This is a *closed book* exam. You have 90 minutes to complete the exam. All intended answers will fit in the space provided. You may use the back of the preceding page for additional space if necessary, but be sure to mark you answers clearly.

Be sure to give yourself enough time to answer each question— the points should help you manage your time.

In some cases, there may be a variety of implementation choices. The most credit will be given to the most elegant and efficient solutions.

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I have neither given nor received aid on this examination.

Signature: ____________________________

Name: ____________________________
1. (14 points) ................................................................. True/False
   Justify each answer with a sentence or two.
   a. Two instances of class Association in the structure package are equal if and only if their keys are equal, regardless of their values.

   b. An instance variable declared as protected can be accessed by any method of the class in which it is declared.

   c. A binary search can locate a value in a sorted Vector in \( O(\log n) \) time.

   d. A binary search can locate a value in a sorted SinglyLinkedList in \( O(\log n) \) time.

   e. A method with no precondition should return with its postcondition true every time it is called.

   f. The Unix command cp /path/to/directory changes your current working directory to /path/to/directory.

   g. Instance variables can be specified in an interface file.
Consider the following Java program:

```java
class Container {
    protected int count;
    protected static int staticCount;

    public Container(int initial) {
        count = initial;
        staticCount = initial;
    }

    public void setValue(int value) {
        count = value;
        staticCount = value;
    }

    public int getCount() {
        return count;
    }

    public int getStaticCount() {
        return staticCount;
    }
}

class WhatsStatic {

    public static void main(String[] args) {
        Container c1 = new Container(17);
        System.out.println("c1 count=" + c1.getCount() + ", staticCount=" + c1.getStaticCount());

        Container c2 = new Container(23);
        System.out.println("c1 count=" + c1.getCount() + ", staticCount=" + c1.getStaticCount());
        System.out.println("c2 count=" + c2.getCount() + ", staticCount=" + c2.getStaticCount());

        c1.setValue(99);
        System.out.println("c1 count=" + c1.getCount() + ", staticCount=" + c1.getStaticCount());
        System.out.println("c2 count=" + c2.getCount() + ", staticCount=" + c2.getStaticCount());

        c2.setValue(77);
        System.out.println("c1 count=" + c1.getCount() + ", staticCount=" + c1.getStaticCount());
        System.out.println("c2 count=" + c2.getCount() + ", staticCount=" + c2.getStaticCount());
    }
}
```

Answer the following questions (next page) about this code.
a. What will the output be when the program is run (`java WhatsStatic`)? Assume no exceptions occur. (4 points)

b. What memory is allocated for `Containers c1 and c2` at the time the line `c1.setValue(99)` is executed? Show any existing local variables and instance variables. (6 points)
3. (26 points) ........................................................................................................... Creating a Set class

In this problem you are to design a Java interface and class for a data structure which represents sets of Strings. As usual for sets, no repeated elements are allowed. Thus, the collection "Propser", "Any", "Lisa", "Karl", "Isabella" is a legal set, but "Bill", "Duane", "Bill" is not. This data structure will have two methods:

- void insert(String myString) adds myString to the set.
- boolean contains(String myString) returns a boolean value indicating if myString is an element of the set.

a. Write a legal Java interface called StringSetInterface for this data structure. Include preconditions and postconditions for the methods. (6 points)
b. Suppose we decide to implement `StringSetInterface` by a class in which a singly-linked list holds the elements. Write the definition of this class. This should be a full and legal Java class definition with all method bodies filled in. Don't forget to declare instance variables, include a constructor, and use qualifiers such as `public` and `protected` when appropriate. You need not repeat your pre- and post-conditions from part a. Please call your class `StringSet`. (10 points)
c. If StringSet is implemented as in part b, what would the worst-case time complexity be for the insert operation when the set has $n$ elements? (Use “Big O” notation.) (4 points)

d. Suppose we design an alternative implementation in which the set is represented by a Vector<String> called strVec. What is the worse-case complexity of insert with this representation? (6 points)
4. (20 points) Recursion on Lists

(20 points) Consider the following class, `ReversibleList`, that extends the `SinglyLinkedList` class by adding a method for reversing the list.

```java
public class ReversibleList<E> extends SinglyLinkedList<E> {

    public ReversibleList() {
        super();
    }

    public void reverse() {
        // Post: list is reversed.
        if (head != null) head = recReverse(head);
    }

    private static SinglyLinkedListNode<E> recReverse(SinglyLinkedListElement<E> current) {
        // Pre: current is not null.
        // Post: list headed by current is reversed; and first Node in that list is returned.
        if (current.next() == null) // Single-node list
            return current;
        else {
            SinglyLinkedListElement<E> newHead = recReverse(current.next()); // Explain
            // current.next() now points to final node in reversed list!
            current.next().setNext(current); // Explain
            current.setNext(null); // Explain
            return newHead;
        }
    }
}
```

a. Prove by induction that `reverse()` behaves correctly. (Hint: focus on `recReverse(current)`)
Include a brief explanation of each step labeled with the "Explain" comment. (8 points).
b. What is the running time of reverse() (2 points)?

c. Prove using mathematical induction that your answer to part b is correct. (10 points)
5. (12 points) ........................................................................... Big-O

Growth of functions. Using “Big O” notation, give the rate of growth for each of these functions. Your answer should represent the tightest bound possible and should be in as simple a form as possible. Justify your answers. (3 points each, 12 total)

a. \( f(n) = n^2 + 17n + 2001 \)

b. \( f(n) = 3n + 5 \log_2 n \)

c. \( f(n) = 7n \) when \( n \) is odd, \( f(n) = \frac{n}{2} \) when \( n \) is even.

d. \( f(n) = 5n^3 \) for \( n < 23 \), \( f(n) = 37 \) otherwise.
6. (13 points) ............................................. Searching and Sorting

(a) (5 points) SelectionSort and Insertion both take \(O(n^2)\) in the worst case. However, they have different best-case running times. Explain why this difference occurs; include a description of examples that have best-case performance.

(b) (8 points) When applied to an array, a MergeSort has three phases:

- **Split**: Find the middle element of the array
- **Recursively Solve**: MergeSort each half of the array
- **Combine**: Merge the two sorted halves of the array into a single sorted array

As we’ve seen, the Split phase takes \(O(1)\) time while the Combine phase takes \(O(n)\) time. Suppose we want to implement MergeSort for a SinglyLinkedList data structure (with tail pointers). Describe what would be involved in implementing the Split and Combine phases and how much time (in the \(O()\) worst-case sense) each phase would take. Would such a MergeSort still take \(O(n \log n)\) time? Why?