Computer Science 136 Exam 2
Sample exam

Show all work. No credit will be given if necessary steps are not shown or for illegible answers. Partial credit for partial answers. Be clear and concise. Write your name on each page of the exam. There are 102 points available, but the maximum score is 100.

You may use one page of handwritten notes (turn this in also). The use of other reference materials or electronic devices is a violation of the honor code. Good luck!!

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Your name (please print): ____________________________

I have neither given nor received aid on this examination:

__________________________________________________
(sign here)
1. (23 points) Short answers. Show your work and justify answers where appropriate.

a. *Free bonus points given in class.* What is the answer given in class for the first question on exam 2? (2 points)

Skip this question.

b. When is it advantageous to use a splay tree instead of a regular binary search tree? (3 points)

Skip this question.

c. A tree with $n$ elements is both a min-heap and a binary search tree. What does it look like? (3 points)

Skip this question.

d. Which tree traversal would you use to print an expression tree in human-readable form? (2 points)

e. Which tree traversal would you use to evaluate an expression tree? (2 points)
f. Given a SinglyLinkedList containing $n$ elements, what is the complexity (Big O) of removeFirst()? of removeLast()? (4 points)

g. Given a DoublyLinkedList containing $n$ elements, what is the complexity (Big O) of removeFirst()? of removeLast()? (4 points)

i. We applied sorting methods primarily to arrays and Vectors. Of the following sort algorithms, which are most appropriate to sort a SinglyLinkedList: insertion sort, selection sort, quicksort, merge sort? (3 points)
2. (15 points) A circular doubly linked list with four elements is represented as in the picture below:

Suppose we have an implementation of such a list, class `CircularDoublyLinkedList`, which includes instance variables:

```java
protected DoublyLinkedListElement head;
protected int count;
```

Relevant parts of the class `DoublyLinkedListElement` from structure5 are on page 6.

Consider the `addLast()` method of the `CircularDoublyLinkedList` class:

```java
// pre: value is not null
// post: adds value to the tail of the list
public void addLast(Object value);
```

a. What special cases must be considered when writing this method? (5 points)
b. Write Java code for this method. You need not replicate the pre- and post-conditions specified on the previous page. You may not use `addFirst()` in your code. (10 points)

```java
public void addLast(Object value) {
}
```
public class DoublyLinkedListElement {
    protected Object data;
    protected DoublyLinkedListElement nextElement;
    protected DoublyLinkedListElement previousElement;

    public DoublyLinkedListElement(Object v,
            DoublyLinkedListElement next,
            DoublyLinkedListElement previous) {
        // post: constructs new element with list prefix referenced by
        // previous and suffix referenced by next
        data = v;
        nextElement = next;
        if (nextElement != null) nextElement.previousElement = this;
        previousElement = previous;
        if (previousElement != null) previousElement.nextElement = this;
    }

    public DoublyLinkedListElement(Object v) {
        // post: constructs a single element
        this(v,null,null);
    }

    public DoublyLinkedListElement next() {
        // post: returns the element that follows this
        return nextElement;
    }

    public DoublyLinkedListElement previous() {
        // post: returns element that precedes this
        return previousElement;
    }

    public Object value() {
        // post: returns value stored here
        return data;
    }

    public void setNext(DoublyLinkedListElement next) {
        // post: sets value associated with this element
        nextElement = next;
    }

    public void setPrevious(DoublyLinkedListElement previous) {
        // post: establishes a new reference to a previous value
        previousElement = previous;
    }
}
3. (15 points) Recall that the Queue interface may be implemented using an array to store the queue elements. Suppose that two int values are used to keep track of the ends of the queue. We treat the array as circular: adding or deleting an element may cause the head or tail to “wrap around” to the beginning of the array.

You are to provide a Java implementation of class CircularQueueArray by filling in the bodies of the methods below. Note that there is no instance variable which stored the number of elements current in the queue; you must compute this from the values of head and tail. You may not add any additional instance variables.

```java
public class CircularQueueArray {
    // instance variables
    protected int head, tail;
    protected Object[] data;

    // constructor: build an empty queue of capacity n
    public CircularQueueArray(int n) {
    }

    // pre: queue is not fill
    // post: adds value to the queue
    public void enqueue(Object value) {
    }
}
```
3. (continued)

    // pre: queue is not empty
    // post: removes value from the head of the queue
    public Object dequeue() {
    }

    // post: return the number of elements in the queue
    public int size() {
    }

    // post: returns true iff queue is empty
    public boolean isEmpty() {
    }

    // post: returns true iff queue is full
    public boolean isFull() {
    }
4. (15 points) The StackSort. Suppose you need to sort a stream of Comparable elements, and the only data structure available to you is an implementation of the Stack interface in the structure5 package (say, a StackList). The elements are available only through an Iterator, so you must process each item as it is returned by the next() method of the Iterator. The sort method should return a Stack containing the sorted elements, with the smallest at the top of the stack. Please fill in the body of the method.

    public static Stack StackSort(Iterator iter) {
        // pre: iter is an Iterator over a structure containing Comparables
        // post: a Stack is returned with the elements sorted, smallest on top
    }
5. (16 points) Recall the definition of a min-heap, a binary tree in which each node is smaller than any of its descendants. For the rest of this question, we presume the Vector implementation of heaps (class VectorHeap). Consider the following tree, which is a min-heap.

![Image of a min-heap diagram]

a. Show the order in which the elements would be stored in the Vector underlying our VectorHeap. (4 points)

b. Show the steps involved in adding the value 4 to the heap. Use drawings of the tree, not the vector. (4 points)
c. Using the original tree (not the one with the 4 added), show the steps involved in removing the minimum value of the heap. (4 points)

d. Why is the `VectorHeap` implementation of a priority queue better than one that uses a linked list implementation of regular queues, modified to keep all items in order by priority? Hint: Your answer should compare the complexities of the add and remove operations. (4 points)
6. (18 points) Suppose we have a **BinaryTree** that contains only **Comparable** values.

a. It is often useful to find the minimum and maximum values in the tree. Implement the method `maximum` as a member of class **BinaryTree**. Relevant sections of **BinaryTree.java** from the structure5 package are included on pages 14–16 to guide you. Your method should return the **Comparable** that is the maximum value in the tree. It should return **null** if called on an empty tree. (6 points)

```java
public Comparable maximum() {
    // pre: the values in this tree are all Comparable
    // post: the maximum value in the tree is returned
}
```

b. What is the worst-case complexity of `maximum` on a tree containing $n$ values? (2 points)

c. What is the complexity of `maximum` on a full tree containing $n$ values? (2 points)
d. Consider the following method, which I propose as a member of class `BinaryTree`:

```java
public boolean isBST() {
    // post: returns true iff the tree rooted here is a binary search tree
    if (this == EMPTY) return true;
    return left().isBST() && right().isBST();
}
```

This method will not always return the correct value. Explain why, then provide a correct method. You may use `minimum()` and `maximum()` from part (a), as well as any other methods of `BinaryTree`. (6 points)

```java
public boolean isBST() {
    // post: returns true iff the tree rooted here is a binary search tree
    if (this == EMPTY) return true;
    return left().isBST() && right().isBST();
}
```

e. In class `BinaryTree`, why is the `setLeft()` method `public`, but the `setParent()` method is `protected`? (2 points)
public class BinaryTree {
  protected Object val; // value associated with node
  protected BinaryTree parent; // parent of node
  protected BinaryTree left; // left child of node
  protected BinaryTree right; // right child of node
  // The unique empty node
  public static final BinaryTree EMPTY = new BinaryTree();

  // A one-time constructor, for constructing empty trees.
  private BinaryTree() {
    val = null; parent = null; left = right = this;
  }

  // Constructs a tree node with no children. Value of the node
  // is provided by the user
  public BinaryTree(Object value) {
    val = value; parent = null; left = right = EMPTY;
  }

  // Constructs a tree node with no children. Value of the node
  // and subtrees are provided by the user
  public BinaryTree(Object value, BinaryTree left, BinaryTree right) {
    this(value);
    setLeft(left);
    setRight(right);
  }

  // Get left subtree of current node
  public BinaryTree left() {
    return left;
  }

  // Get right subtree of current node
  public BinaryTree right() {
    return right;
  }

  // Get reference to parent of this node
  public BinaryTree parent() {
    return parent;
  }

  // Update the left subtree of this node. Parent of the left subtree
  // is updated consistently. Existing subtree is detached
  public void setLeft(BinaryTree newLeft) {
    if (isEmpty()) return;
    if (left.parent() == this) left.setParent(null);
    }
}
left = newLeft;
left.setParent(this);
}

// Update the right subtree of this node. Parent of the right subtree
// is updated consistently. Existing subtree is detached
public void setRight(BinaryTree newRight) {
    if (isEmpty()) return;
    if (right.parent() == this) right.setParent(null);
    right = newRight;
    right.setParent(this);
}

// Update the parent of this node
protected void setParent(BinaryTree newParent) {
    parent = newParent;
}

// Returns the number of descendants of node
public int size() {
    if (this == EMPTY) return 0;
    return left().size() + right.size() + 1;
}

// Returns reference to root of tree containing n
public BinaryTree root() {
    if (parent() == null) return this;
    else return parent().root();
}

// Returns height of node in tree. Height is maximum path
// length to descendant
public int height() {
    if (this == EMPTY) return -1;
    return 1 + Math.max(left.height(), right.height());
}

// Compute the depth of a node. The depth is the path length
// from node to root
public int depth() {
    if (parent() == null) return 0;
    return 1 + parent.depth();
}

// Returns true if tree is full. A tree is full if adding a node
// to tree would necessarily increase its height
public boolean isFull() {
if (this == EMPTY) return true;
if (left().height() != right().height()) return false;
return left().isFull() && right().isFull();
}

// Returns true if tree is empty.
public boolean isEmpty() {
    return this == EMPTY;
}

// Return whether tree is complete. A complete tree has minimal height
// and any holes in tree would appear in last level to right.
public boolean isComplete() {
    int leftHeight, rightHeight;
    boolean leftIsFull, rightIsFull, leftIsComplete, rightIsComplete;
    if (this == EMPTY) return true;
    leftHeight = left().height();
    rightHeight = right().height();
    leftIsFull = left().isFull();
    rightIsFull = right().isFull();
    leftIsComplete = left().isComplete();
    rightIsComplete = right().isComplete();

    // case 1: left is full, right is complete, heights same
    if (leftIsFull && rightIsComplete &&
        (leftHeight == rightHeight)) return true;
    // case 2: left is complete, right is full, heights differ
    if (leftIsComplete && rightIsFull &&
        (leftHeight == (rightHeight + 1))) return true;
    return false;
}

// Return true iff the tree is height balanced. A tree is height
// balanced iff at every node the difference in heights of subtrees is
// no greater than one
public boolean isBalanced() {
    if (this == EMPTY) return true;
    return (Math.abs(left().height()-right().height()) <= 1) &&
        left().isBalanced() && right().isBalanced();
}

// Returns value associated with this node
public Object value() {
    return val;
}