CSCI 136 Data Structures & Advanced Programming

Jeannie Albrecht Lecture 10 Mar 3, 2014

Administrative Details

- Lab 3 is due today at noon
 - Run-time (big-O notation) in comments above methods
 - Don't forget to define "n" if you say O(n)
- I have to cancel office hours today...sorry! $\ensuremath{\textcircled{\ensuremath{\Theta}}}$
- I might be in tomorrow afternoon instead
- Check your email

Last Time

- Discussed runtime analysis techniques
- Reviewed and discussed recursion
 - Looked at factorial in class
 - Looked at digit sum and subset sum in lab

Today's Outline

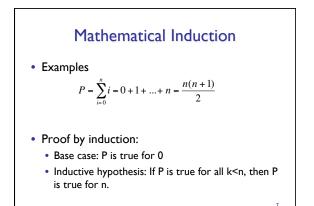
- Begin reviewing mathematical induction
- Begin learning about searching and sorting
 Two of the most important classes of algorithms
- We'll discuss two searches:
 - Linear search
 - Binary search

Recursion Tradeoffs

- Advantages
 - Often easier to construct recursive solution
 - Code is usually cleaner
 - Some problems do not have obvious nonrecursive solutions
- Disadvantages
 - Overhead of recursive calls
 - Can use lots of memory (need to store state for each recursive call until base case is reached)

Mathematical Induction

- The mathematical equivalent of recursion is called **induction**
- Induction is a proof technique
- Comes from how natural numbers are defined:
 - A is the set of natural numbers such that
 0 is an element of A
 - 2. For each n, if 0, 1, 2, ..., n-1 are in A, than n is in A.



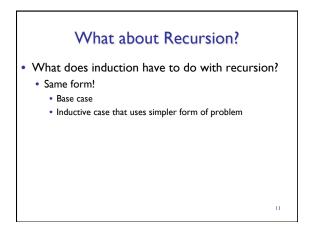
Mathematical Induction
• Prove:
$$\sum_{i=0}^{n} 2^{i} = 2^{0} + 2^{1} + 2^{2} + \dots + 2^{n} = 2^{n+1} - 1$$
• Prove:
$$0^{3} + 1^{3} + \dots + n^{3} = (0 + 1 + \dots + n)^{2}$$

Proof:
$$0^3 + 1^3 + \dots + n^3 = (0 + 1 + \dots + n)^2$$

- Base case: $n = 0, 0^3 = (0)^2$.
- Ind. Hyp.: For k < n assume true. • $(0^3 + 1^3 + ... + k^3) = (0 + 1 + ... + k)^2$
- Ind Step: Show true for *n*.

Proof:
$$0^3 + 1^3 + \dots + n^3 = (0 + 1 + \dots + n)^2$$

 $(0^3 + 1^3 + \dots n^3) = (0^3 + 1^3 + \dots + (n - 1)^3) + n^3$
 $= (0 + 1 + \dots + (n - 1))^2 + n^3$
 $= \left(\frac{n(n - 1)}{2}\right)^2 + n^3$
 $= n^2 \left(\frac{(n - 1)^2 + 4n}{4}\right)$
 $= n^2 \left(\frac{n^2 + 2n + 1}{4}\right)$
 $= n^2 \left(\frac{(n + 1)^2}{4}\right)$
 $= \left(\frac{n(n + 1)}{2}\right)^2$
 $= (0 + 1 + \dots + n)^2$



What about Recursion?

Example: factorial

- Prove that fact(n) requires (n-1) multiplications
 - Base case: n = 1 returns 1, 0 multiplications
 - Assume true for all k<n, so fact(k) requires k-1 multiplications.
 - fact(n) performs one multiplication (n*fact(n-1)). We know that fact(n-1) requires n-2 multiplications by inductive hypothesis.

I+n-2 = n-1, therefore fact(n) requires n-1 multiplications.

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