The problems on this assignment ask you to determine whether a language described in the problem is or is not regular. In these questions, if you believe the language in question is regular you should justify this belief by either describing and explaining a DFA, NFA or regular expression that would correspond to the language and/or using the closure properties of regular languages. If you conclude that a language is not regular, use the Pumping Lemma, the closure properties of regular languages, or the Myhill-Nerode Theorem to show that it is not regular. Be clear but concise in either case.

1. Is \( L_{2^n} = \{ 1^k \mid k \text{ is a power of 2} \} \) regular?

2. Consider the languages:
   - \( L_{\text{least}} = \{ 1^k y \mid y \in \{0,1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for } k \geq 1 \} \)
   - \( L_{\text{most}} = \{ 1^k y \mid y \in \{0,1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1 \} \)
   At least one of these two languages is not regular. Identify one that you believe is not regular and justify that this is the case.

3. Consider the languages:
   - \( L_1 = \{ \# w_1 \# \ldots \# w_m \# \mid w_i \in \{0,1\}^n, n > 1 \text{ and } \exists j, k \ w_j \neq w_k \} \)
   - \( L_2 = \{ \# w_1 \# \ldots \# w_n \# \mid w_i \in \{0,1\}^m, n > 1 \text{ and } \exists j, k \ w_j \neq w_k \} \)
   where in both cases \( m \) is a fixed constant greater than 1 (i.e., your argument should not rely on any assumptions about \( m \) other than that it is fixed) and \( \{0,1\}^k \) is meant to describe the set of all binary strings of length \( k \). As in an earlier problem, the \( \# \) is not a special operator. It is just a symbol in the alphabet of the languages.
   One of these languages is not regular and the other is regular. Identify which is which and justify your belief. Hint: Although the introduction to this problem set says you can describe a DFA or NFA to show a language is regular, for this problem I would highly recommend you focus on the use of regular expressions and closure properties instead.

4. For any language \( L \) over alphabet \( \Sigma \), consider the derived languages
   \[ L_{-\frac{1}{2}} = \{ w \mid \text{for some } x, y \in \Sigma^*, |x| = |y| = |w| \text{ and } xyw \in L \} \]
   \[ L_{-\frac{1}{3}} = \{ w \mid \text{for some } x, y \in \Sigma^*, |x| = |y| = |w| \text{ and } xwy \in L \} \]
   \[ L_{\frac{1}{2}} = \{ w \mid \text{for some } x, y, z \in \Sigma^*, |x| = |y| = |z|, w = xz \text{ and } xyz \in L \} \]
   (These transformations should be familiar from last week’s assignment.)
   Regular languages are closed under at least one of these three transformations. At the same time, regular languages are not closed under at least one of these transformations. Identify one of the transformations that does not preserve regularity and justify your claim that regular languages are not closed under this transformation by identifying a regular language
such that performing the transformation you have identified produces a non-regular language. Your answer should include formal descriptions of the language $L$ and the language into which it is transformed. If the transformed language is not a language we have already shown to not be regular, justify your claim that it is not a regular language.