CSCI 136
Data Structures &
Advanced Programming

Jeannie Albrecht
Lecture 23
April 16, 2014

Administrative Details

- Lab 8 is today
  - Can work with a partner again
  - We'll briefly go over design in lab
  - Faculty meeting at 4 today
- Looking ahead:
  - Lab 9 – Darwin, due 5/7 (2 weeks)
  - Wed 4/30: Midterm 2 (during lab again)
- One (or two) more labs after that (last one is probably optional)
- Office hours on Thursday: 2ish – 3:30ish

Last Time

- Looked at ways to prove tree properties using induction
- Started discussing decision trees

BT Questions/Proofs

- (A) Prove that number of nodes at level n <= 2^n.
- (B) Prove that number of nodes in tree of height n is <= $2^{(n+1)} - 1$.
  - Base case n=0: Tree of height = 0 only contains root. Thus only 1 node when height=0.
    - $2^{(0+1)} - 1 = 1$. Base case holds.
  - IH: Assume true for all k<n.
    - That is, the number of nodes in tree of height k is <= $2^{(k+1)} - 1$
  - IS: Suppose k=n – 1. (We will show it holds for k=n.)
    - By our IH, we know that the number of nodes is <= $2^{(n)} - 1$.
    - By (A), we also know that the number of nodes at level n <= 2^n.
    - So at height n, the number of nodes in tree is at most (<=) $2^{(n)} + 2^{(n)} - 1 = 2 \times 2^{(n)} - 1 = 2^{(n+1)} - 1$.

Today's Outline

- Continue discussing decision trees
- Learn about tree traversal
  - In-order, pre-order, post-order, level-order
  - Learn how to implement tree iterators

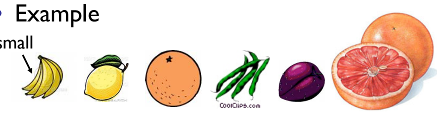
Recap: Representing Knowledge

- Trees can be used to represent knowledge
  - Example: InfiniteQuestions game
- We often call these trees **decision trees**
  - Leaf: object
  - Internal node: question to distinguish objects
- Move down decision tree until we reach a leaf node
- Check to see if the leaf is correct
  - If not, add another question, make new and old objects children

Building Decision Trees

- Gather/obtain data
- Run correlation analysis
  - Make greedy choices: Find good questions that divide data into halves (or as close as possible)
- Construct tree with shortest height
- Example

small



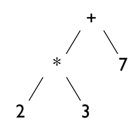
large and yellow

Moving on…

Tree Traversals

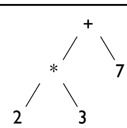
- In linear structures, there are only a few logical (useful) ways to traverse the data structure
  - Start at one end and visit each element
  - Start at the other end and visit each element
- How do we traverse binary trees?
  - (At least) four potential mechanisms

Tree Traversals



- In-order: 2*3+7
- Pre-order: +*237
- Post-order: 23*7+ (look familiar?)
- Level-order: +*723

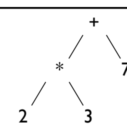
Tree Traversals



- Pre-order
  - Each node is visited before any children. Visit node, then each node in left subtree, then each node in right subtree. (node, left, right)
    - +*237
- In-order
  - Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree. (left, node, right)
    - 2*3+7

(Look at "pseudocode")

Tree Traversals



- Post-order
  - Each node is visited after its children are visited. Visit all nodes in left subtree, then all nodes in right subtree, then node itself. (left, right, node)
    - 23*7+
- Level-order (not recursive!)
  - All nodes of level i are visited before nodes of level i+1. (visit nodes left to right on each level)
    - +*723

(Look at "pseudocode")

Iterators

- We need to provide iterators that implement the different tree traversal algorithms
- Methods provided by BT class:
  - preorderIterator()
  - inorderIterator()
  - postorderIterator()
  - levelorderIterator()

PreOrder Iterator

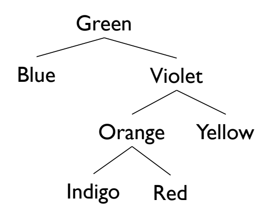
- Basic idea
  - Should return elements in same order as processed by pre-order traversal method
  - Recursive method won't work for iteration, must phrase in terms of next() and hasNext()
  - But we "simulate recursion" with stack
    - Maintain list of subtrees left to traverse
    - Todo stack: Roots of trees left to process
  - Stack is *frontier*: nodes left to traverse

PreOrder Iterator

- Outline: node - left - right
  1. Push root onto todo stack
  2. On call to next():
     - Pop node from stack
     - Push right and then left nodes of popped node onto stack
     - Return node's value
  3. On call to hasNext():
     - return !stack.isEmpty()

PreOrder Iterator

Visit node, then each node in left subtree, then each node in right subtree.



PreOrder Iterator

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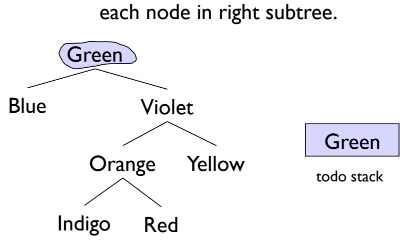

PreOrder Iterator

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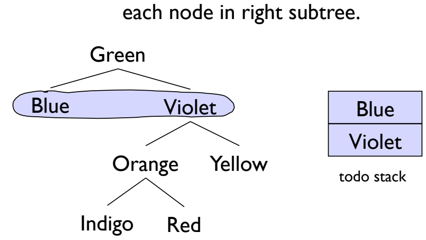

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PreOrder Iterator

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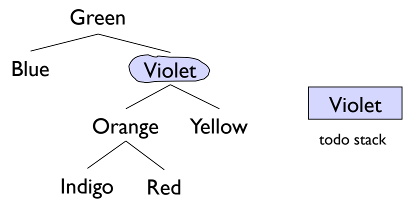

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PreOrder Iterator

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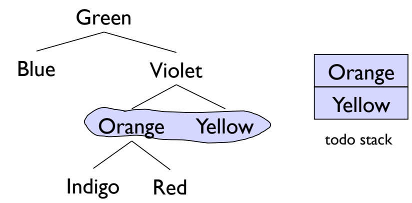

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PreOrder Iterator

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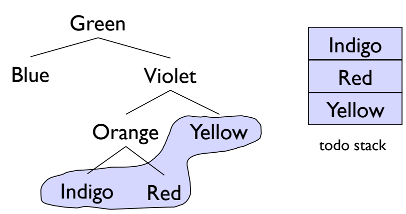

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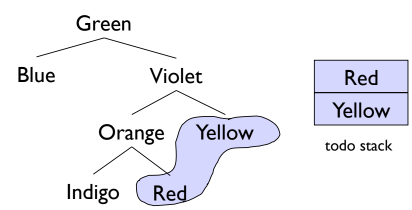

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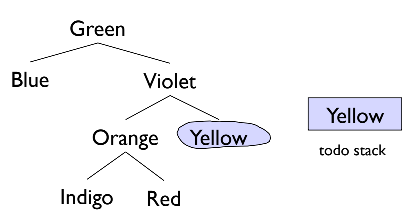

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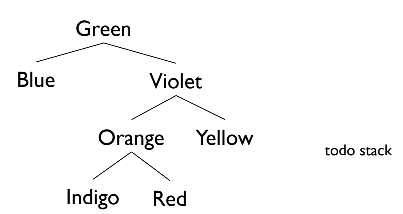
PreOrder Iterator

Visit node, then each node in left subtree, then each node in right subtree.



G B V O I R Y

Now let's look at the code…