

## Administrative Details

- Lab 8 - due Monday
- Any questions?


## Last Time

- Wrapped up decision trees
- Discussed tree traversal
- Looked closely at code for pre-order

```
public E next() {
    BinaryTree top = todo.pop();
    E result = top.value();
    if(!top.right().isEmpty()) {
        todo.push(top.right());
    }
        (!top.left().isEmpty())
        todo.push(top.left());
    }
    return result;
}
```

            if (root.isEmpty()) return;
            process(root.value());
            preOrder(root.left())
            preOrder(root.right());
        \}
    - In real code, we need to keep track of our own stack!


## Today's Outline

- Finish discussing tree iterators
- In-order, level-order, post-order
- Wrap up chapter 12 (Binary Trees) and start chapter 13 (Priority Queues)
- Briefly discuss Huffman codes


## Tree Traversal Recap

- Pre-order: +*237
- Each node is visited before any children. Visit node, then each node in left subtree, then each node in right subtree.
- In-order: 2*3+7
- Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.
- Post-order: 23*7+
- Each node is visited after its children are visited. Visit all nodes in left subtree, then all nodes in right subtree, then node itself.
- Level-order: +*723
- All nodes of level $i$ are visited before nodes of level $i+1$.


## InOrder Iterator

- Outline: left - node - right
I. Push left children (as far as possible) onto todo stack

2. On call to next():

- Pop node from stack
- Push right child and follow left children as far as possible
- Return node's value

3. On call to hasNext():

- return !stack.isEmpty)



## InOrder Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.


B

## InOrder Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.


B G


## InOrder Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.


B G I


## InOrder Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.


Code?

| Level-order <br> - Let's take a closer look at LevelOrder... <br> - Level-order: +*723 <br> - All nodes of level i are visited before nodes of level $\mathrm{i}+\mathrm{I}$. |
| :---: |
|  |  |

## LevelOrder Iterator

- Do we want to use a stack??
- No! Use a queue instead.
- Outline
I. Enqueue root

2. On call to next():

- Dequeue node
- Enqueue left and right child
- Return node

3. On hasNext()

- Return !queue.isEmpty()




## PostOrder Iterator

- Left as an exercise...

Moving on...

- Note:
- Code for PostOrder is similar to PreOrder with minor differences
- Please see Bailey for details (preferably before your midterm!)


## An Aside: Tree Search Strategies

- Two main approaches
- Breadth-first search (BFS)
- Search across tree before searching down to another level
- Level-order traversal
- Depth-first search (DFS)
- Search down tree (to leaf) before search across tree
- Pre-order traversal
- DFS is more efficient if solution is "far away" from root (i.e., many edges between root and solution)


## Next up: Huffman Codes

- Normally, I character = 8 bits (I byte)
- Allows for $2^{8}=256$ different characters
- ‘A’ = 0100000I, ‘B’ = 01000010
- Space to store "AN ANTARCTIC PENGUIN"
- 20 characters -> $20 * 8$ bits $=160$ bits
- Is there a better way?
- Only II symbols are used (ANTRCIPEGU_)
- Only need 4 bits per symbol (since $2^{4}>1$ I)! - $20 * 4=80$ bits instead of 160 !
- Can we still do better??


## Huffman Codes

- General idea
- Use less bits for most common letters
- AN ANTARCTIC PENGUIN
- Compute letter frequencie

| $\mathrm{A}: 3$ | $\mathrm{~N}: 4$ |  |
| :--- | :--- | :--- |
| $\mathrm{~T}:$ | 2 | $\mathrm{R}:$ |

C: $2 \quad \mathrm{I}:$
P: $1 \quad \mathrm{E}: ~ 1$
: 2 U:

- Build tree by recursively creating trees of smallest weighted components


## How Many Bits?

A: $100 \times 3$ N: 101 x 4

T: 001 x 2 R: 0000 x
C: $010 \times 2$ I: $011 \times 2$
P: $0001 \mathrm{x} 1 \quad \mathrm{E}: 1100 \mathrm{x} 1$
$\mathrm{G}: 1101 \mathrm{x} 1 \quad \mathrm{U}: 1110 \mathrm{x} 1$
_: 1111 x 2

- So total number of bits $=67$
- Note: There may be multiple possible Huffman trees
- All trees should use same total number of bits


## Other Compression Techniques

- Examine larger pieces of data for patterns
- AAAAA BBBBBBBBB CC AAAAAAA
- $(5, \mathrm{~A})(9, B)(2, C)(7, A)$
- Lempel-Ziv-Welch (LZW)
- Huffman code for longer substrings
- ABCABCABC
- 0-255: ASCII characters
- 256: AB
- 257: ABC

Alternative Tree Representations


- Total \# "slots" = 4n
- Since each BinaryTree maintains a reference to left, right, parent, value
- Much more overhead than vector, SLL, array, ...
- But trees capture successor and predecessor relationships that other data structures don't...


## Using Arrays to Store Trees

- Encode structure of tree in array indexes
- Where are children of node i?
- Children of node $i$ are at $2 i+1$ and $2 i+2$
- Look at example
- Where is parent of node $j$ ?
- Parent of node j is at $(\mathrm{j}-\mathrm{I}) / 2$


## ArrayTree Tradeoffs

- Why are ArrayTrees good?
- Save space for links
- No need for additional memory allocated/garbage collected
- Works well for full or complete trees
- Complete: All levels except last are full and all gaps are at right
- "A complete binary tree of height h is a full binary tree with 0 or more of the rightmost leaves of level $h$ removed"
- Why bad?
- Could waste a lot of space
- Height of $n$ requires $2^{n+1}-I$ array slots even if only $O(n)$ elements

