CS134: Sorting

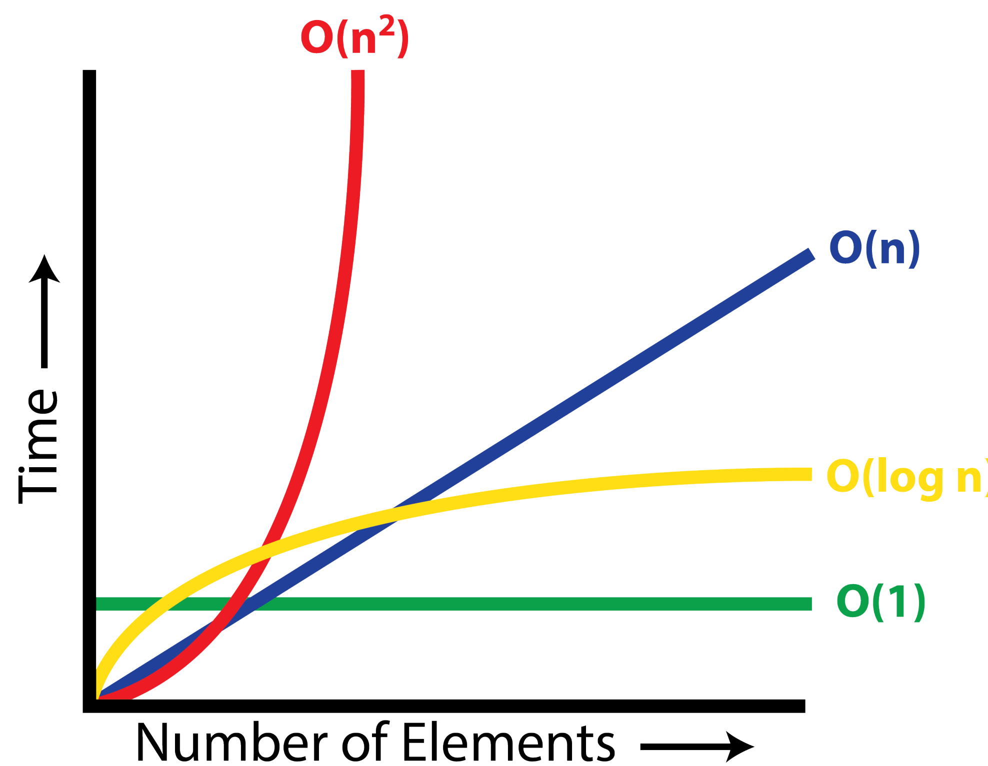
Announcements & Logistics

- **Lab 9 Boggle**
  - Work on Boggle again in lab this week today/tomorrow
  - **All three parts** are due Wed/Thur at 10 pm
- **HW 9** will be released on Wed, due next Mon @ 10 pm (last one!)
- Last lab (**Lab 10**) will be a very short Java program
- We will discuss Java in last few lectures after we wrap up sorting today

Do You Have Any Questions?

Last Time: Efficiency & Searching

- Measured efficiency as number of steps taken by algorithm on worst-case inputs of a given size
- Introduced Big-O notation which captures the rate at which the number of steps taken by the algorithm grows wrt size of input n, "as n gets large"
- Compared array lists vs linked lists
- Compared linear vs binary search

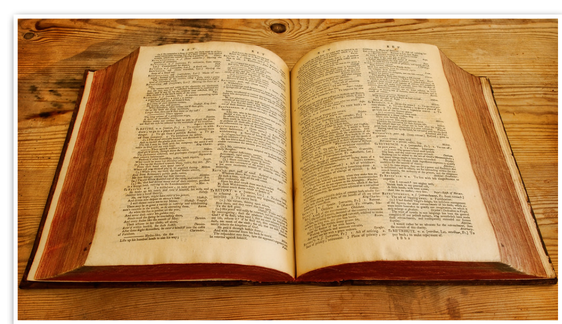


Today: Searching and Sorting

- Wrap up our discussion of binary search including a runtime analysis
- Discuss some classic sorting algorithms:
    - ***Selection sorting*** in $O(n^2)$ time
    - A brief (high level) discussion of how we can improve it to $O(n \log n)$
    - Overview of recursive ***merge sort*** algorithm

Review: Logarithms

- Logarithms are the inverse function to exponentiation
- $\log_2 n$ describes the exponent to which 2 must be raised to produce n
- That is, $2^{\log_2 n} = n$
- Alternatively:
  - $\log_2 n$ (essentially) describes the number of times n must be divided by 2 to reduce it to below 1
- For us, here's the important takeaway:
  - How many times can we divide n by 2 until we get down to 1
  - $\approx \log_2 n$

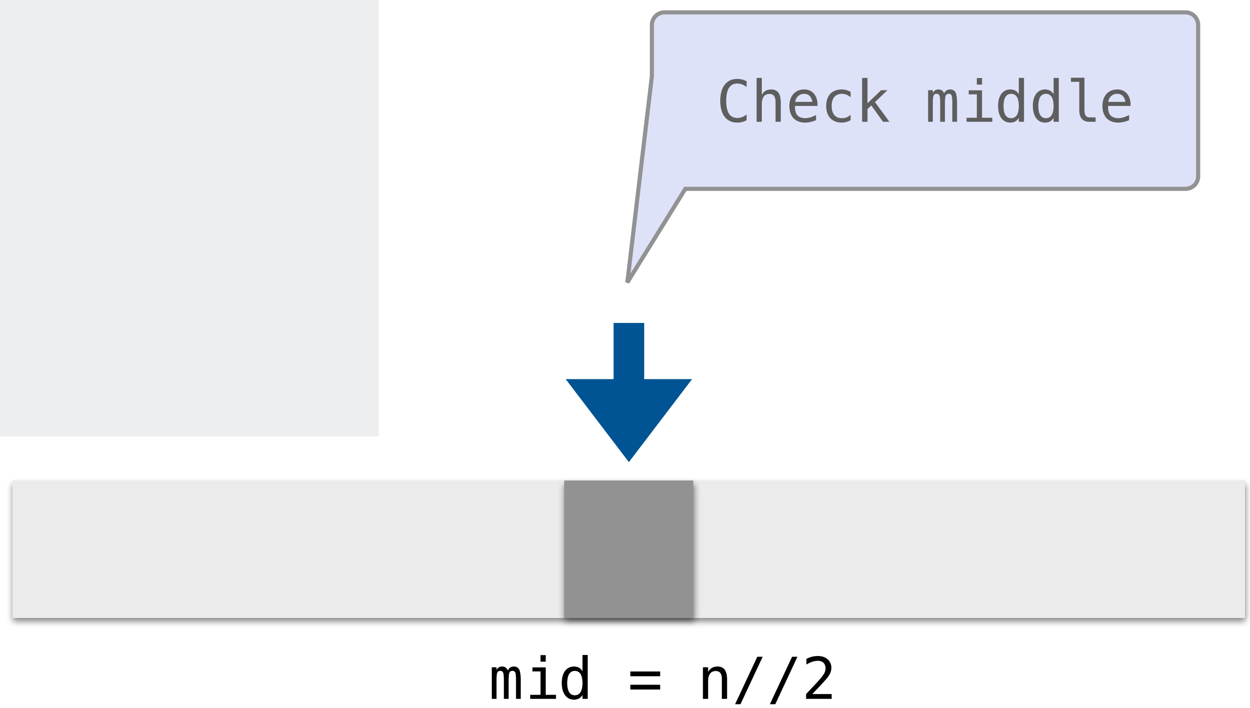


Review: Binary Search

- Base cases? When are we done?
  - If list is too small (or empty) to continue searching
  - If item we're searching for is the middle element

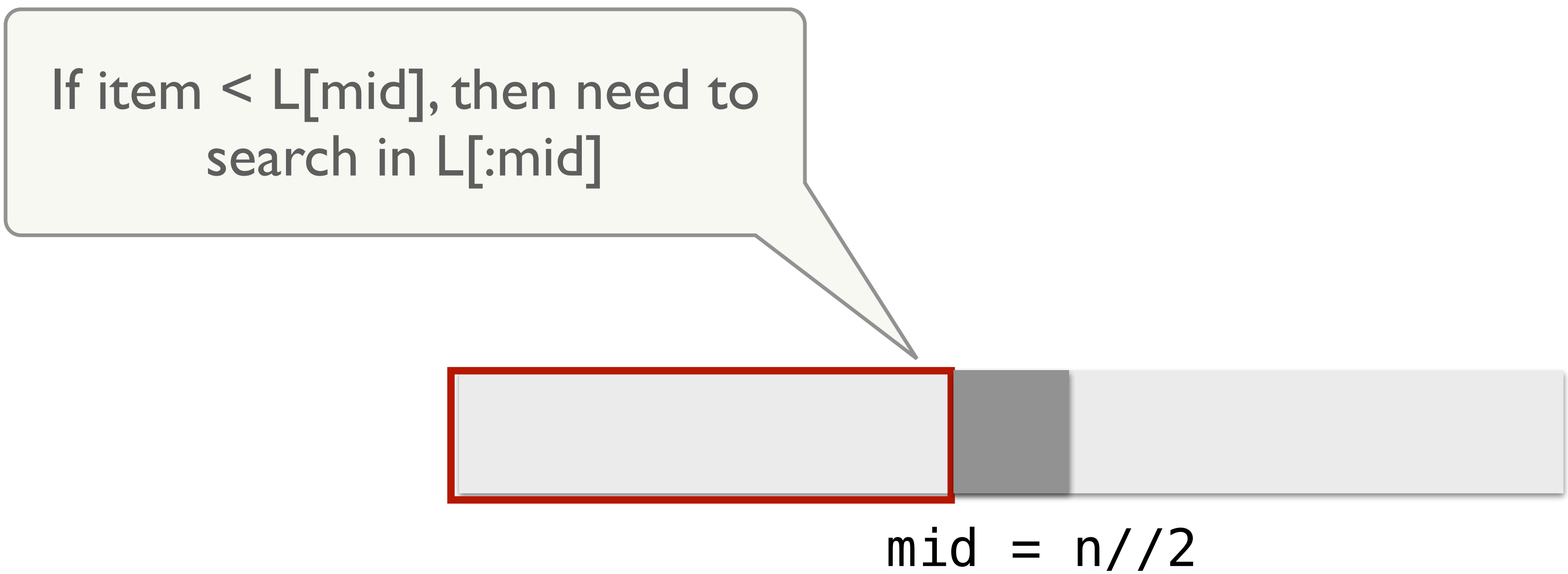
```python
def binarySearch(aList, item):
    """Assume aList is sorted."""
    n = len(aList)
    mid = n // 2
    # base case 1
    if n == 0:
        return False
    # base case 2
    elif item == aList[mid]:
        return True

    # recursive cases...
```



Review: Binary Search

- Recursive case:
    - Recurse on left side if item is smaller than middle
    - Recurse on right side if item is larger than middle



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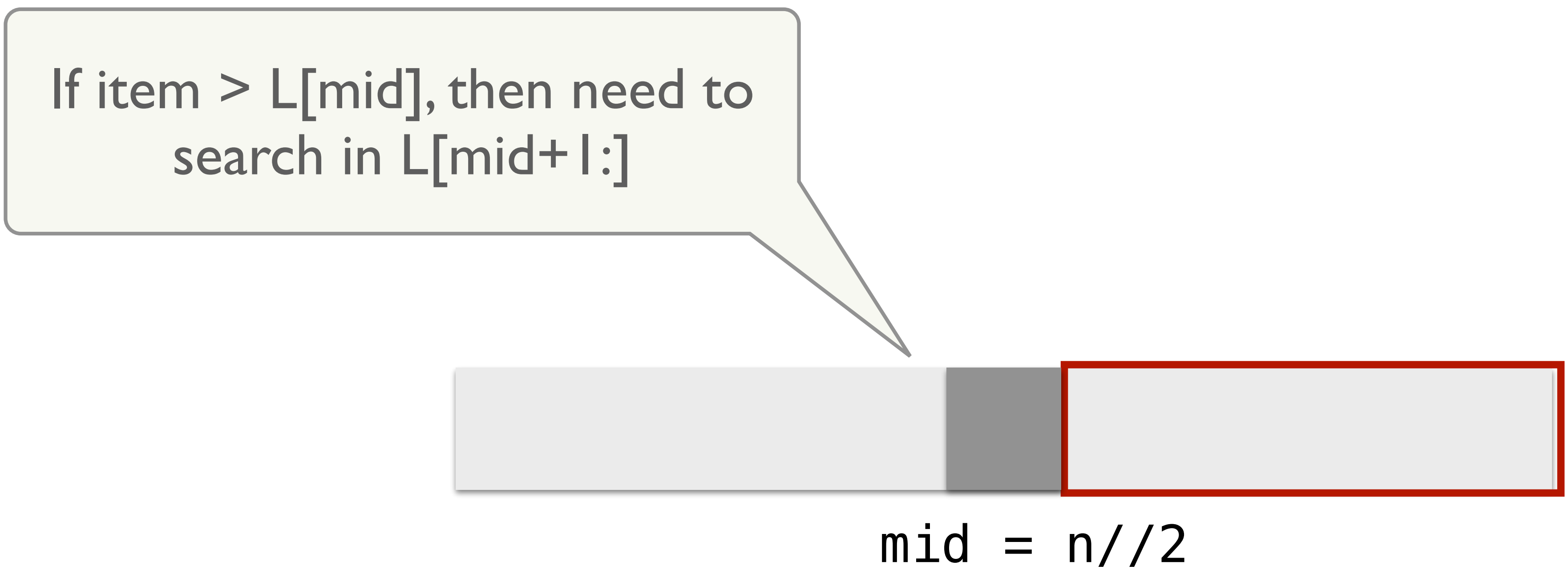

Review: Binary Search

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    # base case 1
    if n == 0:
        return False
    # base case 2
    elif item == aList[mid]:
        return True

    # recurse on left
    elif item < aList[mid]:
        return binarySearch(aList[:mid], item)
    # recurse on right
    else:
        return binarySearch(aList[mid + 1:], item)
```

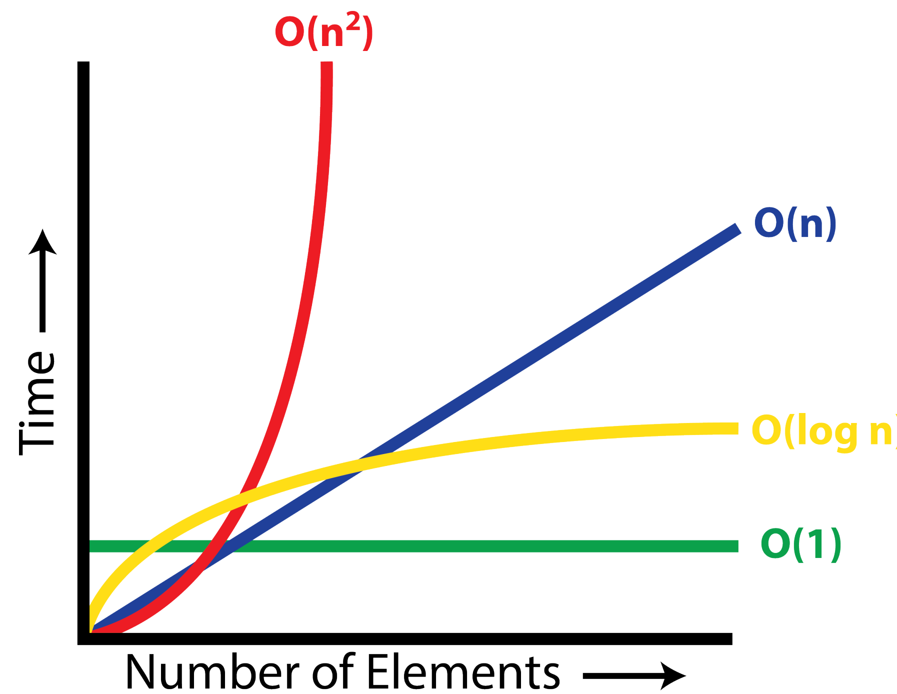
Technically, there is one *small* problem with our implementation. List splicing is actually O(n)! See Jupyter for improvement.

Review: Binary Search

- **Binary search:** recursive search algorithm to search in a **sorted array list**
  - Similar to how we search for a word in a (physical) dictionary
  - Takes $O(\log n)$ time since we are reducing half of the search space on each step: $n \rightarrow n/2 \rightarrow n/4 \rightarrow n/8 \rightarrow \cdots \rightarrow n/2^i = 1$
- Much more efficient than a **linear search**
- **Note:** $\log n$ grows much more slowly compared to n as n gets large

But how expensive is sorting??

$\log_2$ (1 billion) ~ 30



Sorting

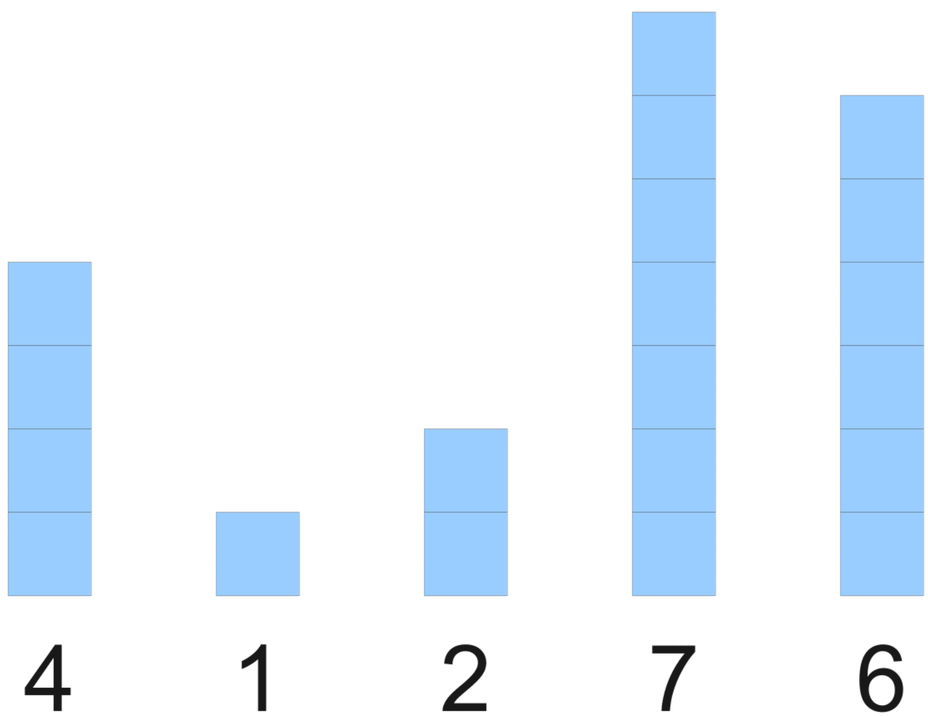
Selection Sort

Sorting

- **Problem:** Given a sequence of unordered elements, we need to sort the elements in ascending order.
- There are many ways to solve this problem!
- Built-in sorting functions/methods in Python
    - `sorted()`: function that returns a new sorted list
    - `sort()`: method that mutates and sorts the list it's called on
- **Today:** how do we design our own sorting algorithm?
- **Question:** What is the best (most efficient) way to sort n items?
- We will use Big-O to find out!

Selection Sort

- A possible approach to sorting elements in a list/array:
  - Find the smallest element and move (swap) it to the first position
  - Repeat: find the second-smallest element and move it to the second position, and so on



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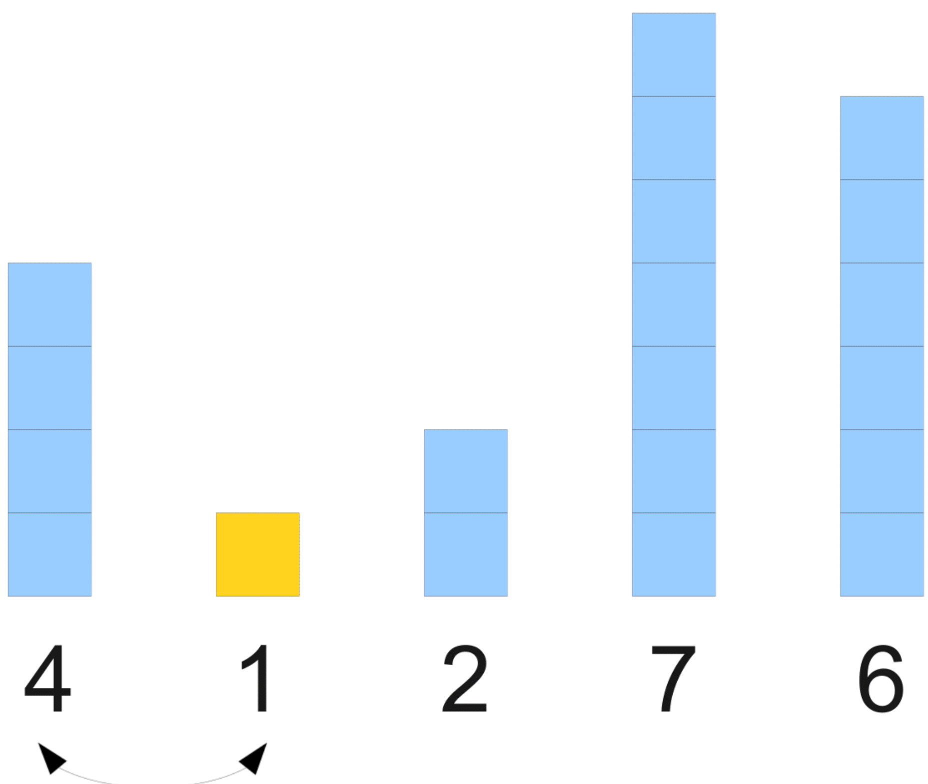

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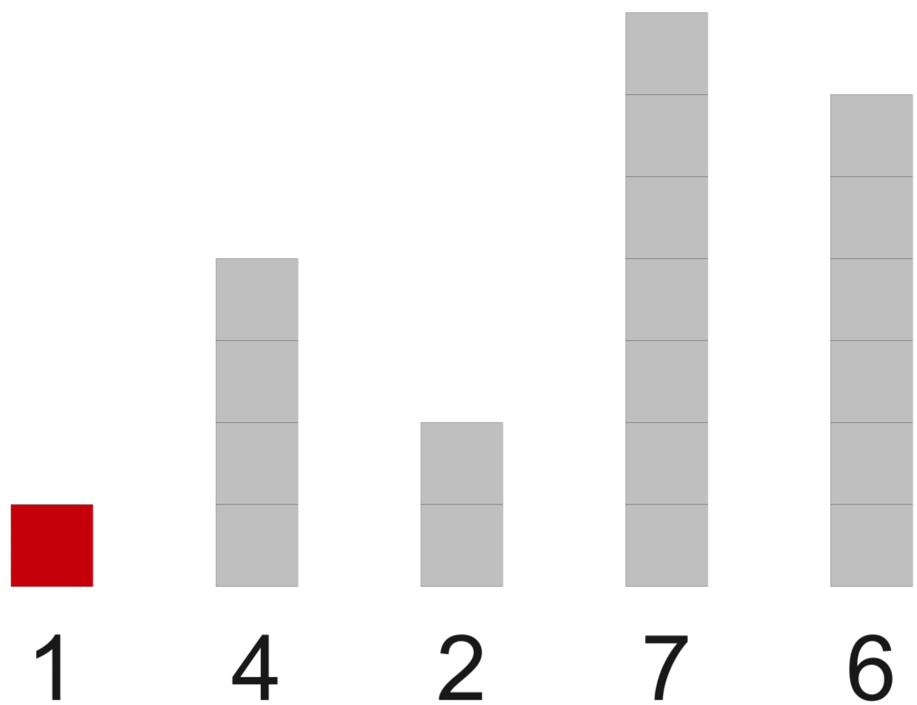

Selection Sort

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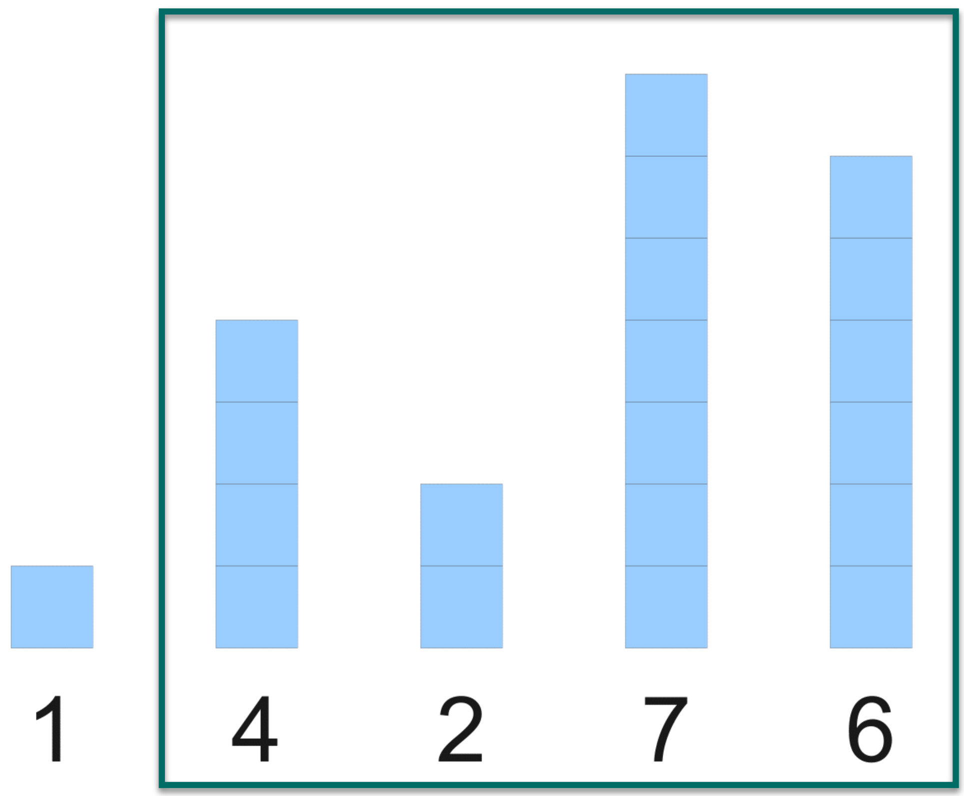

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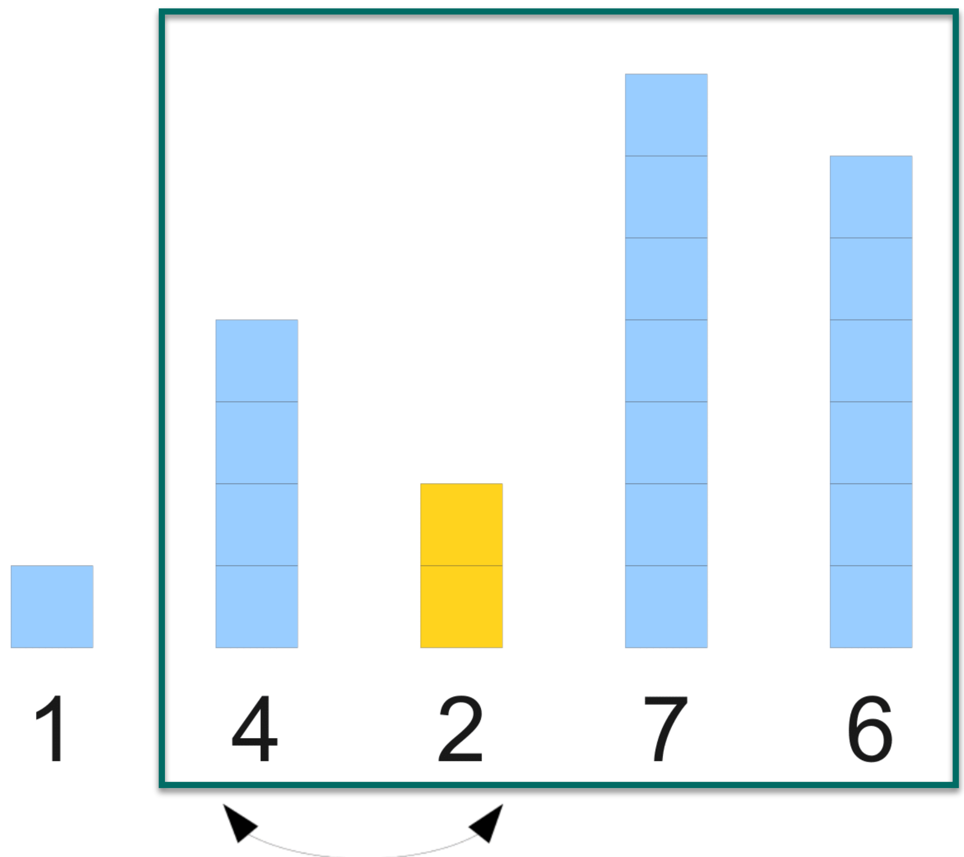

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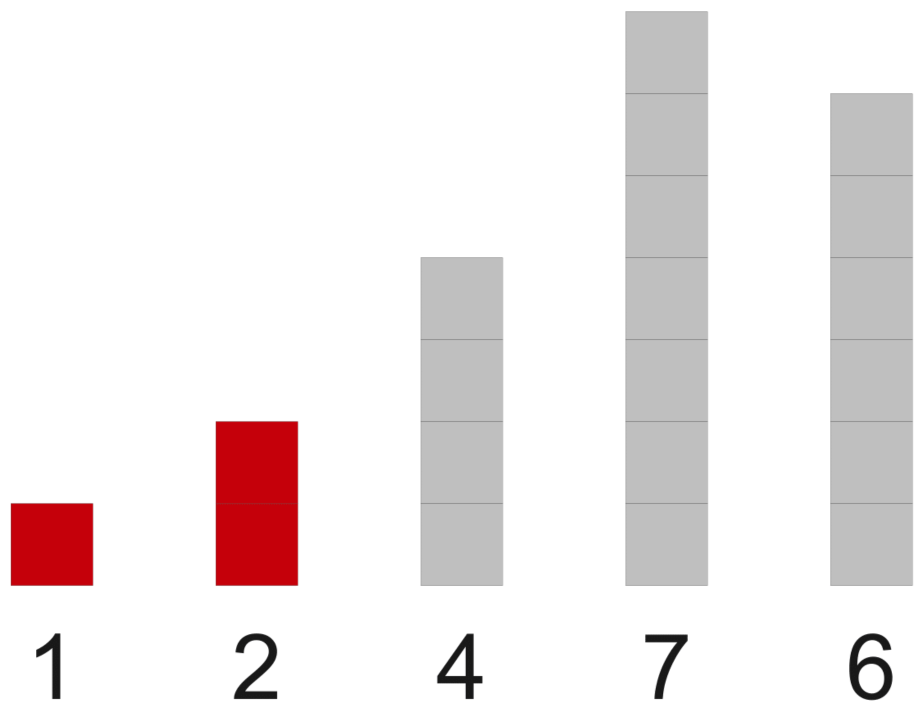

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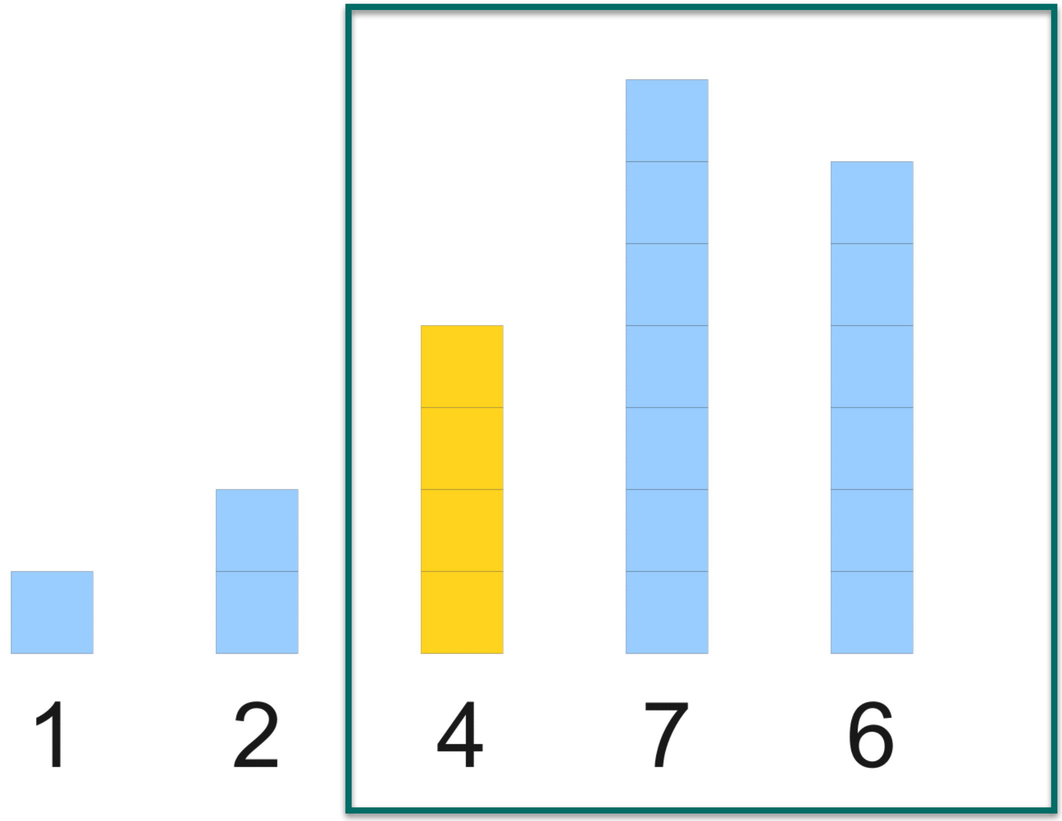

Selection Sort

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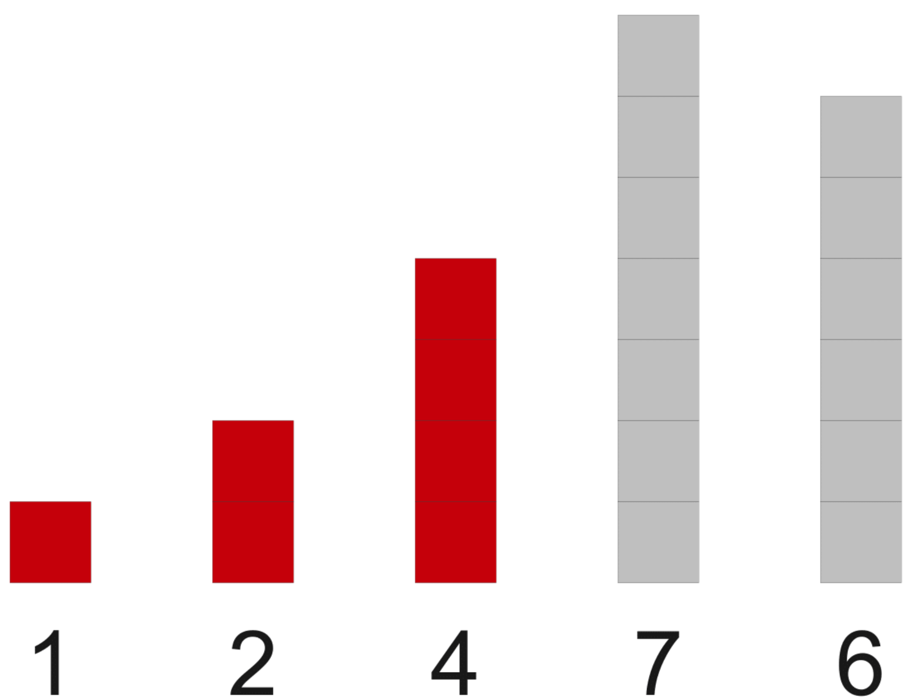

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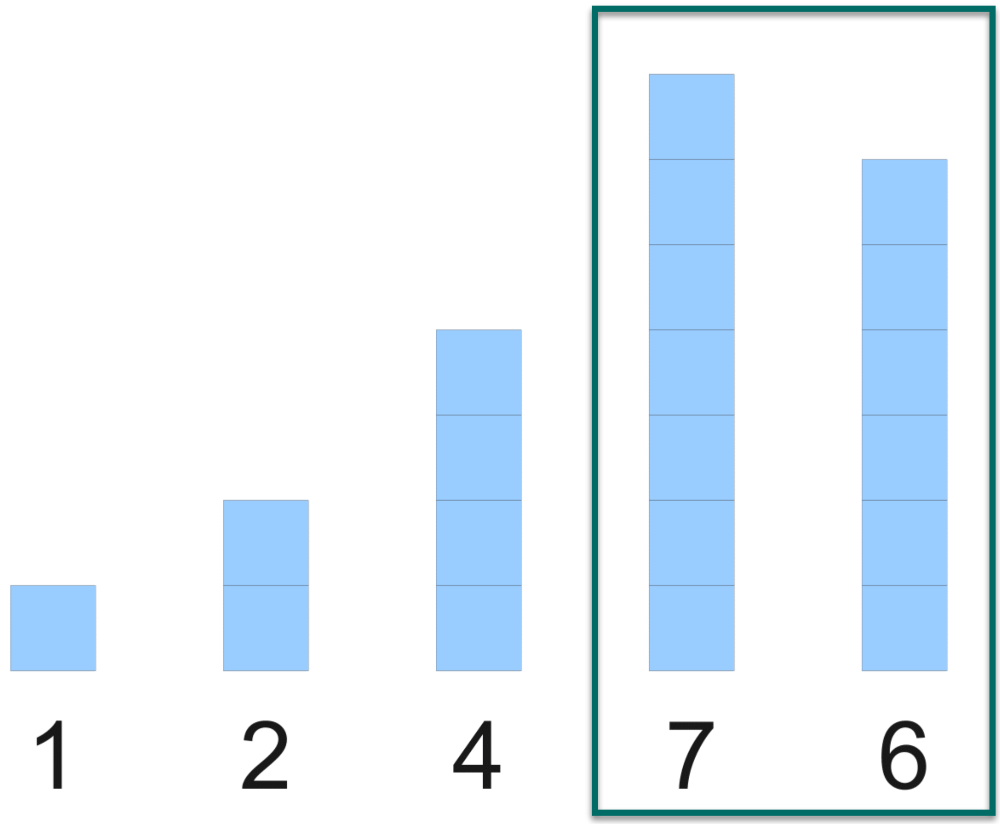

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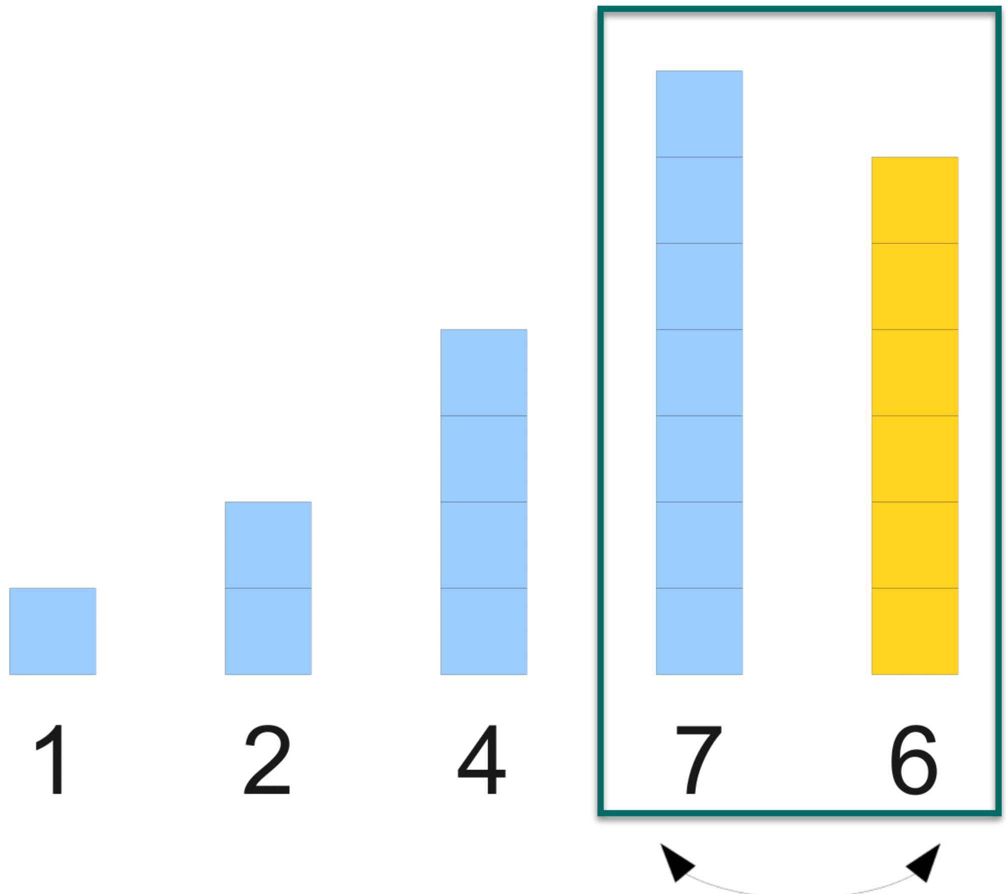

Selection Sort

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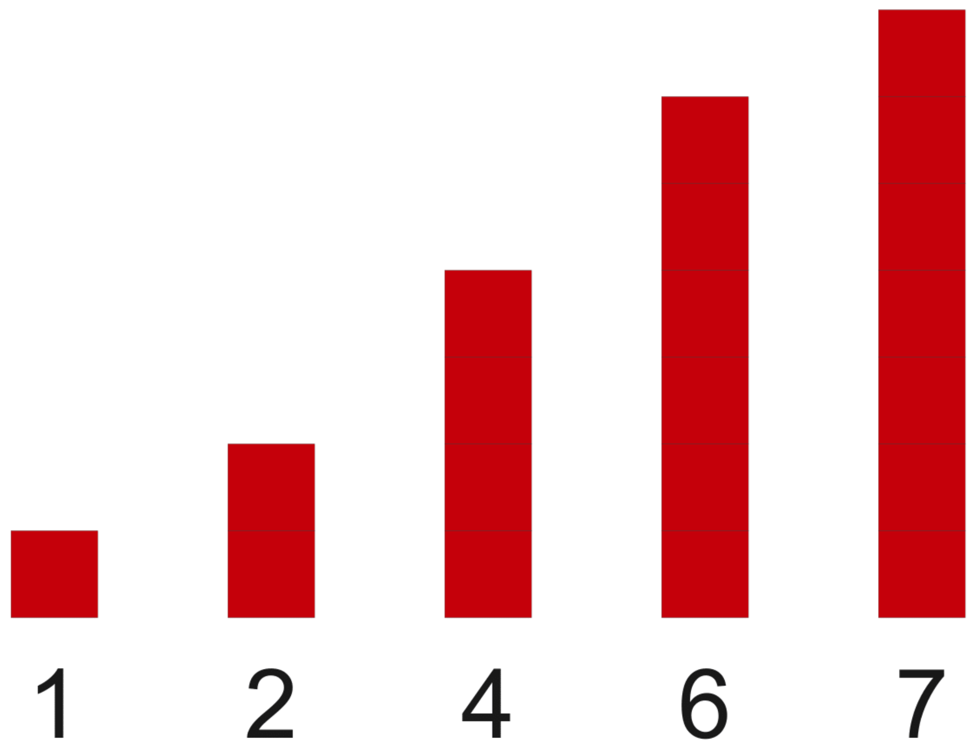

Selection Sort

- Generalize: For each index *i* in the list `L`, we need to find the **min** item in `L[i:]` so we can replace `L[i]` with that item
- In fact we need to find the position `minIndex` of the item that is minimum in `L[i:]`
- **Reminder:** how to swap values of variables `a` and `b`?
  - Using tuple assignment in Python: `a, b = b, a`
  - Or using a temp variable: `temp = a; a = b; b = temp`
- Let's implement this algorithm! (We won't use recursion this time, although we could…)

Selection Sort Code

```python
def selectionSort(myList):
    """Selection sort of given list myList,
    mutates list and sorts using selection sort."""
    # find size
    n = len(myList)

    # traverse through all elements
    for i in range(n):

        # find min element in remaining unsorted list
        minIndex = i
        for j in range(i + 1, n):
            if myList[minIndex] > myList[j]:
                minIndex = j

        # swap min element with element at i
        myList[i], myList[minIndex] = myList[minIndex], myList[i]
```

```
>>> myList = [12, 2, 9, 4, 11, 3, 1, 7, 14, 5, 13]
>>> selectionSort(myList)
>>> print(myList)
```

```
[1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14]
```

Selection Sort Analysis

- For $i = 0$, inner loop checks $n - 1$ items
- For $i = 1$, inner loop checks $n - 2$ items
- ...
- For $i = n - 1$, inner loop checks 0 items

```python
# traverse through all elements
for i in range(n):

    # find min element in remaining unsorted list
    minIndex = i
    for j in range(i + 1, n):
        if myList[minIndex] > myList[j]:
            minIndex = j

    # swap min element with element at i
    myList[i], myList[minIndex] = myList[minIndex], myList[i]
```

Selection Sort Analysis

- Within the inner loop we have $O(1)$ steps - just 1 comparison (constant)
- Thus overall number of steps is sum of inner loop steps
  $(n-1) + (n-2) + \cdots + 0 \leq n + (n-1) + (n-2) + \cdots + 1$
- What is this sum? (Math 200??)

```python
# traverse through all elements
for i in range(n):

    # find min element in remaining unsorted list
    minIndex = i
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        if myList[minIndex] > myList[j]:
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    # swap min element with element at i
    myList[i], myList[minIndex] = myList[minIndex], myList[i]
```

Selection Sort Analysis

$$
\begin{array}{rl}
 & S = n + (n-1) + (n-2) + \cdots + 2 + 1 \\
+ & S = 1 + 2 + \cdots + (n-2) + (n-1) + n \\
\hline
\end{array}
$$

$$2S = (n+1) + (n+1) + \cdots + (n+1) + (n+1) + (n+1)$$

$$2S = (n+1) \cdot n$$

$$S = (n+1) \cdot n \cdot 1/2$$

- Total number of steps taken by selection sort is thus:
  - $O(n(n+1)/2) \ = O(n(n+1)) \ = O(n^2+n) \ = O(n^2)$

Sorting

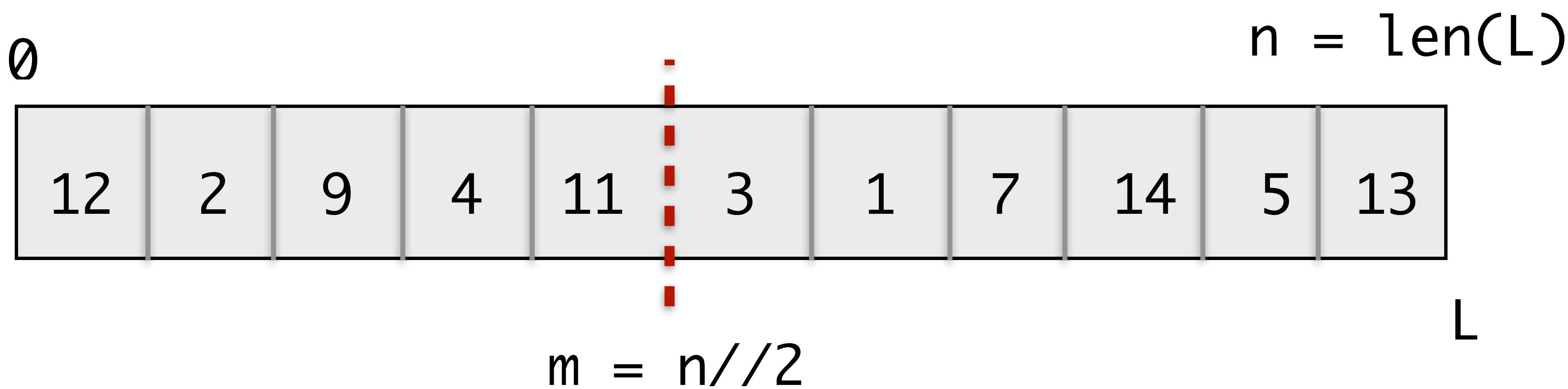
Merge Sort

Towards an $O(n \log n)$ Algorithm

- There are other sorting algorithms that compare and rearrange elements in different ways, but are still $O(n^2)$ steps
  - Any algorithm that takes n steps to move each item n positions (in the worst case) will take at least $O(n^2)$ steps
  - To do better than n^2, we need to move an item in fewer than n steps
- We can sort in $O(n \log n)$ time if we are clever: ***Merge sort algorithm*** (Invented by John von Neumann in 1945)

Merge Sort: Basic Idea

- If we split the list in half, sorting the left and right half are smaller versions of the same problem
- **Algorithm:**
  - **(Divide)** Recursively sort left and right half (O(log n))
  - **(Conquer)** Merge the sorted halves into a single sorted list (O(n))
  - (More info in extra slides at the end of this lecture!)



Selection vs Merge Sort in Practice

- Selection sort is $O(n^2)$ and merge sort is $O(n \log n)$ time
- How different is the performance in practice?
- Example: `wordList` is 12,000 words from the book *Pride & Prejudice*
- `miniList` and `medList` are the first 500 and 7000 words respectively

```
wordList = []
with open('prideandprejudice.txt') as book:
    for line in book:
        line = line.strip().split()
        wordList.extend(line)
print(len(wordList))
```

```
122089
```

```
>>> miniList = wordList[:500]
>>> medList = wordList[:7000]
```

Selection vs Merge Sort in Practice

- miniList: 500 words
- medList: 7000 words
- wordList: ~12000 words

```
timedSorting(miniList)
```

```
Selection sort takes 0.005692720413208008 secs
Merge sort takes 0.0005681514739990234 secs
```

```
timedSorting(medList)
```

```
Selection sort takes 1.0527238845825195 secs
Merge sort takes 0.009032011032104492 secs
```

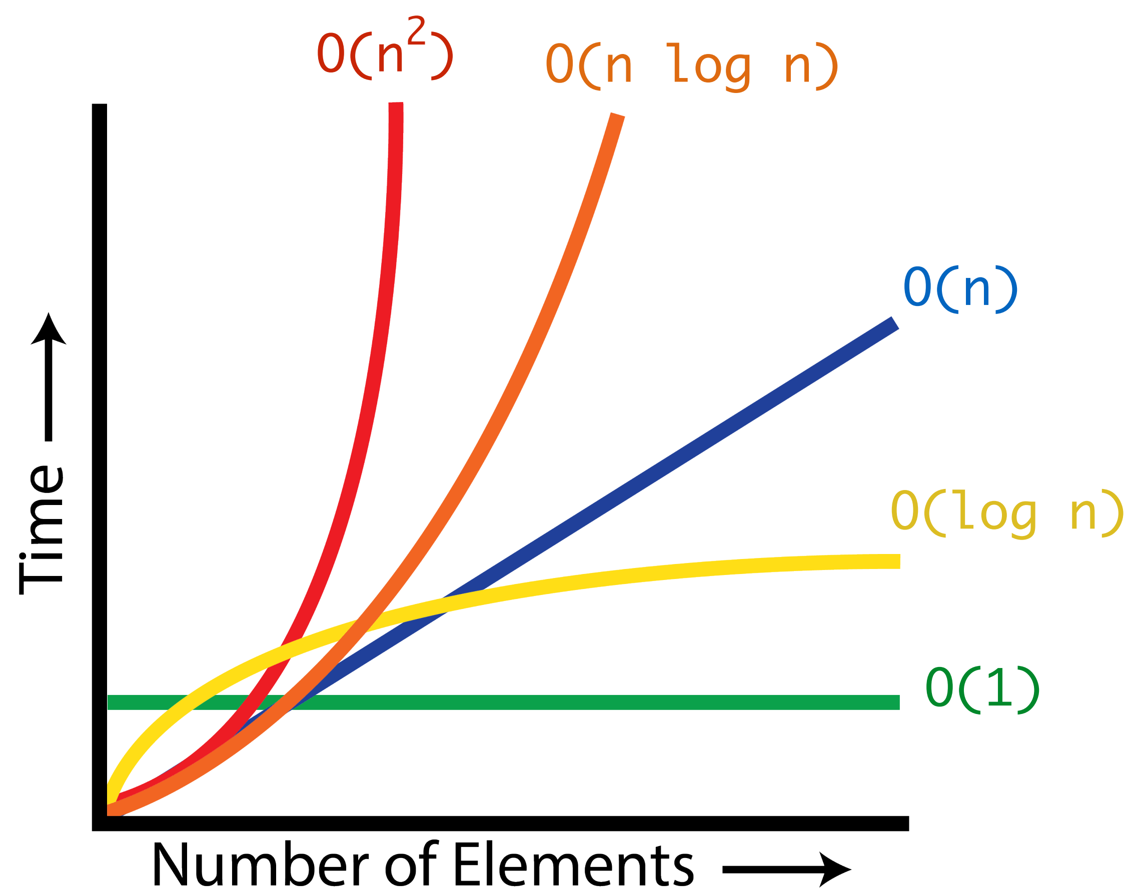
```
timedSorting(wordList)
```

```
Selection sort takes 322.0893268585205 secs
Merge sort takes 0.1942448616027832 secs
```

~5 mins vs 1/5 sec!

Summary: Searching and Sorting

- We have seen algorithms that are
  - $O(\log n)$: binary search in a sorted list
  - $O(n)$: linear searching in an unsorted list
  - $O(n \log n)$: merge sort
  - $O(n^2)$: selection sort
- Important to think about efficiency when writing code!

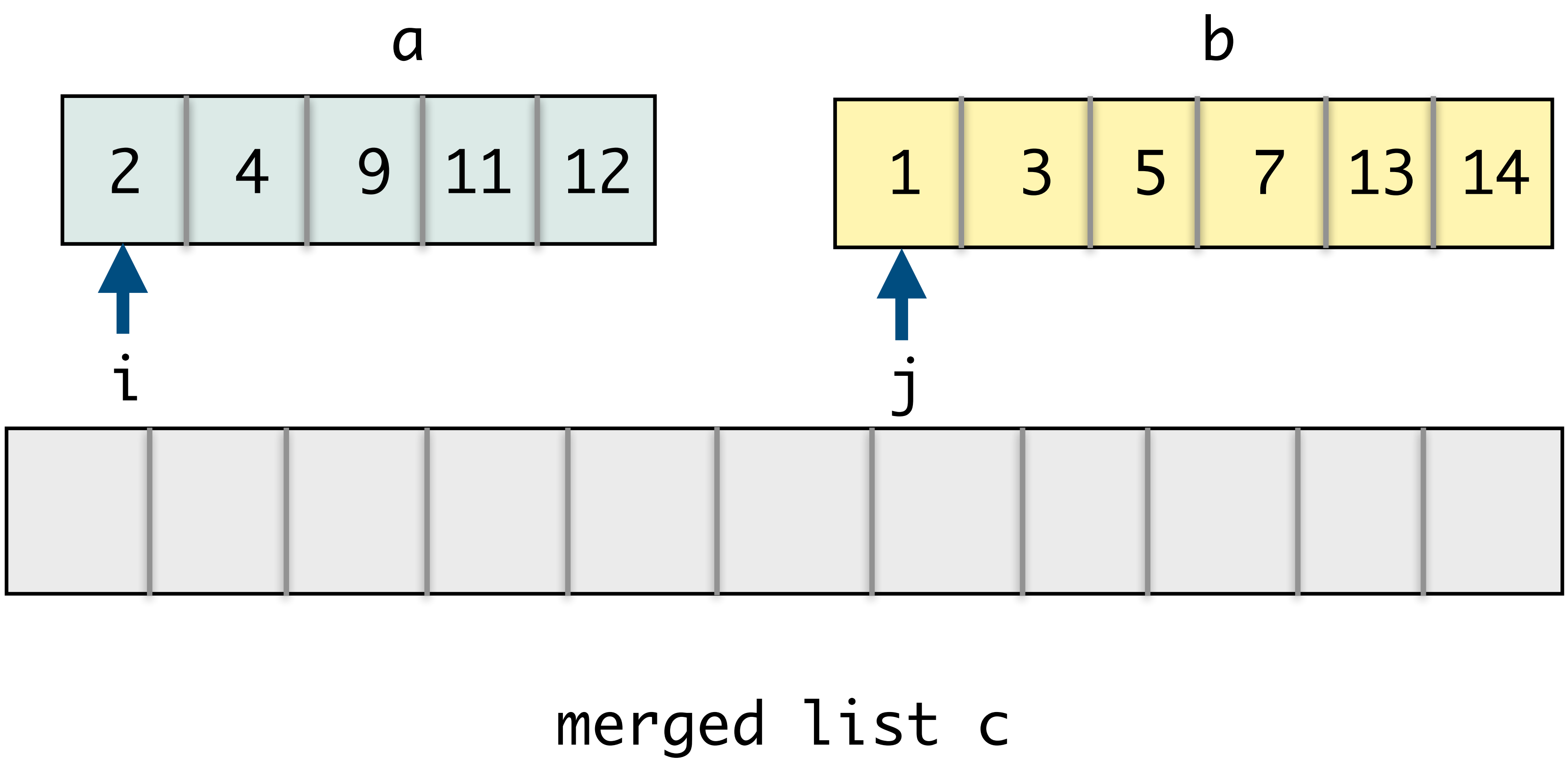


The end!

Leftover Slides

Merging Sorted Lists

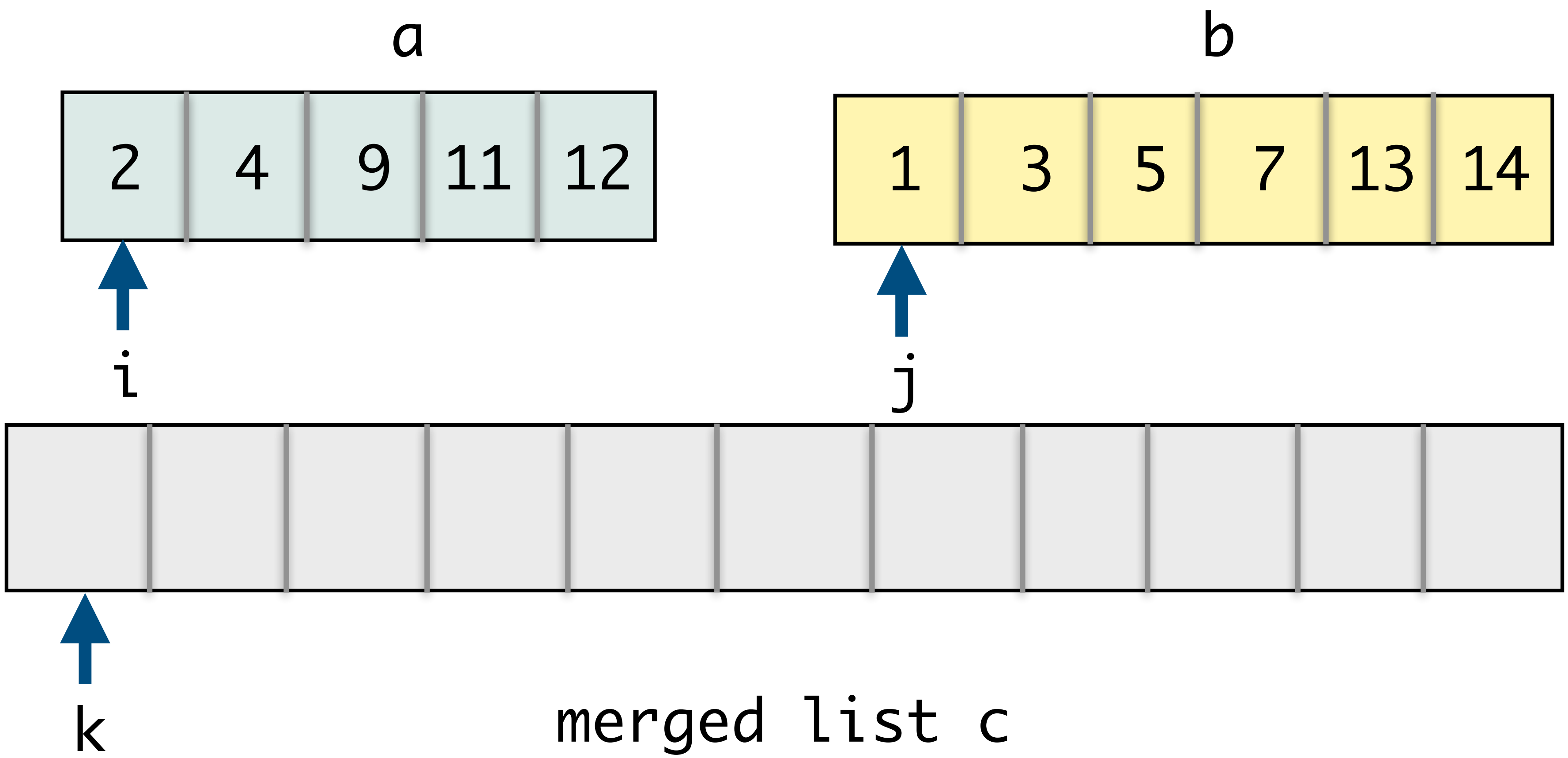
- **Problem.** Given two sorted lists `a` and `b`, how quickly can we merge them into a single sorted list?



Merging Sorted Lists

Is `a[i] <= b[j]` ?

- Yes, `a[i]` appended to `c`
- No, `b[j]` appended to `c`



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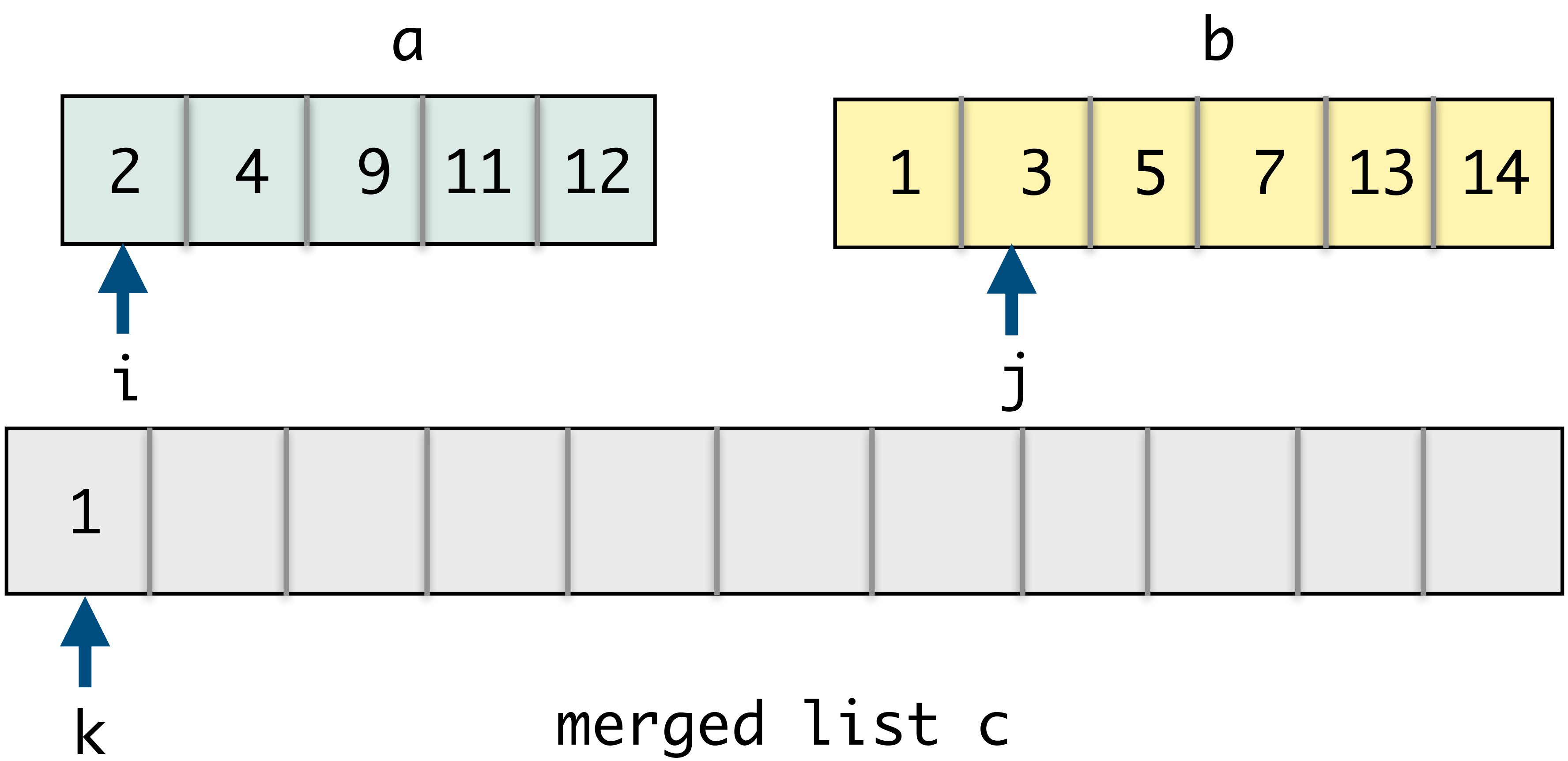

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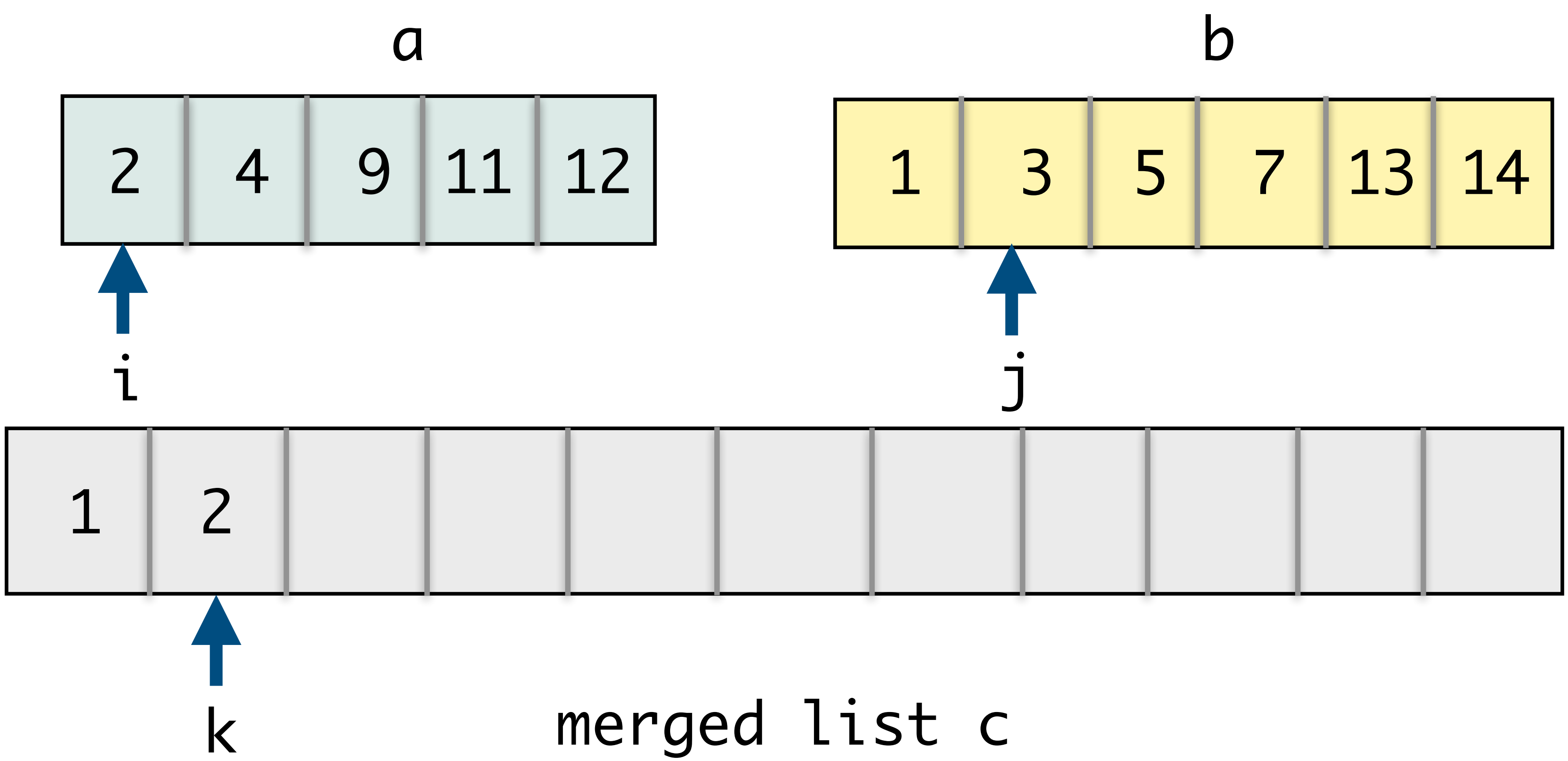

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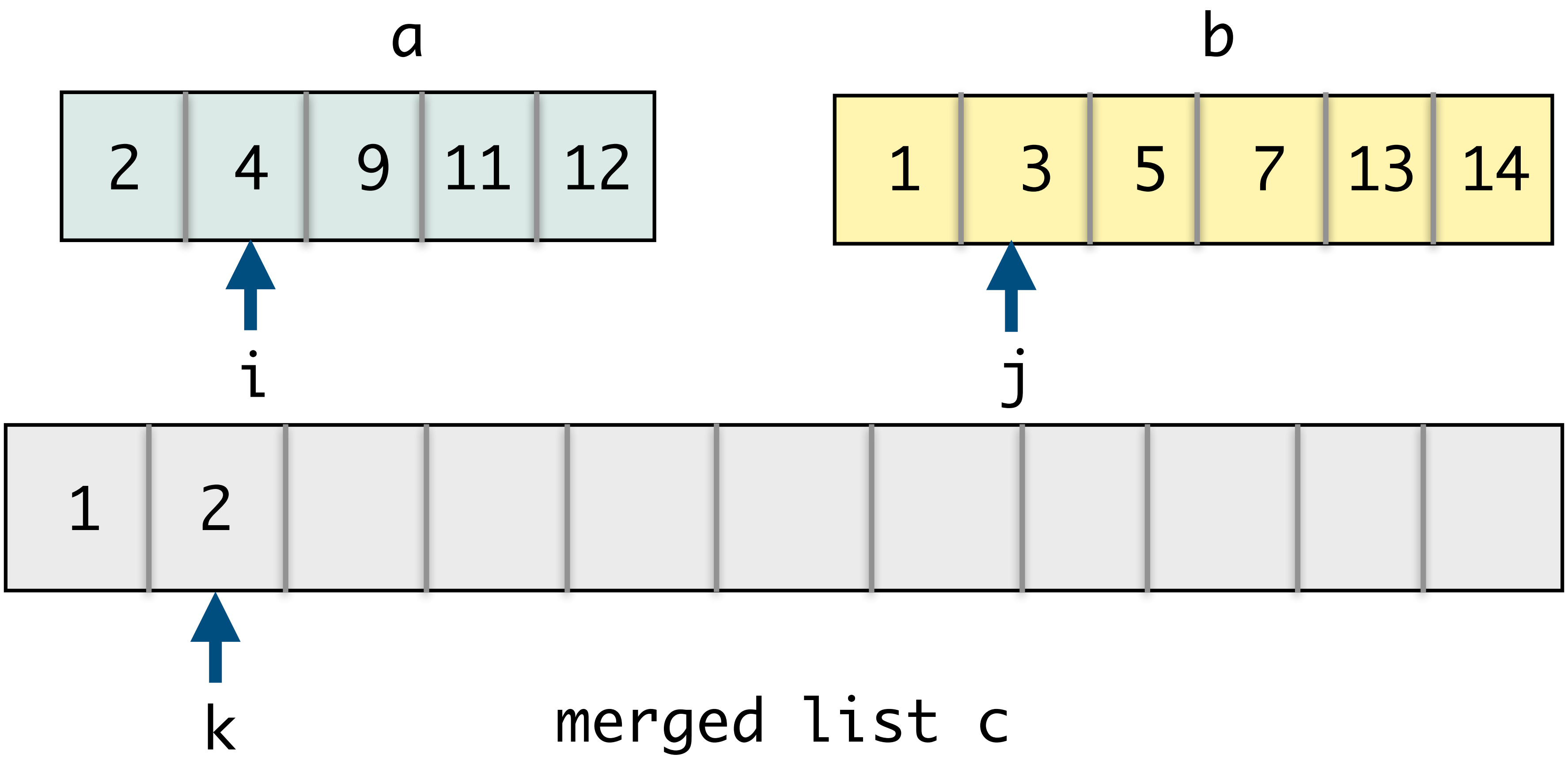

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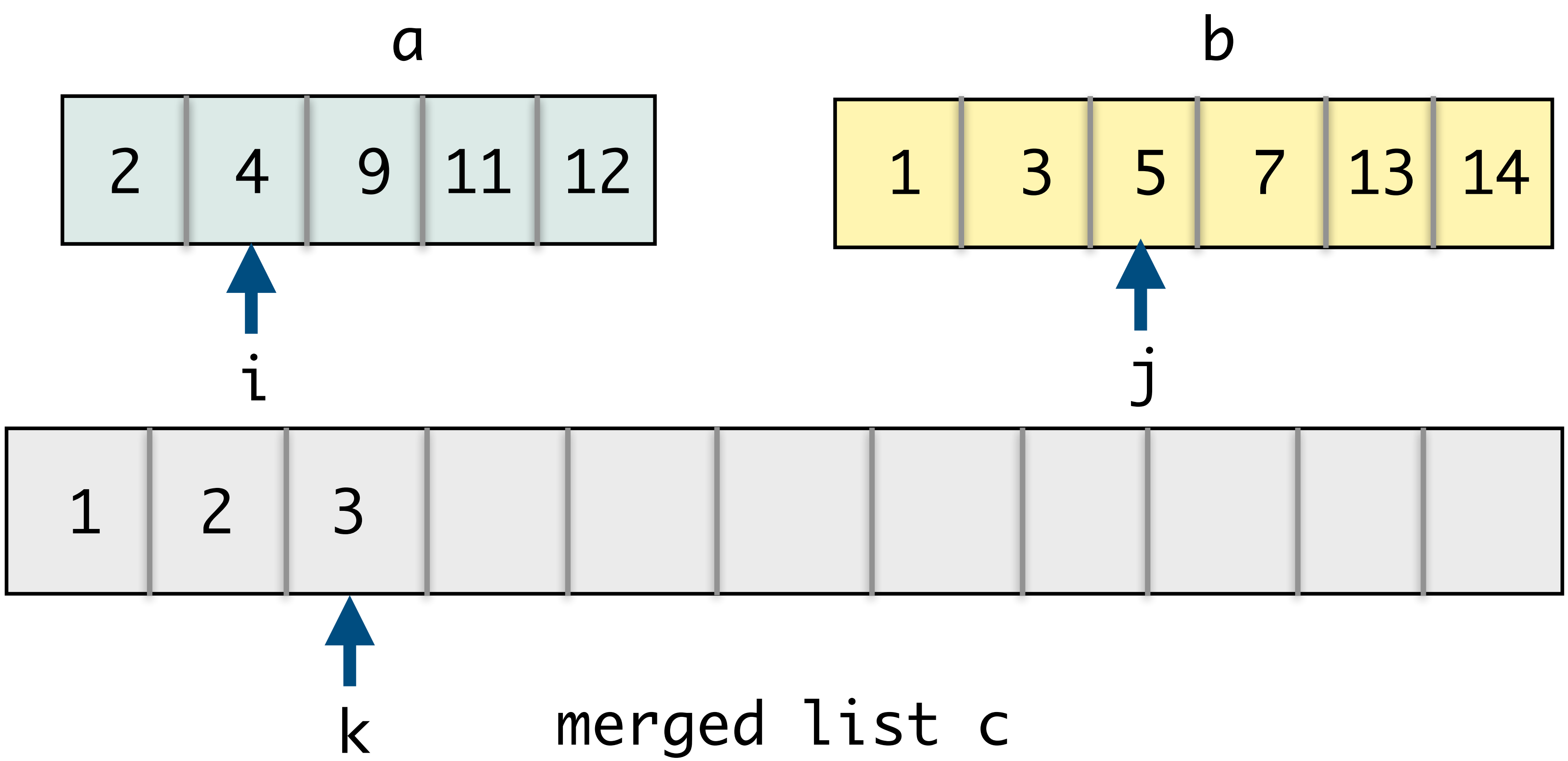

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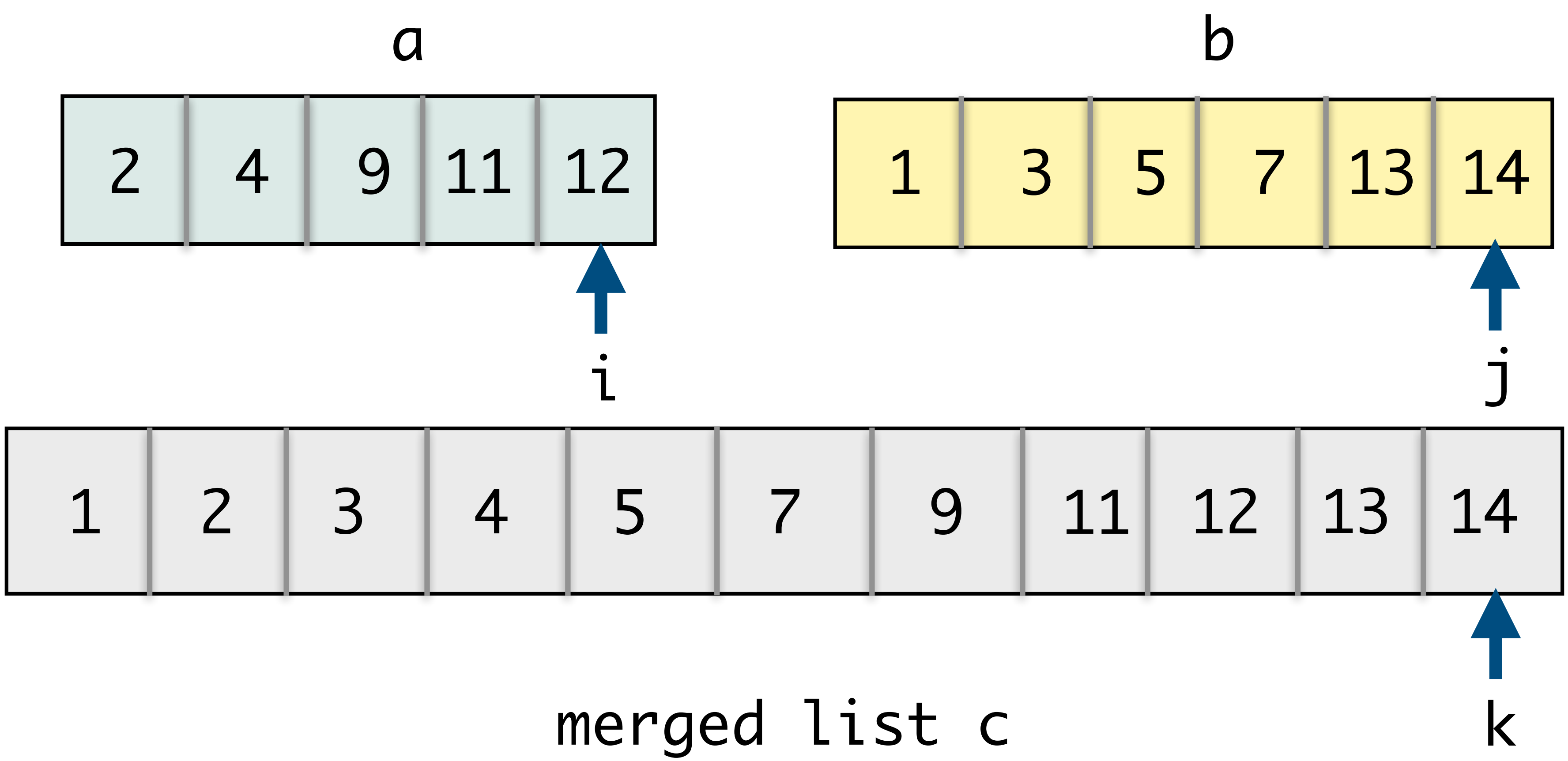


Merging Sorted Lists

- Walk through lists a, b, c maintaining current position of indices i, j, k
- Compare $a[i]$ and $b[j]$, whichever is smaller gets put in the spot of $c[k]$
- Merging two sorted lists into one is an $O(n)$ step algorithm!
- Can use this merge procedure to design our recursive merge sort algorithm!

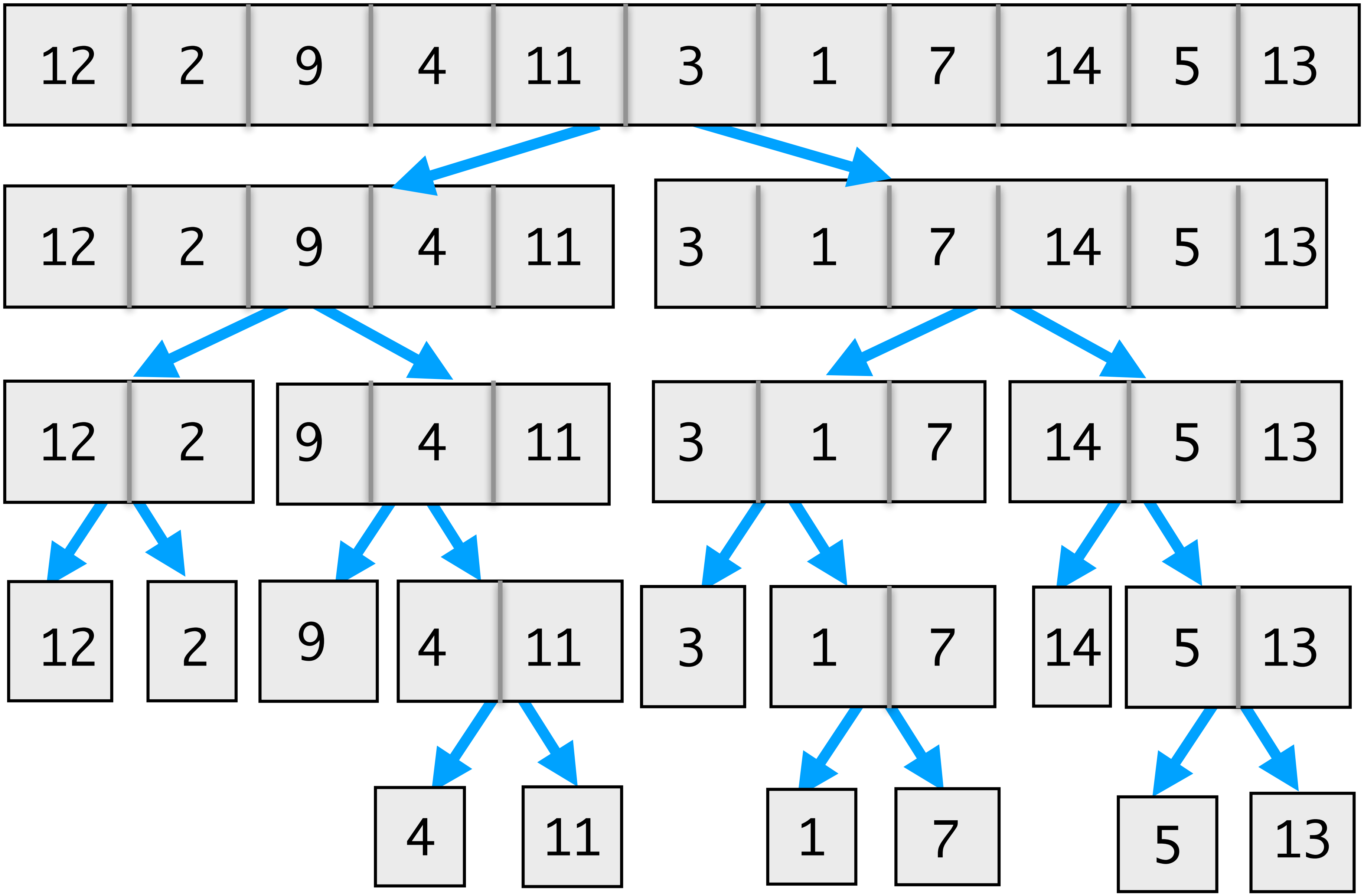
```python
def merge(a, b):
    """Merges two sorted lists a and b,
    and returns new merged list c"""
    # initialize variables
    i, j, k = 0, 0, 0
    lenA, lenB = len(a), len(b)
    c = []

    # traverse and populate new list
    while i < lenA and j < lenB:

        if a[i] <= b[j]:
            c.append(a[i])
            i += 1
        else:
            c.append(b[j])
            j += 1
        k += 1

    # handle remaining values
    if i < lenA:
        c.extend(a[i:])

    elif j < lenB:
        c.extend(b[j:])

    return c
```

Merge Sort Algorithm

- **Base case:** If list is empty or contains a single element: it is already sorted
- **Recursive case:**
  - Recursively sort left and right halves
  - Merge the sorted lists into a single list and return it
- **Question:**
  - Where is the ***sorting*** actually taking place?

```python
def mergeSort(L):
    """Given a list L, returns
    a new list that is L sorted
    in ascending order."""
    n = len(L)

    # base case
    if n == 0 or n == 1:
        return L

    else:
        m = n//2 # middle

        # recurse on left & right half
        sortLt = mergeSort(L[:m])
        sortRt = mergeSort(L[m:])

        # return merged list
        return merge(sortLt, sortRt)
```

Merge Sort Example



Merge Sort Example