1. On page 297, Sipser briefly mentions the class coNP. coNP is the class of complements of problems in NP. For example, since the subset sum problem is in NP, its complement, \( \{ \{x_1, \ldots, x_k \}, N \} \mid \text{all } x_i \text{ and } N \text{ are integers and no subset of the } x_i\text{s adds up to } N \} \) is in coNP. Sipser also mentions that it is unknown whether NP and coNP are different or the same.

A problem \( C \) is said to be coNP-complete if it is in coNP and for any problem \( B \in \text{coNP} \), \( B \leq_p C \).

(a) Show that if NP \( \neq \) coNP, then P \( \neq \) NP. (Hint: Recall that P is closed under complement.)

(b) Show that if \( X \) is NP-complete then \( \overline{X} \) is coNP-complete.

2. Given a family of sets of integers \( S_1, S_2, \ldots, S_n \), a subfamily of sets \( S_{i_1}, S_{i_2}, \ldots, S_{i_k} \) such that

\[
\bigcup_{j=1}^{k} S_{i_j} = \bigcup_{j=1}^{n} S_j
\]

is called a k-set cover for \( \{ S_1, S_2, \ldots, S_n \} \). Show that the language

\[
 \text{SCOVER} = \{ \langle S_1, \ldots, S_n, k \rangle \mid \text{there is a k-set cover for } \{ S_1, \ldots, S_n \} \}
\]

is NP-complete by showing that this language is in NP and showing that 3-SAT \( \leq_p \) SCOVER.

3. Complete problem 7.29 in Sipser. As an alternative to the reduction suggested by the hints in the problem, you should assume the solution to problem 7.26 (which was assigned in the last homework) and show that 3COLOR is NP-complete using a reduction from \( \neg\text{-SAT} \).