1. On page 297, Sipser briefly mentions the class coNP. coNP is the class of complements of problems in NP. For example, since the subset sum problem is in NP, its complement, \{\{x_1, \ldots, x_k\}, N\} \mid \text{all } x_i \text{ and } N \text{ are integers and no subset of the } x_i\text{s adds up to } N \} is in coNP. Sipser also mentions that it is unknown whether NP and coNP are different or the same.

A problem \(C\) is said to be coNP-complete if it is in coNP and for any problem \(B \in \text{coNP}\), \(B \leq_p C\).

(a) Show that if \(\text{NP} \neq \text{coNP}\), then \(\text{P} \neq \text{NP}\). (Hint: Recall that P is closed under complement.)

(b) Show that if \(X\) is NP-complete then \(\overline{X}\) is coNP-complete.

2. Given a family of sets of integers \(S_1, S_2, \ldots, S_n\), a subfamily of sets \(S_{i_1}, S_{i_2}, \ldots, S_{i_k}\) such that
\[
\bigcup_{j=1}^{k} S_{i_j} = \bigcup_{j=1}^{n} S_j
\]
is called a k-set cover for \(\{S_1, S_2, \ldots, S_n\}\). Show that the language
\[
\text{SCOVER} = \{(S_1, \ldots, S_n, k) \mid \text{there is a k-set cover for } \{S_1, \ldots, S_n\}\}
\]
is NP-complete by showing that this language is in NP and showing that 3-SAT \(\leq_p\) SCOVER.

3. Complete problem 7.29 in Sipser. (Hint: as an alternative to or in conjunction with Sipser’s hint, you are free to assume the solution to problem 7.26 (which was assigned in the last homework).