Last Time

- Recursive Depth-First Search
  - Tips on writing recursive methods
- Graph Data Structures: Implementation
  - Graph Interface
Today’s Outline

• Graph Data Structures: Implementation
  • Adjacency Array Implementation
  • Adjacency List Implementation
    • Featuring many Iterators!
Recall: Desired Functionality

- What are the basic operations we need to describe algorithms on graphs?
  - Given vertices u and v: are they adjacent?
  - Given vertex v and edge e, are they incident?
  - Given an edge e, get its incident vertices (ends)
  - How many vertices are adjacent to v? (degree of v)
    - The vertices adjacent to v are called its neighbors
  - Get a list of the neighbors of v (or the edges incident with v)
We want to store information at vertices and at edges, but we favor vertices

- Let V and E represent the types of information held by vertices and edges respectively
- Interface Graph<V,E> extends Structure<V>
  - Vertices are the building blocks; edges depend on them
- Type V holds a label for a (hidden) vertex type
- Type E holds a label for an (available) edge type
  - Label: Application-specific data for a vertex/edge
The methods described in the Structure interface deal with *vertices*
  - but also impact edges: e.g., clear()

We’ll want to add a number of similar methods to provide information about edges, and the graph itself
Graph Interface Methods

• void add(V vLabel)
• V remove(V vLabel)
  • Add/remove vertex to graph
• void addEdge(V vLabel1, V vLabel2, E edgeLabel), E removeEdge(V vLabel1, V vLabel2)
  • Add/remove edge between vLabel1 and vLabel2
• boolean containsEdge(V vLabel1, V vLabel2)
  • Returns true iff there is an edge between vLabel1 and vLabel2
• Edge<V,E> getEdge(V vLabel1, V vLabel2)
  • Returns edge between vLabel1 and vLabel2
• void clear()
  • Remove all nodes (and edges) from graph
Graph Interface Methods

- `boolean visit(V vLabel)`
  - Mark vertex as “visited” and return previous value of visited flag
- `boolean visitEdge(Edge<V,E> e)`
  - Mark edge as “visited”
- `boolean isVisited(V vLabel), boolean isVisitedEdge(Edge<V,E> e)`
  - Returns true iff vertex/edge has been visited
- `Iterator<V> neighbors(V vLabel)`
  - Get iterator for all neighbors of vLabel
  - For directed graphs, out-edges only
- `Iterator<V> iterator()`
  - Get vertex iterator
- `void reset()`
  - Remove visited flags for all nodes/edges
Edge Class

• Graph edges are defined in their own public class
  • Edge<V,E>( V vLabel1, V vLabel2, E label, boolean directed)
  • Construct a (possibly directed) edge between two labeled vertices (vLabel1 → vLabel2)
  • vLabel1 : here; vLabel2 : there

• Useful methods:
  label(), here(), there()
  setLabel(), isVisited(), isDirected()
Recursive Depth-First Search

// Before first call to DFS, set all vertices to unvisited
// Then call DFS(G, v)

DFS(G, v)

Mark v as visited; count = 1;
for each unvisited neighbor u of v:
    count += DFS(G, u);
return count;
Recursive Depth-First Search

```java
int DFS(Graph<V,E> g, V src) {
    g.visit(src);
    int count = 1;
    Iterator<V> neighbors = g.neighbors(src);
    while (neighbors.hasNext()) {
        V next = neighbors.next();
        if (!g.isVisited(next))
            count += DFS(g, next);
    }
    return count;
}
```
Representing Graphs

• Two standard approaches
  • Option 1: Array-based (directed and undirected)
  • Option 2: List-based (directed and undirected)

• We’ll look at both
  • Array-based graphs store the edge information in a 2-dimensional array indexed by the vertices
  • List-based graphs store the edge information in a (1-dimensional) array of lists
    • The array is indexed by the vertices
    • Each array element is a list of edges incident with that vertex
Adjacency Array: Directed Graph

Entry (i,j) stores 1 if there is an edge from i to j; 0 otherwise.

E.G.: edges(B,C) = 1 but edges(C,B) = 0
**Adjacency Array: Undirected Graph**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Entry (i,j) store 1 if there is an edge between i and j; else 0

E.G.: \( \text{edges}(B,C) = 1 = \text{edges}(C,B) \)
The vertices are stored in an array $V[]$
$V[]$ contains a linked list of edges having a given source
The vertices are stored in an array $V[]$
$V[]$ contains a linked list of edges incident to a given vertex
Graph Classes in structure5

- Interface
- Abstract Class
- Class

Structure

Graph

- GraphMatrix
  - GraphMatrixDirected
  - GraphMatrixUndirected
- GraphList
  - GraphListDirected
  - GraphListUndirected

Vertex

- GraphMatrixVertex
- GraphListVertex

Edge
**Graph Classes in structure**

Why so many?!  
- There are two types of graphs: undirected & directed  
- There are two implementations: arrays and lists  
- We want to be able to avoid large amounts of identical code in multiple classes  
- We abstract out features of implementation common to both directed and undirected graphs

We’ll tackle array-based graphs first....
### Adjacency Array: Directed Graph

#### Entry (i,j) stores 1 if there is an edge from i to j; 0 otherwise

E.G.: $\text{edges}(B,C) = 1$ but $\text{edges}(C,B) = 0$

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>H</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Adjacency Array: Undirected Graph

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Entry \((i,j)\) store 1 if there is an edge between \(i\) and \(j\); else 0

E.G.: \(\text{edges}(B,C) = 1 = \text{edges}(C,B)\)
Adjacency Array: Undirected Graph

Halving the Space (not in structure5)

(i,j) maps to i*7+j
We need to define a Vertex class

- Unlike the Edge class, Vertex class is not public
- Useful Vertex methods:
  - V label(), boolean visit(),
  - boolean isVisited(), void reset()

- GraphMatrixVertex class adds one more useful attribute to Vertex class
  - Index of node (int) in adjacency matrix
    - int index()
  - Why do we only need one int to represent index?

- In these slides, we write GMV for GraphMatrixVertex
Choosing a Dictionary Structure

• We need a structure that will let us retrieve the index of a vertex given the vertex label (a dictionary)

• Many choices
  • Vector of associations:
    • Vector<Association<V, GraphMatrixVertex<V>>>
  • Ordered Vector of Associations
  • BinarySearchTree of Associations

• Problem: We don’t want to allow multiple vertices with same label.... [Why?]

• We’ll use the Map Interface [Chapter 15]
  • Maps require a unique key for each entry
Digression: Map Interface

- **Methods for Map<K, VAL>**
  - `int size()` - returns number of entries in map
  - `boolean isEmpty()` - true iff there are no entries
  - `boolean containsKey(K key)` - true iff key exists in map
  - `boolean containsValue(VAL val)` - true iff val exists at least once in map
  - `VAL get(K key)` - get value associated with key
  - `VAL put(K key, VAL val)` - insert mapping from key to val, returns value replaced (old value) or null
  - `VAL remove(K key)` - remove mapping from key to val
  - `void clear()` - remove all entries from map

- We’ll study this more in a week or so....
Implementing the Matrix Model

• Abstract class – partially implements Graph
  
  ```java
  public abstract class GraphMatrix<V,E> implements Graph<V,E>
  ```

• This class will implement features common to directed and undirected graphs

• Instance variables
  
  ```java
  protected int size;  //max size of matrix
  protected Object data[][];  //matrix of edges
  protected Map<V, GMV<V>> dict;  //labels -> vertices
  // This is structure5.Map, NOT java.util.Map!
  protected List<Integer> freeList;  //avail indices
  protected boolean directed;
  ```
GraphMatrix Constructor
(Yes, abstract classes can have constructors!)

protected GraphMatrix(int size, boolean dir) {
    this.size = size; // set maximum size
    directed = dir; // fix direction of edges

    // the following constructs a size x size matrix
    // (the “Objects” will be “Edges”)
    // (can’t use generics with arrays!)
    data = new Object[size][size];

    // label→index translation table
    dict = new Hashtable<V,GraphMatrixVertex<V>>(size);

    // put all indices in the free list
    freeList = new SinglyLinkedList<Integer>();
    for (int row = size-1; row >= 0; row--)
        freeList.add(new Integer(row));
}
public void add(V label) {
    // if there already, do nothing
    if (dict.containsKey(label)) return;

    Assert.pre(!freeList.isEmpty(), "Matrix not full");
    // allocate a free row and column
    int row = freeList.removeFirst().intValue();
    // add vertex to dictionary
    dict.put(label, new GraphMatrixVertex<V>(label, row));
}
public V remove(V label) {
    // find and extract vertex
    GraphMatrixVertex<V> vert;
    vert = dict.remove(label);
    if (vert == null) return null;
    // remove vertex from matrix
    int index = vert.index();
    // clear row and column entries
    for (int row=0; row<size; row++) {
        data[row][index] = null;
        data[index][row] = null;
    }
    // add node index to free list
    freeList.add(new Integer(index));
    return vert.label();
}
Neighbors Iterator : GraphMatrix

neighbors Iterator

public Iterator<V> neighbors(V label) {
    GraphMatrixVertex<V> vert = dict.get(label);
    List<V> list = new SinglyLinkedList<V>();
    for (int row=size-1; row>=0; row--) {
        Edge<V,E> e = (Edge<V,E>)data[vert.index()][row];
        if (e != null) {
            if (e.here().equals(vert.label()))
                list.add(e.there());
            else
                list.add(e.here());
        }
    }
    return list.iterator();
}