1. Let \( b(n) \) denote the binary representation of \( n \) without leading 0s (e.g., \( b(10) = 1010 \)). Let \( b(n)^R \) denote the reversal of such a string.

Show that \( A = \{ b(n)^R # b(n + 1) \mid n \geq 1 \} \) is not a regular language. Provide two proofs of this fact, one using the Myhill-Nerode theorem and one using the Pumping Lemma.

2. Consider the DFA \( D = (\{1, \ldots, 8\}, \{a, b\}, \delta, 1, \{1, 8\}) \) with \( \delta \) described by the following table:

<table>
<thead>
<tr>
<th>( q )</th>
<th>( \delta(q, a) )</th>
<th>( \delta(q, b) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

Using the algorithm described in class:

(a) Determine which states of the machine are equivalent. Show your work. That is, produce one or more tables of the form

```
 1
 ▼ ▼ ▼
 2 ▼ ▼ ▼
 ▼ ▼ ▼ ▼
 3 ▼ ▼ ▼
 ▼ ▼ ▼ ▼ ▼
 4 ▼ ▼ ▼
 ▼ ▼ ▼ ▼ ▼
 5 ▼ ▼ ▼
 ▼ ▼ ▼ ▼ ▼
 6 ▼ ▼ ▼
 ▼ ▼ ▼ ▼ ▼
 7 ▼ ▼ ▼
 ▼ ▼ ▼ ▼ ▼
 8
```

with increasingly many \( \neq \)s marking pairs of states that are known to be non-equivalent and show those tables in your solution.

(b) Draw the state diagram for the equivalent minimal DFA for \( L(D) \).

3. Complete exercise 2.13 from Sipser (as modified below):

Let \( G = (V, \Sigma, R, S) \) be the following grammar. \( V = \{S, T, U\}; \Sigma = \{0, \#\}; \) and \( R \) is the set of rules:

\[
\begin{align*}
S & \rightarrow TT \mid U \\
T & \rightarrow 0T \mid T0 \mid \# \\
U & \rightarrow 0U00 \mid \#
\end{align*}
\]

(a) Describe \( L(G) \) in English. (Note: A mix of English and extended regular expression notation may be more effective.)

(b) Prove that \( L(G) \) is not regular using the Myhill-Nerode Theorem.

Due: 12:00. October 12, 2018