O(log n)-time operations for **find** / **insert** / **delete**.

find	insert	delete
O(log n)	O(n)	O(n)

Stored in a sorted array.

find	insert	delete
O(log n)	O(log n)	O(log n)

Goals.

We'll design a binary tree structure that has some of the benefits of sorted arrays and some of the benefits of linked lists.

The Benefits of Doubly Linked Lists and Sorted Arrays

The advantage of a doubly linked list is being able to quickly delete/insert elements.

The advantage of a sorted array is finding elements quickly using binary search.

- Binary search is based on finding the median between two values.

prev	data	next
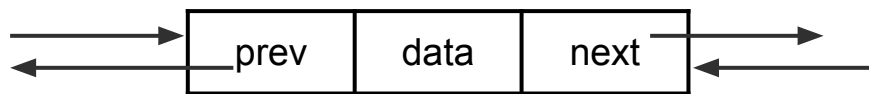
The pointers enable quick additions and deletions.

-3	0	7	10	15	42	44	88
1	2	3	4	5	6	7	8

x   y
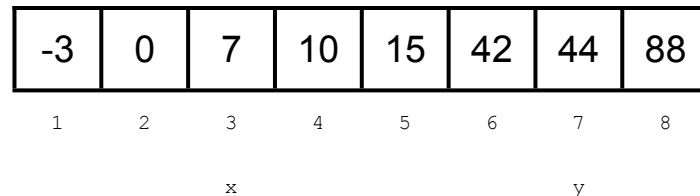
The median value between the xth and yth largest values can be found in position $(x+y)/2$.

How can we combine these two benefits?

- We will attempt to adapt the doubly linked list.
- Add links to the median elements instead of to the immediate left and right.

Intuition

Suppose that the n elements are sorted. We want to be able to access the middle element in position n/2, so make it the root.

Depending on the value we are search for, we then want access to element that is in position n/4 or 3n/4, so make these two nodes the children of the root.

This creates a binary tree structure. Furthermore, there are restrictions on the values of the nodes. What are they?

-3	0	7	10	15	42	44	88	94
1	2	3	4	5	6	7	8	9
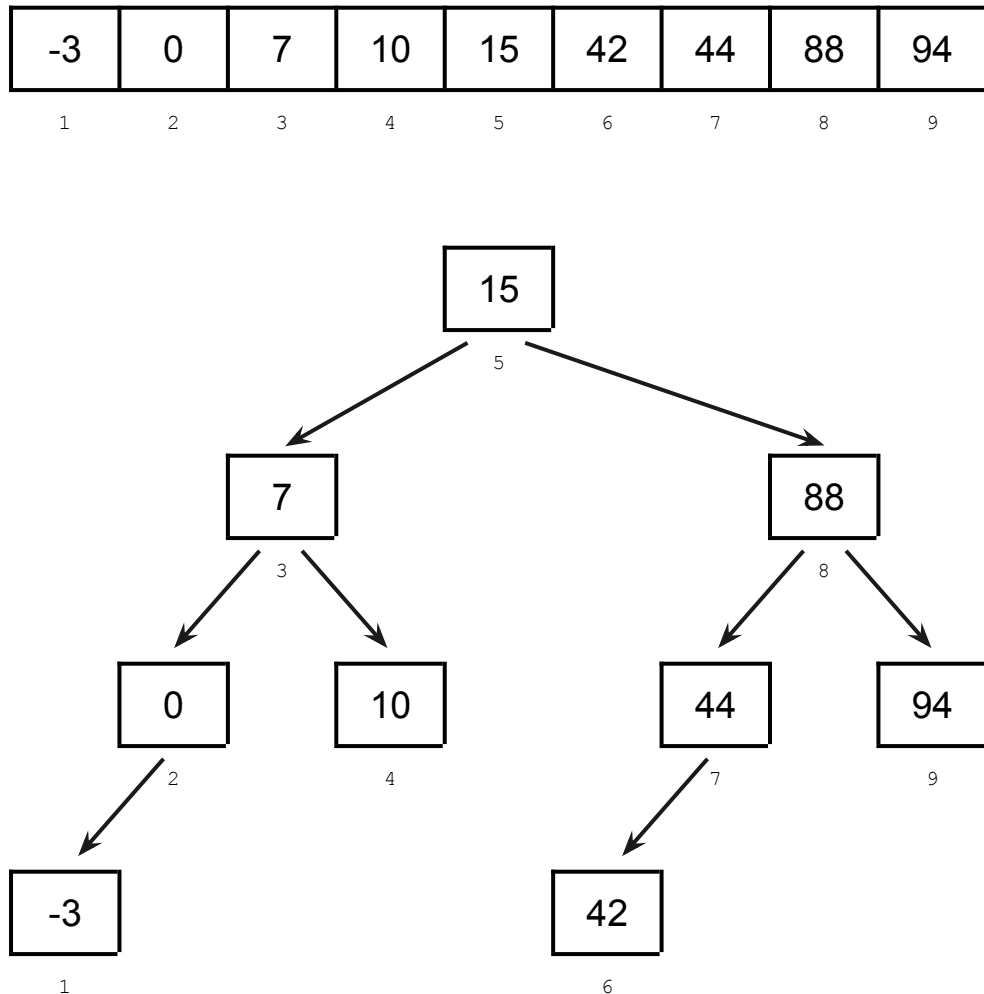
Subtree Conditions

Binary Search Tree

A *binary search tree* (*BST*) is a binary tree with values at each node that satisfy the following two *subtree conditions*:

- If b is the value of a node and a is a value in its left subtree, then a ≤ b.
- If b is the value of a node and c is a value in its right subtree, then b ≤ c.

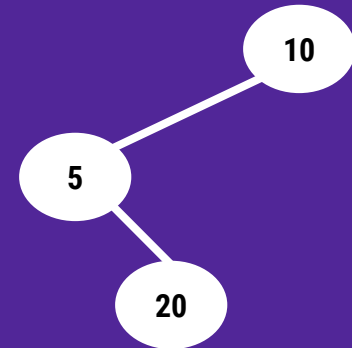This is true for every node b, and not just the root.
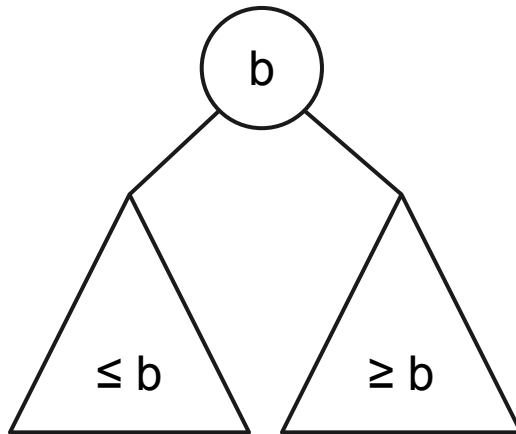
Example: Binary Search Tree
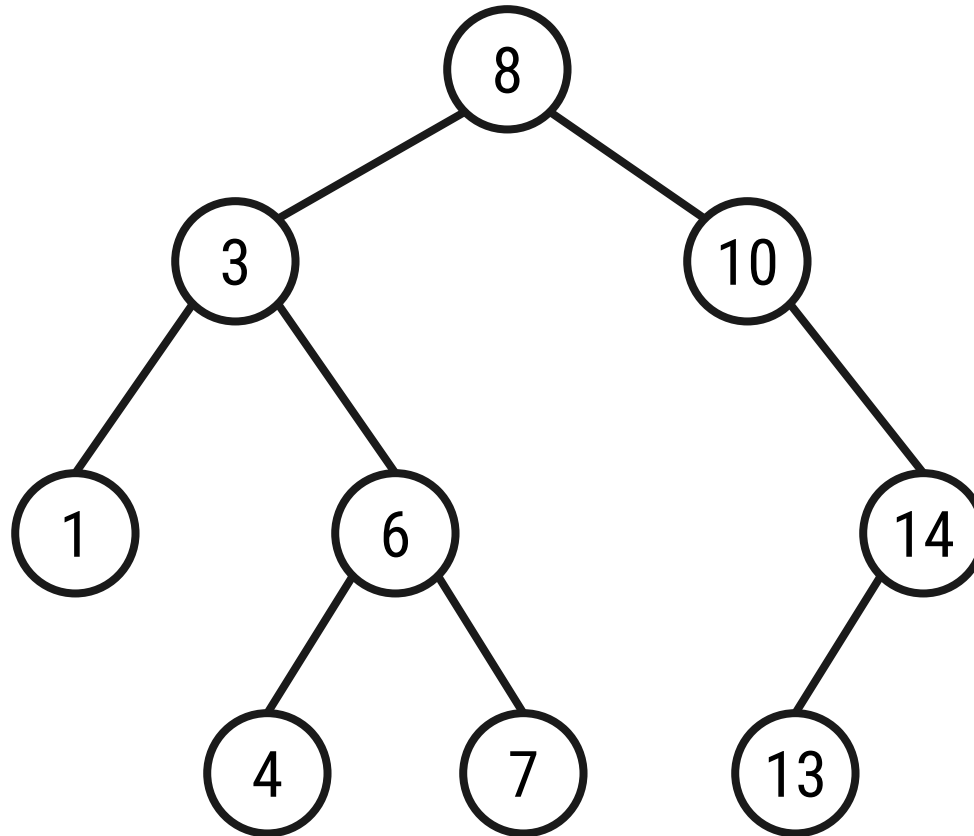
Question: Child vs Subtree Conditions

Consider these two edge conditions for a binary tree with values in its nodes.
- If b is the value of a node and a is the value of its left child, then a ≤ b.
- If b is the value of a node and c is the value of its right child, then b ≤ c.

Let's refer to these as the *child conditions*, since they apply only to children and not to subtrees.

Question: Are the child conditions equivalent to the subtree conditions? It's clear that the subtree conditions imply the child conditions, but is the converse true? Do the child conditions imply the subtree conditions?
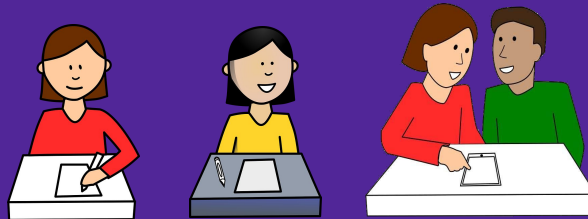
Think about this for 2 minutes.
Then discuss it with your neighbor for 2 minutes.

If the answer is yes, then provide a brief explanation.
If the answer is no, then provide a small example.

The answer is no.
This tree satisfies the child conditions
but it doesn't satisfy the subtree conditions.

Operations
(Part 1)

Operations for Binary Search Trees

Binary Search Trees are typically associated with the following three operations.

(Side note: When working with data structures, you'll find that a thesaurus can come in handy!
This is because different terms are used for the same operations. We'll practice this for BSTs.
It will also simplify comparisons between these slides and the textbook + `structure` package.)

- *Find / Contains.* Return **yes** or **no** depending on whether the given element is present.
  In these slides, we'll refer to the method as `find`, while the textbook names it `contains`.

- *Insert / Add.* Add a new element into the binary search tree.
  In these slides, we'll refer to the method as `insert`, while the textbook names it `add`.

- *Delete / Remove*. Delete the provided element from the binary search tree.
  In these slides, we'll refer to the method as `delete`, while the textbook names it `remove`.
  (We will see this operation in the next lecture.)

In each case, we refer to the binary search tree by providing a reference (or pointer) to its root.

Binary Search Tree: Find
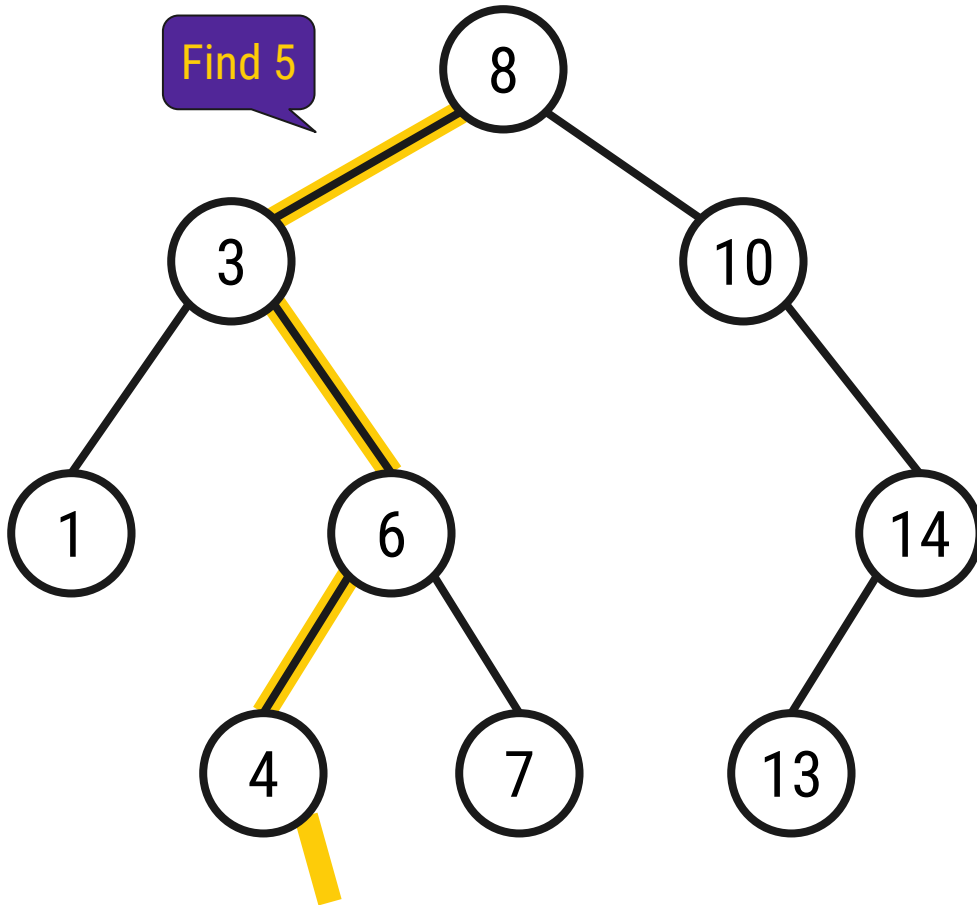
To determine if a value is in the binary search tree we repeatedly do the following:

- If the value is found at the root, then the answer is **yes**.
- Otherwise, start the search again at the root's left child or right child depending on the relative order of the value in the root and the value being searched for.
- Answer **no** if the root is empty.

This is most easily implemented using recursion, but it can also be implemented using iteration (i.e., a loop).

Binary Search Tree: `Find`

The following pseudocode implements the find operation recursively.

```
function find(root, target)
    if root is empty then
        return no

    if root.value == target then
        return yes

    if target < root.value then
        return find(root.left, target)
    else
        return find(root.right, target)
```
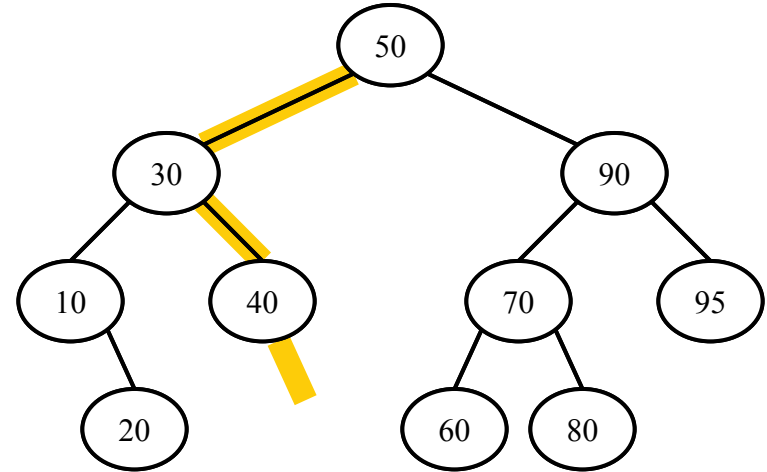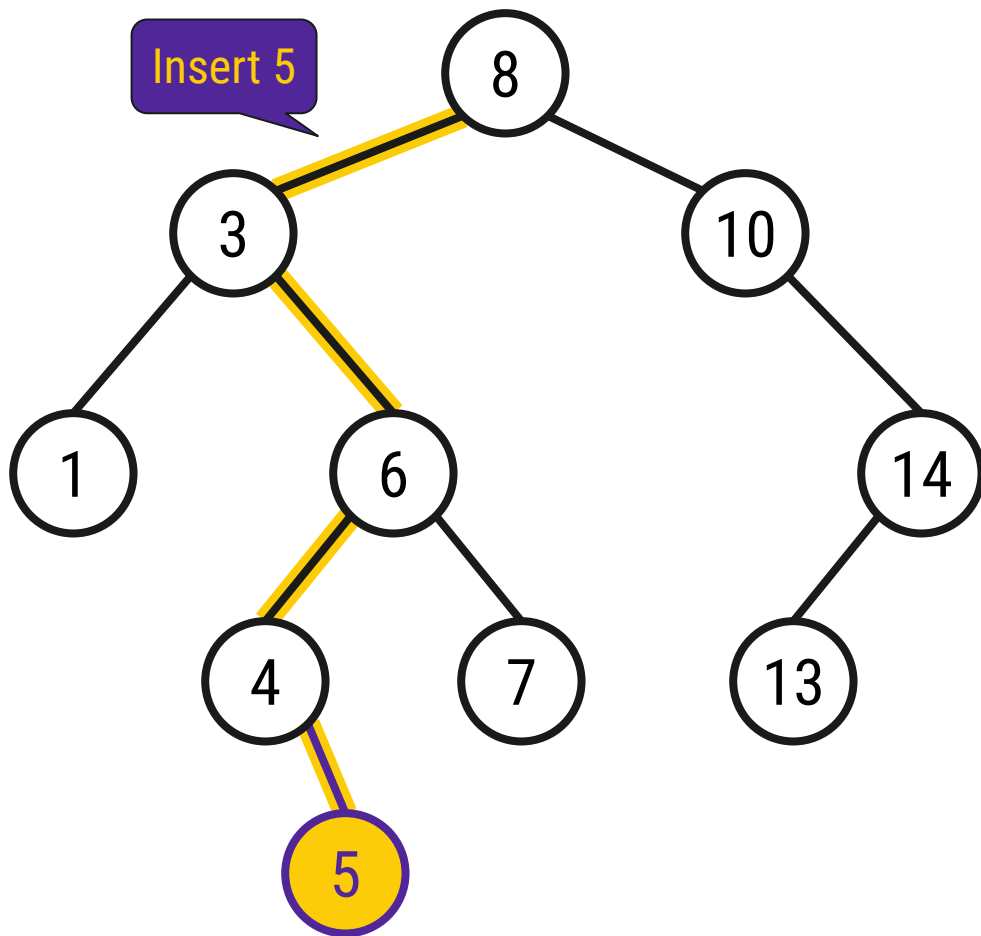

Example: Find value 40 (yes).
Example: Find value 45 (no).

Question: How long does this algorithm take?

Answer: O(h)-time where h is the height of the tree.

Binary Search Tree: Insert

To insert into a binary search tree we first perform a find.

The location where the node does not appear is the location where we will place the new node.

This makes sense, since the insertion will be in the same location that a subsequent find operation will check.

Also note that this approach will not violate any of the subtree conditions.

# Binary Search Tree: `Insert`

The following pseudocode implements the insert operation recursively.

● We'll discuss the issue of repeated values in the next lecture.

```
// Assumes that value is not in the tree.
function insert(root, value)
    if root is empty then
        root = new node(value)
        return

    if root.value < value then
        if root.left is empty then
            root.left = new node(value)
        else
            insert(root.left, value)
    else
        if root.right is empty then
            root.right = new node(value)
        else
            insert(root.right, value)
```
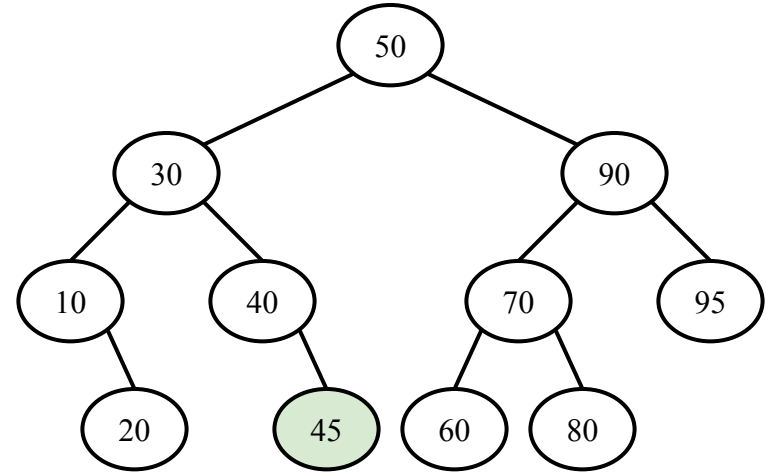

Example: Insert 45 into the tree.

This algorithm also takes O(h)-time where h is the height of the tree.