1. Let $b(n)$ denote the binary representation of $n$ without leading 0s (e.g., $b(10) = 1010$). Let $b(n)^R$ denote the reversal of such a string.

Show that $A = \{b(n)^R\#b(n + 1) \mid n \geq 1\}$ is not a regular language. Provide two proofs of this fact, one using the Myhill-Nerode theorem and one using the Pumping Lemma.

2. Consider the DFA $D = (\{1, \ldots, 8\}, \{a, b\}, \delta, 1, \{1, 8\})$ with $\delta$ described by the following table:

<table>
<thead>
<tr>
<th>$q$</th>
<th>$\delta(q, a)$</th>
<th>$\delta(q, b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

Using the algorithm described in class:

(a) Determine which states of the machine are equivalent. Show your work. That is, produce one or more tables of the form

```
1  2
3  4
5  6
7  8
```

with increasingly many Xs marking pairs of states that are known to be non-equivalent and show those tables in your solution.

(b) Draw the state diagram the equivalent minimal state DFA for $L(D)$.

3. (From D. Kozen) For any set $A$ of natural numbers, define

**binary** $A = \{w \mid w \in \{0, 1\}^* \text{ and the number represented by } w \text{ in binary is an element of } A \}$

**unary** $A = \{1^n \mid n \in A\}$

For example, if $A = \{4, 9, 16\}$, then

**binary** $A = \{100, 1001, 10000\}$

**unary** $A = \{1111, 11111111, 1111111111111111\}$

Due: 12:00, October 13, 2017
Consider the following two propositions:

- For all $A$, if binary $A$ is regular, then so is unary $A$.
- For all $A$, if unary $A$ is regular, then so is binary $A$.

One of these statements is true and the other is false. Which is which? Justify your claims by either providing a proof or a counterexample.

Some hints:

- Think about closure properties. If you can describe a language as a union or intersection of a finite collection of languages that are clearly regular, then the language is regular.
- When thinking about DFAs for unary languages, consider what happens if you limit yourself to examples of DFAs that only contain a single final state.

4. Complete exercise 2.13 from Sipser (as modified below):

Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}; \Sigma = \{0, \#\};$ and $R$ is the set of rules:

- $S \rightarrow TT \mid U$
- $T \rightarrow 0T \mid T0 \mid \#$
- $U \rightarrow 0U00 \mid \#$

a. Describe $L(G)$ in English. (Note: A mix of English and extended regular expression notation may be more effective.)

b. Prove that $L(G)$ is not regular using the Myhill-Nerode Theorem.