Lecture 21

Trees III

- Exploring Binary Trees
  - Challenge 2
  - Challenge 3
  - Challenge 4
- `structure` Package
- Huffman Codes

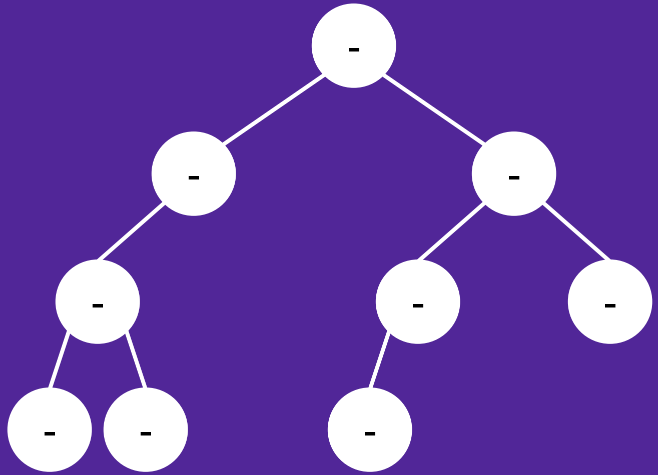
Exploring Binary Trees

Challenge: Exploring Binary Trees (Part 2)

How can we determine the following value in a binary tree?

- The height of the tree.

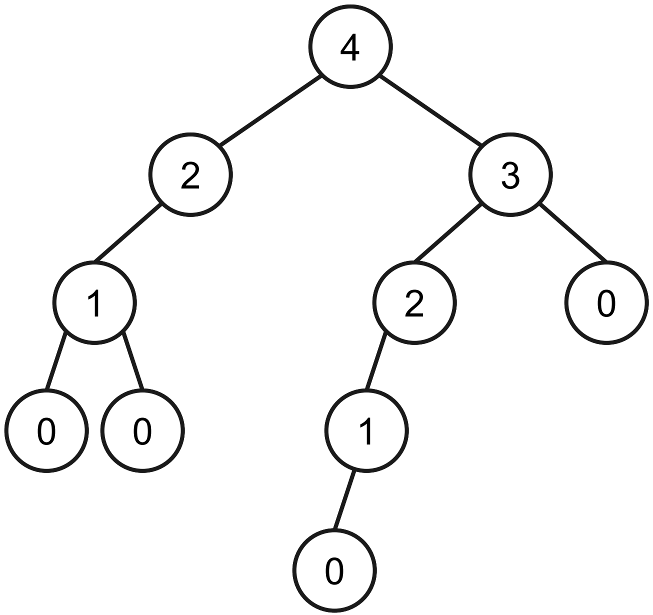
Think about this for 1 minute.
Then discuss it with your neighbor for 2 minutes.



This binary tree has height 3 (counting from 0).

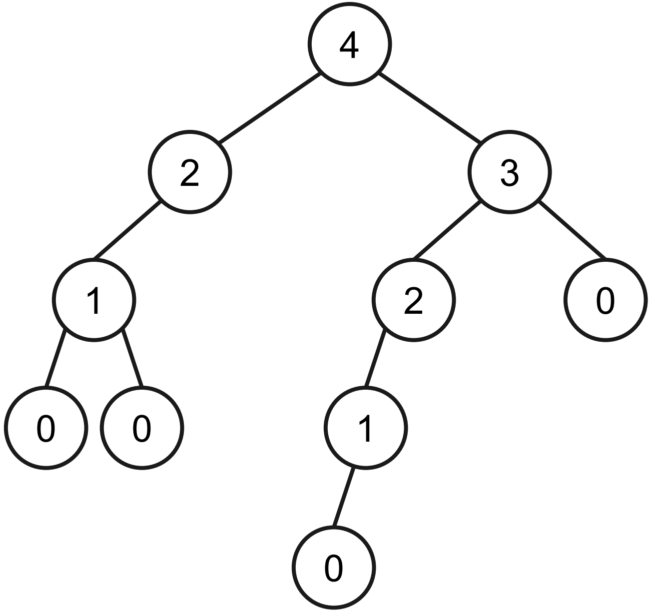
Hints:

- Consider a recursive algorithm.
- Remember that a parent and child are not at the same level.




```
// Return the height of the tree rooted at node.
// Note: A tree with one node has height 0.
height(node)
  // todo

// Main method: Run the algorithm from the tree's root.
answer = height(root)
```

Determining the height of a binary tree.




```
// Return the height of the tree rooted at node.
// Note: A tree with one node has height 0.
height(node)
  // Base case: the root node is null
  if node is null then
     return 0

  // Base case: the tree consists only of the root
  if node.left is null and node.right is null then
    return 0

  // Determine the height of the two subtrees.
  heightLeft  = height(node.left)
  heightRight = height(node.right)

  // Return the maximum plus one.
  return max(heightLeft, heightRight) + 1

// Main method: Run the algorithm from the tree's root.
answer = height(root)
```

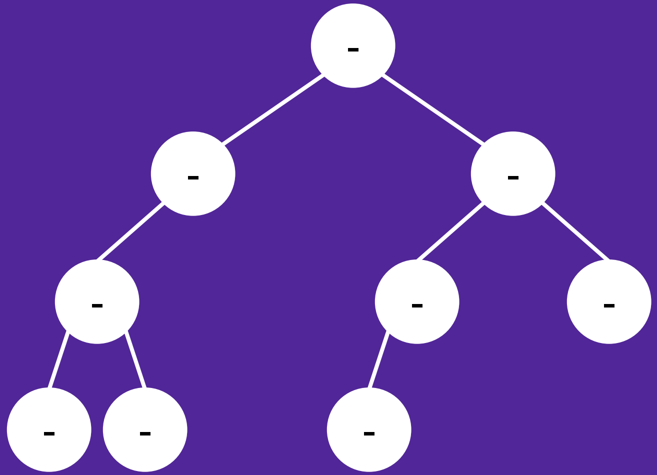
Right: Determining the height of a binary tree. Left: The return values shown in each node.

Challenge: Exploring Binary Trees (Part 3)

How can we determine the following values in a binary tree?

- The total number of nodes.
- The smallest level that has a leaf.
- The number of left links that are used.

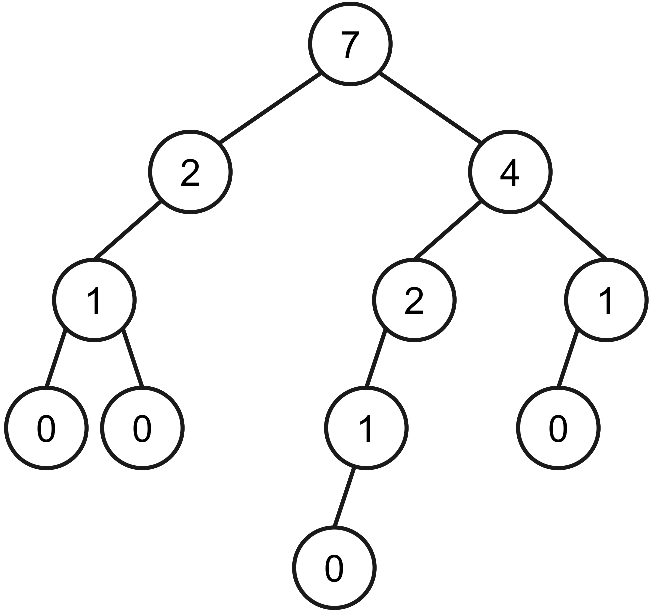
Think about this for 1 minute.
Then discuss it with your neighbor for 4 minutes.



This binary tree has 8 total nodes.
The smallest level of a leaf is 2.
It has 5 left links in total.

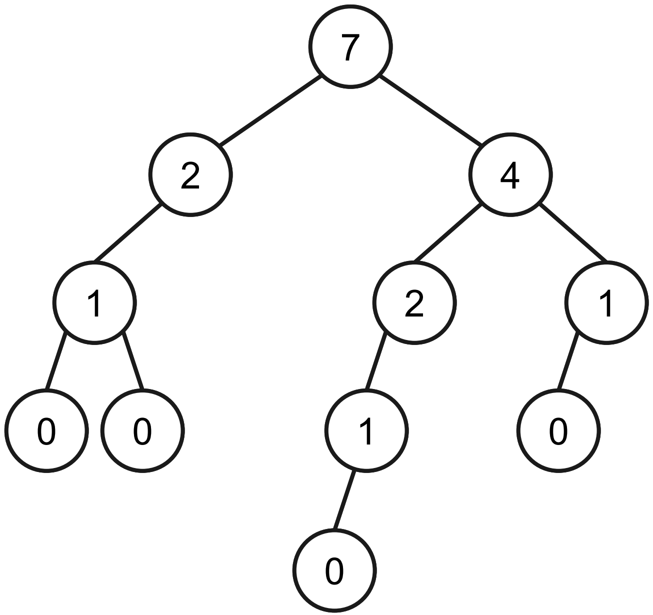
What other quantities could we try to count?

-




```
// Return the number of left links in a binary tree that is
// rooted at a given node.
left(node)
  // todo




// Main method: Run the algorithm from the tree's root.
answer = left(root)
```

Determining the number of left links in a binary tree.



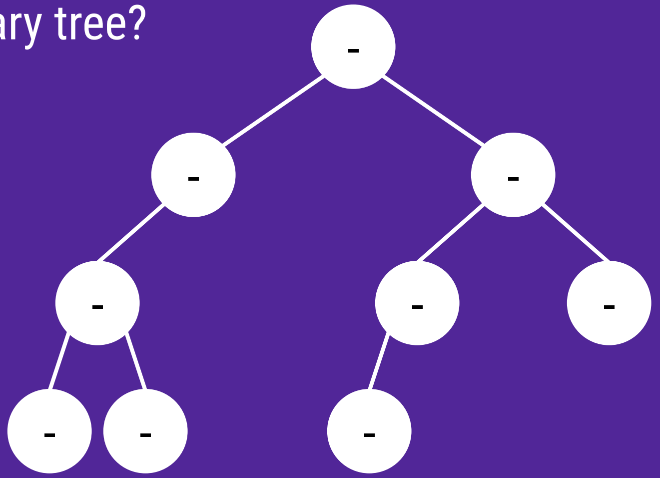

```
// Return the number of left links in a binary tree that is
// rooted at a given node.
left(node)
  // todo




// Main method: Run the algorithm from the tree's root.
answer = left(root)
```

Determining the number of left links in a binary tree.

Challenge: Exploring Binary Trees (Part 4)

How could we print out a nice text representation of a binary tree?

Think about this for 1 minute.



This binary tree could be printed out as

```
       o
     /   \
    o     o
   /     / \
  o     o   o
 / \   /
o   o o
```

Questions:

- What do you interpret *nice* to mean?
- What values would you want to compute?

`structure` Package

```
  GNU nano 5.8                        BinaryTree.java
package structure5;
import java.util.Iterator;

public class BinaryTree<E>
{
    // The value associated with this node
    protected E val; // value associated with node

    // The parent of this node
    protected BinaryTree<E> parent; // parent of node

    // The left child of this node, or an "empty" node
    protected BinaryTree<E> left, right; // children of node

    // A one-time constructor, for constructing empty trees.
    public BinaryTree()
    {
        val = null;
        parent = null; left = right = this;
    }
```

```
    // Constructs a tree node with no children.  Value of the node
    // and subtrees are provided by the user
    public BinaryTree(E value)
    {
        Assert.pre(value != null, "Tree values must be non-null.");
        val = value;
        right = left = new BinaryTree<E>();
        setLeft(left);
        setRight(right);
    }

    // Constructs a tree node with two children.  Value of the node
    // and subtrees are provided by the user.
    public BinaryTree(E value, BinaryTree<E> left, BinaryTree<E> right)
    {
        Assert.pre(value != null, "Tree values must be non-null.");
        val = value;
        if (left == null) { left = new BinaryTree<E>(); }
        setLeft(left);
        if (right == null) { right = new BinaryTree<E>(); }
        setRight(right);
    }
```

Are you surprised by anything?

- Everything is a tree!
- There is not a separate class for nodes (as was the case with `structure`'s linked lists).

```
GNU nano 5.8                    BinaryTree.java
    // Returns the number of descendants of node
    public int size()
    {
        if (isEmpty()) return 0;
        return left().size() + right().size() + 1;
    }

    // Returns reference to root of tree containing n
    public BinaryTree<E> root()
    {
        if (parent() == null) return this;
        else return parent().root();
    }

    // Returns height of node in tree.  Height is maximum path
    // length to descendant
    public int height()
    {
        if (isEmpty()) return -1;
        return 1 + Math.max(left.height(),right.height());
    }
```

```
    // Compute the depth of a node.  The depth is the path length
    // from node to root
    public int depth()
    {
        if (parent() == null) return 0;
        return 1 + parent.depth();
    }

    // Returns true if tree is full.  A tree is full if adding a node
    // to tree would necessarily increase its height
    public boolean isFull()
    {
        if (isEmpty()) return true;
        if (left().height() != right().height()) return false;
        return left().isFull() && right().isFull();
    }

    // Returns true if tree is empty.
    public boolean isEmpty()
    {
        return val == null;
    }
```

Implementations for `size` (i.e. number of nodes) and `height` and more.

Huffman Codes

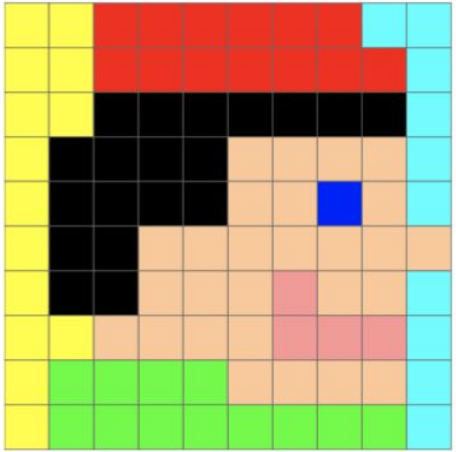
Encoding an Image

How can we encode an image in binary (i.e., in a file)?

- Assign a code word for each color.
- Write the code words for each pixel's color in *row major order* (i.e., from left-to-right starting at the top row).

How should we assign the code words? Several options below.

1. Use `00000001` for yellow, `00000010` for red, etc.
   This works, but it is wasteful.
2. Use `0` for yellow, `1` for red, `10` for green, `11` for teal, etc.
   This is compact, but it results in a *prefix problem*.
   - Suppose that the file starts with `11`.
     That could indicate two red pixels or one blue pixel.
     The problem is that code `1` is prefix of code `11`.
3. Use binary strings of the same length. Length: $\lceil \log(n) \rceil$
   Use `000` for yellow, `001` for red, etc.



An image with n = 8 colors.

1. 00000001 00000001 00000010 …
2. 0 0 1 1 1 1 1 1 11 11 0 0 …
3. 000 000 001 001 001 001 …

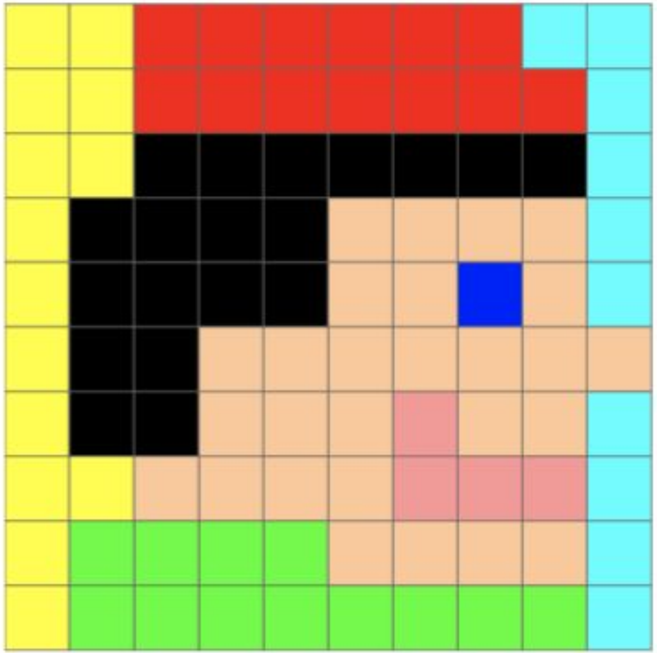
The start of the file for
each of the three encoding schemes.

Example: Huffman Codes

Let's try to improve upon the 3rd encoding scheme from the prefix slide.

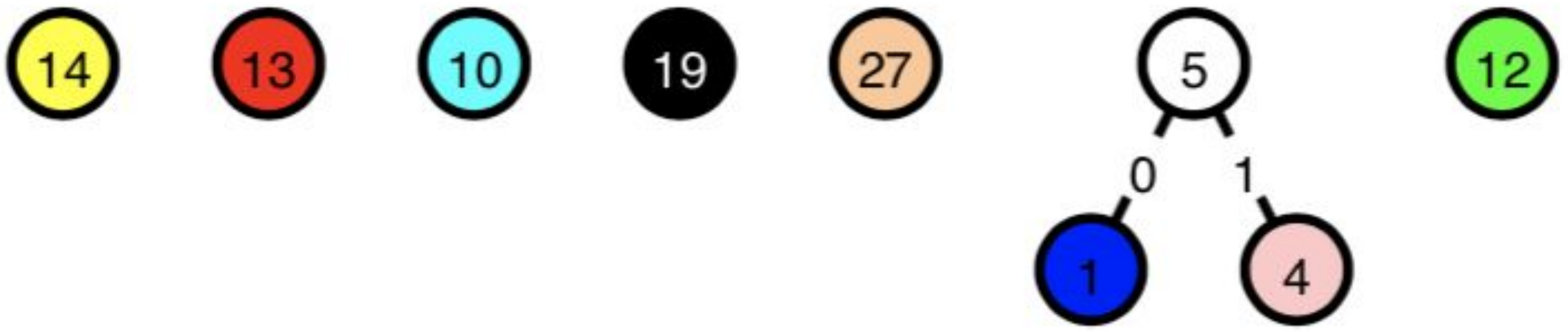
We want to represent each color using as few bits as possible.

- Frequent colors should use fewer bits.



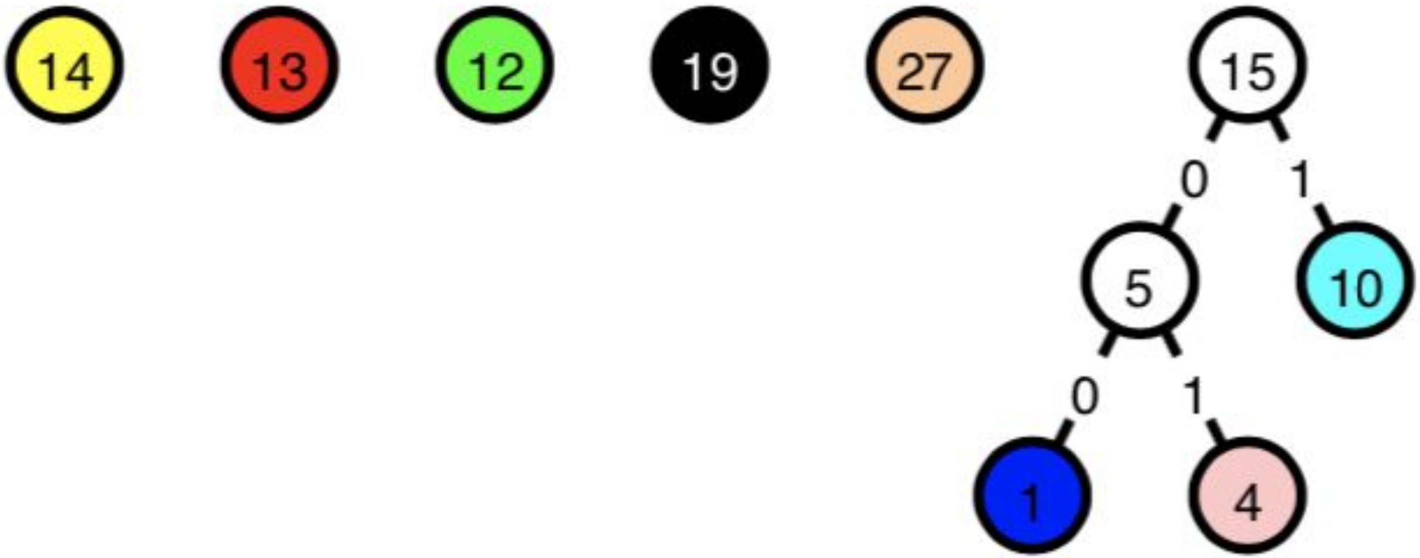
Repeatedly merge the two lowest frequency nodes under a parent node.

- The left / right edges have label 0 / 1.
- The parent's node is the sum of the frequencies.



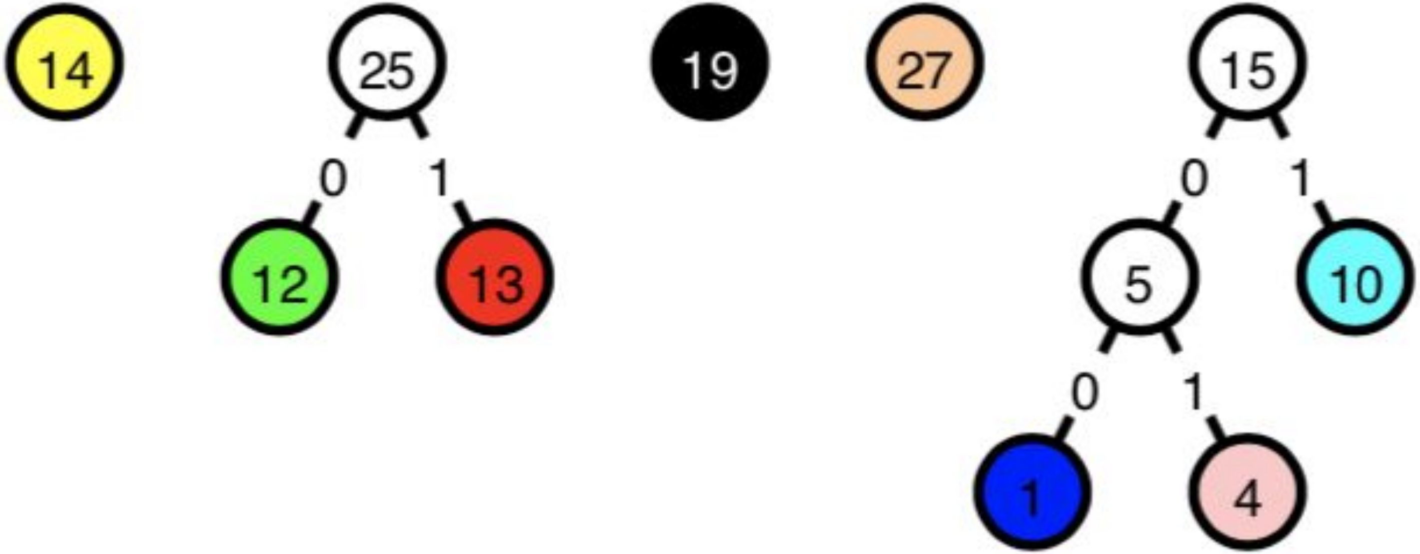
Repeatedly merge the two lowest frequency nodes under a parent node.

- The left / right edges have label 0 / 1.
- The parent's node is the sum of the frequencies.



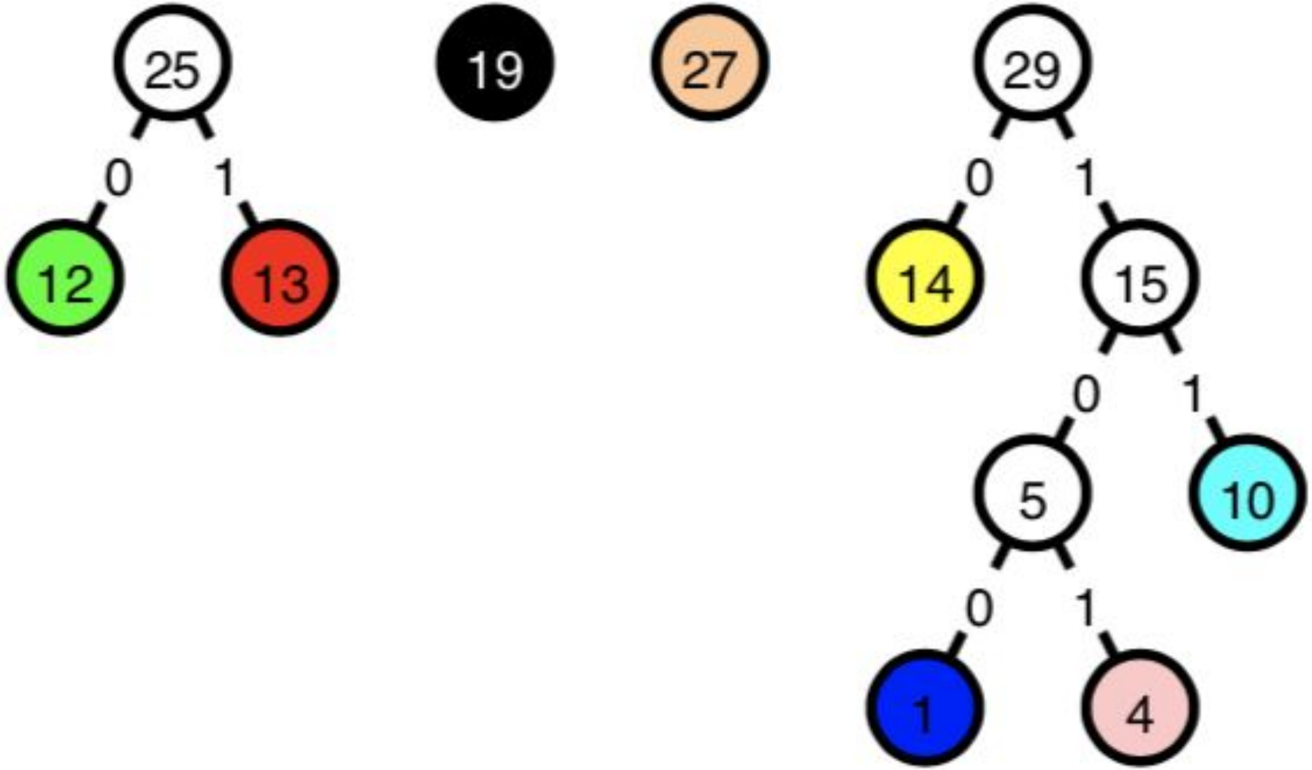
Repeatedly merge the two lowest frequency nodes under a parent node.

- The left / right edges have label 0 / 1.
- The parent's node is the sum of the frequencies.



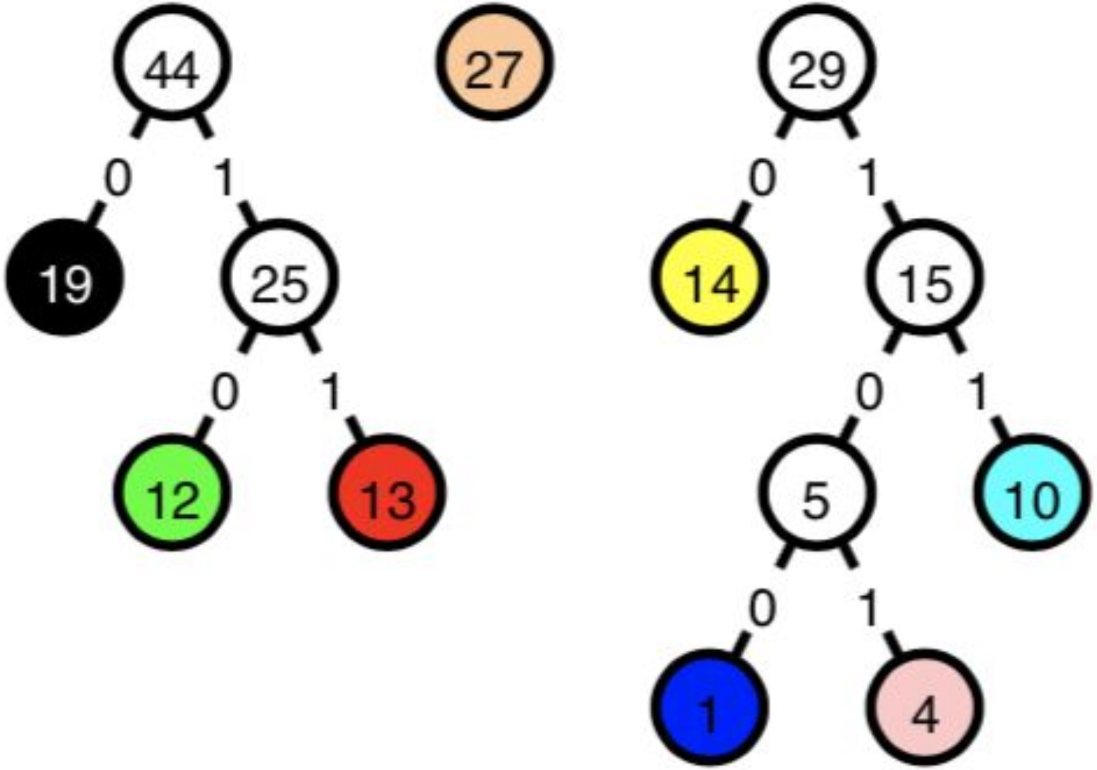
Repeatedly merge the two lowest frequency nodes under a parent node.

- The left / right edges have label 0 / 1.
- The parent's node is the sum of the frequencies.



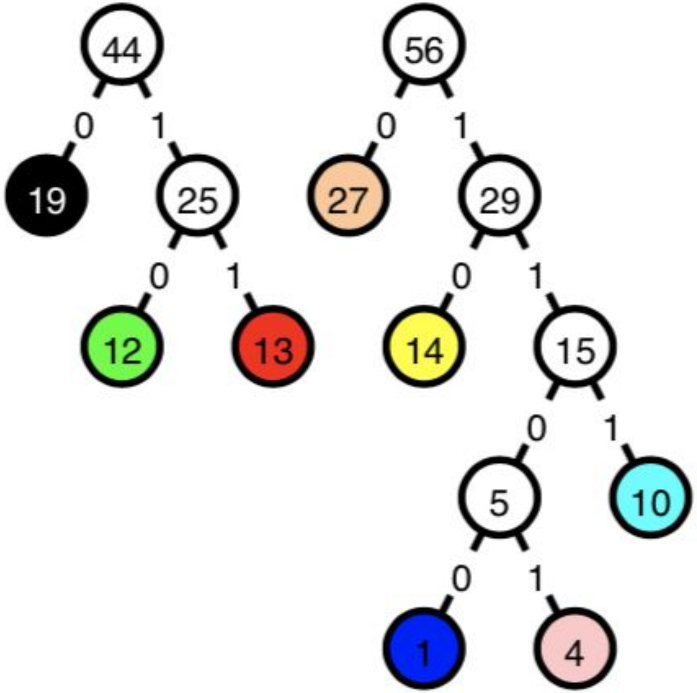
Repeatedly merge the two lowest frequency nodes under a parent node.

- The left / right edges have label 0 / 1.
- The parent's node is the sum of the frequencies.



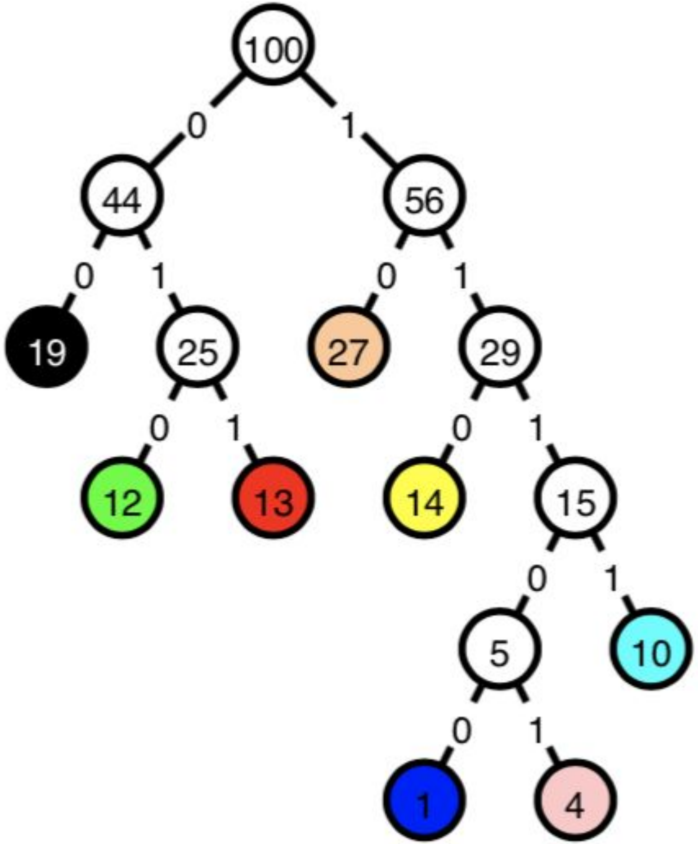
Repeatedly merge the two lowest frequency nodes under a parent node.

- The left / right edges have label 0 / 1.
- The parent's node is the sum of the frequencies.

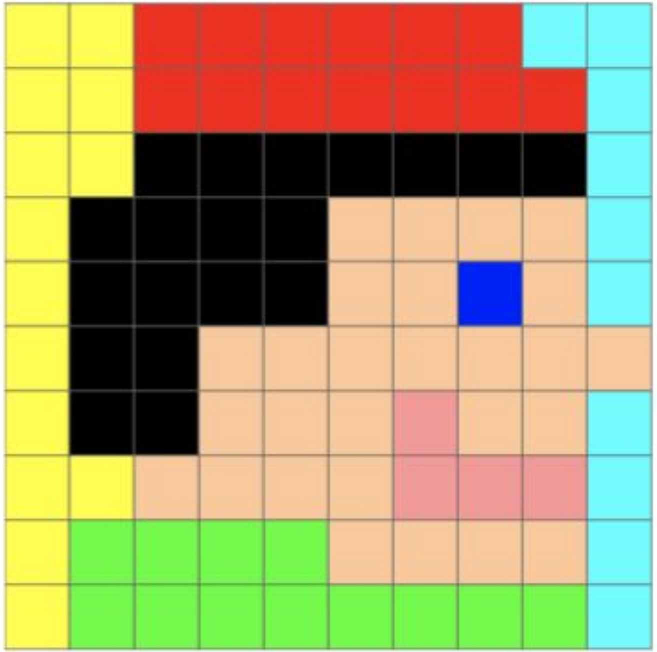


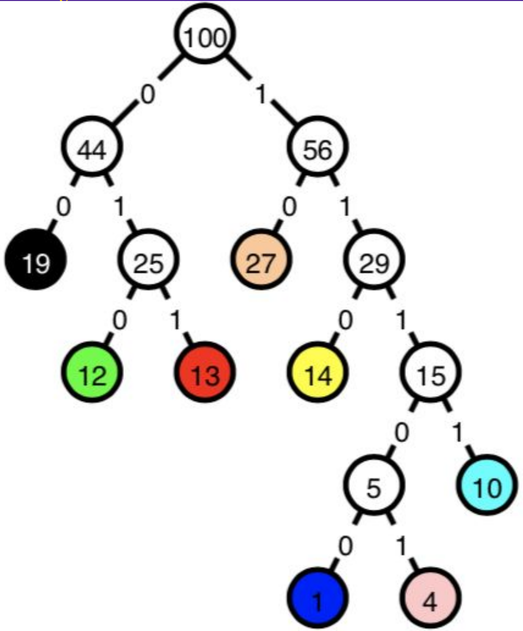
Repeatedly merge the two lowest frequency nodes under a parent node.

- The left / right edges have label 0 / 1.
- The parent's node is the sum of the frequencies.



The path to each color gives its encoding. e.g. green is 010.



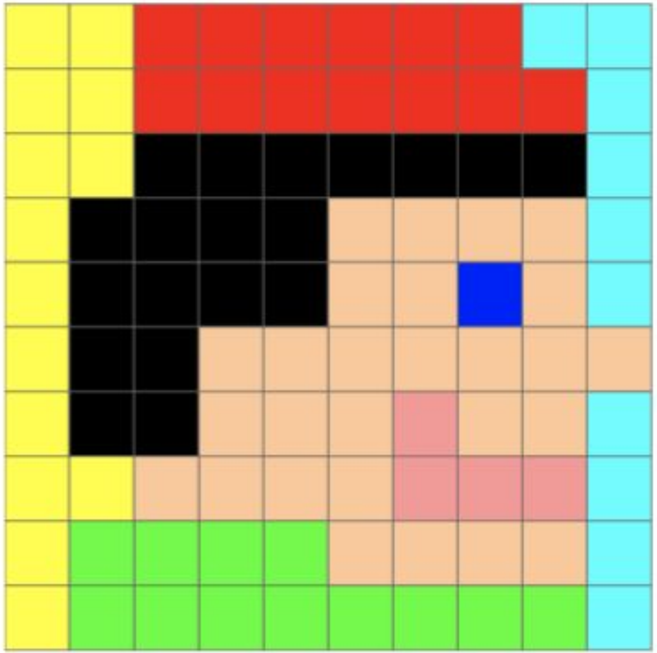


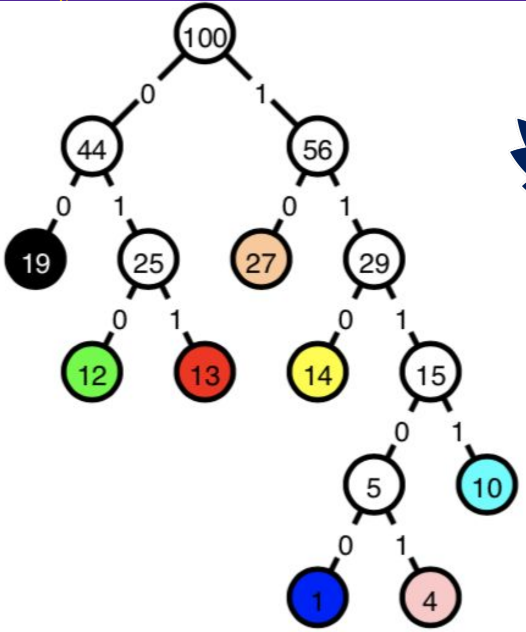
The resulting image can then be encoded as below. (Bit saving occurs for the black and peach colors.)

110 110 011 011 011 011 011 011 1111 1111 110…

Notes:

- We also need to store the codes and the image dimensions. Otherwise, this stream of colors could be interpreted as a 10-by-10 or 5-by-20 image, since $10 \cdot 10 = 5 \cdot 20 = 100$.
- This is an example of a *greedy algorithm*. You'll see many more of these in CSCI 256.





How can we be sure that the bit stream is uniquely unencodable?

110 110 011 011 011 011 011 011 1111 1111 110...

The code words satisfy the *prefix property* (i.e., no code word is the prefix of another code word). This is due to the fact that every color is stored in a leaf in this tree. For example, the codeword for yellow is `110`. Since it is in a leaf, there cannot be another code word starting with `110`.