The first three questions on this assignment should be completed as group work. One copy of the solution to each problem should be submitted by noon on Wednesday or Thursday, 5/6 or 5/6 depending on your group's meeting time.

The final question on this assignment should be submitted independently by each student by Tuesday 5/5. You may discuss approaches to the question with members of your working group, but the final writeup you submit should represent your own work in the same sense that was expected for all homework submissions during the first half of the semester.

1. Consider the language
   $SUBSET_{DFA}^{CFL} =$
   $$\{\langle G, M\rangle \mid G \text{ is a CFG and } M \text{ is a DFA and } L(M) \subset L(G)\}$$
   Determine whether this language is decidable, not decidable but recognizable, not decidable but its complement is recognizable or neither it nor its complement is recognizable. Precisely justify your conclusion for the category you choose by giving an algorithm for deciding or recognizing the language and/or a reduction argument.

2. Consider the two languages
   $TOTAL_{TM} =$
   $$\{\langle M\rangle \mid M \text{ is a TM and for all } w \in \Sigma_M^*, M \text{ halts on input } w\}$$
   $TOTAL_{LBA} =$
   $$\{\langle M\rangle \mid M \text{ is a LBA and for all } w \in \Sigma_M^*, M \text{ halts on input } w\}$$
   Determine whether each of these languages is decidable, not decidable but recognizable, not decidable but its complement is recognizable or neither it nor its complement is recognizable. Precisely justify your conclusion for the category you choose by giving an algorithm for deciding or recognizing the language and/or using a mapping reduction induced by a computable function.

3. Consider the language
   $CONTAINSPAL_{CFG} =$
   $$\{\langle G\rangle \mid G \text{ is a CFG and } ww^R \in L(G) \text{ for some } w \in \Sigma_G^*\}$$
   Determine whether this language is decidable, not decidable but recognizable, not decidable but its complement is recognizable or neither it nor its complement is recognizable. Precisely justify your conclusion for the

category you choose by giving an algorithm for deciding or recognizing the language and/or a reduction argument.

Hint/warning: I believe that this problem is quite challenging. Do your best to make progress on it, but do not panic if you cannot precisely categorize this language.

4. Show that a language L is Turing-recognizable if and only if $L \leq_M A_{TM}$.