[TAP:XLKTU] Balanced Trees

- Which of the following are not guaranteed to be “balanced”?
  A. AVL Tree
  B. Red-black Tree
  C. Heap
  D. They are all balanced
  E. Whatever

Administrative Details

- Lab 9 Today: Gardner's Hex-a-Pawn
  - Another partner lab!
  - Challenging to design & debug
  - Design doc is worth 2 points of your lab grade

Today's Outline

- Balanced Binary Search Trees
  - AVL Tree
  - Red-Black Tree
  - (Splay Tree)
- Game Trees

Splay Trees

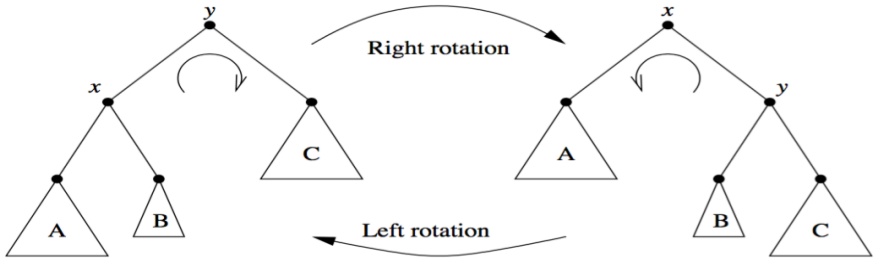
Splay trees are self-adjusting binary trees

- Each time a node is accessed, it is moved to root position via rotations
- No guarantee of balance (or shallow height)
- But good *amortized* performance

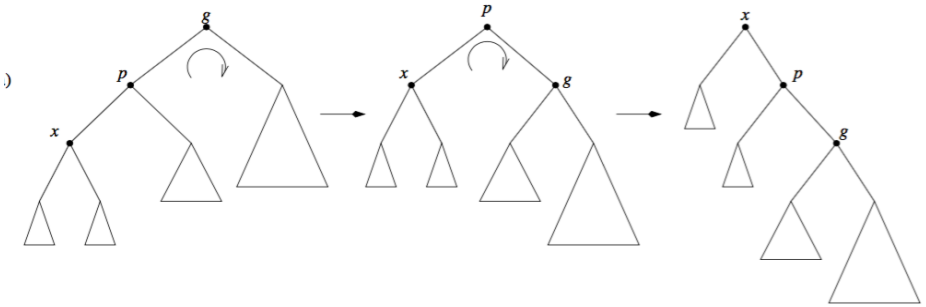
Theorem: Any set of m operations (add, remove, contains, get) on an n-node splay tree take at most O(m log n) time.

Splay Tree Rotations

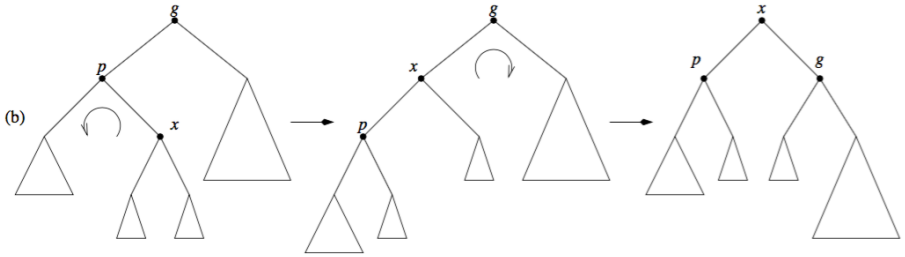
Right Zig Rotation (left version too)



Right Zig-Zig Rotation (left version too)



Right Zig-Zag Rotation (left version too)



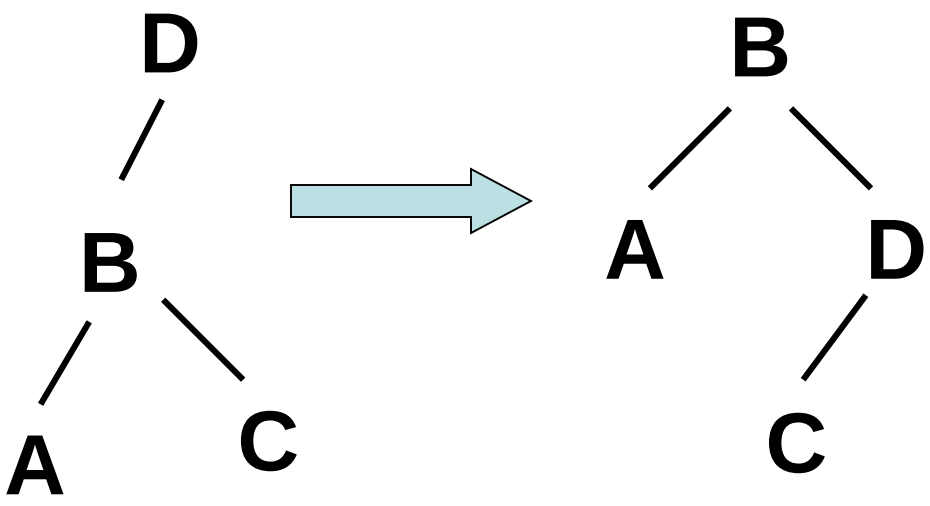
AVL Tree Facts

- A tree that is AVL except at root, where root balance factor equals ± 2 can be rebalanced with at most 2 rotations
- add(v) requires at most O(log n) balance factor changes and one (single or double) rotation to restore AVL structure
- remove(v) requires at most O(log n) balance factor changes and O(log n) (single or double) rotations to restore AVL structure
- An AVL tree on n nodes has height O(log n)

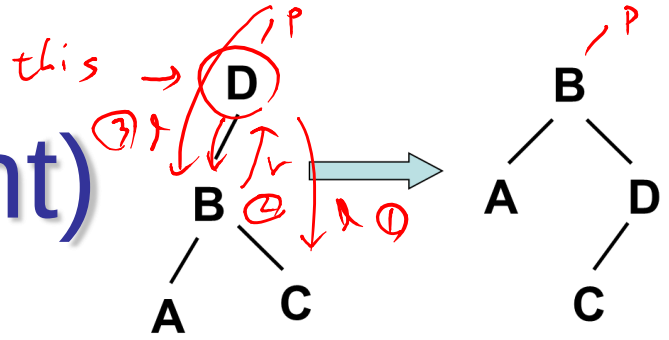
Red-Black Tree Facts

- The coloring rules lead to the following result
  - No leaf has depth more than twice that of any other leaf.
  - A Red-Black tree with n nodes has height $O(\log n)$

Single Rotation (Right)



Single Rotation (Right)




```
// pre: this has a left subtree
// post: rotates local portion of tree so left child is root
protected void rotateRight() {
```

BinaryTree<E> p = parent;
boolean wasLeftChild = isLeftChild();

BinaryTree<E> newRoot = left;

setLeft(newRoot.right); // ①
newRoot.setRight(this); // ②

if (parent != null) {
  if (wasLeftChild)
    parent.setLeft(newRoot); ← ③
  } else
    parent.setRight(newRoot);

~~newRoot.setParent(parent);~~
~~setParent(newRoot);~~ ~~left.setParent(this);~~

purple if setLeft() and setRight()
updates the parent

```
}
```

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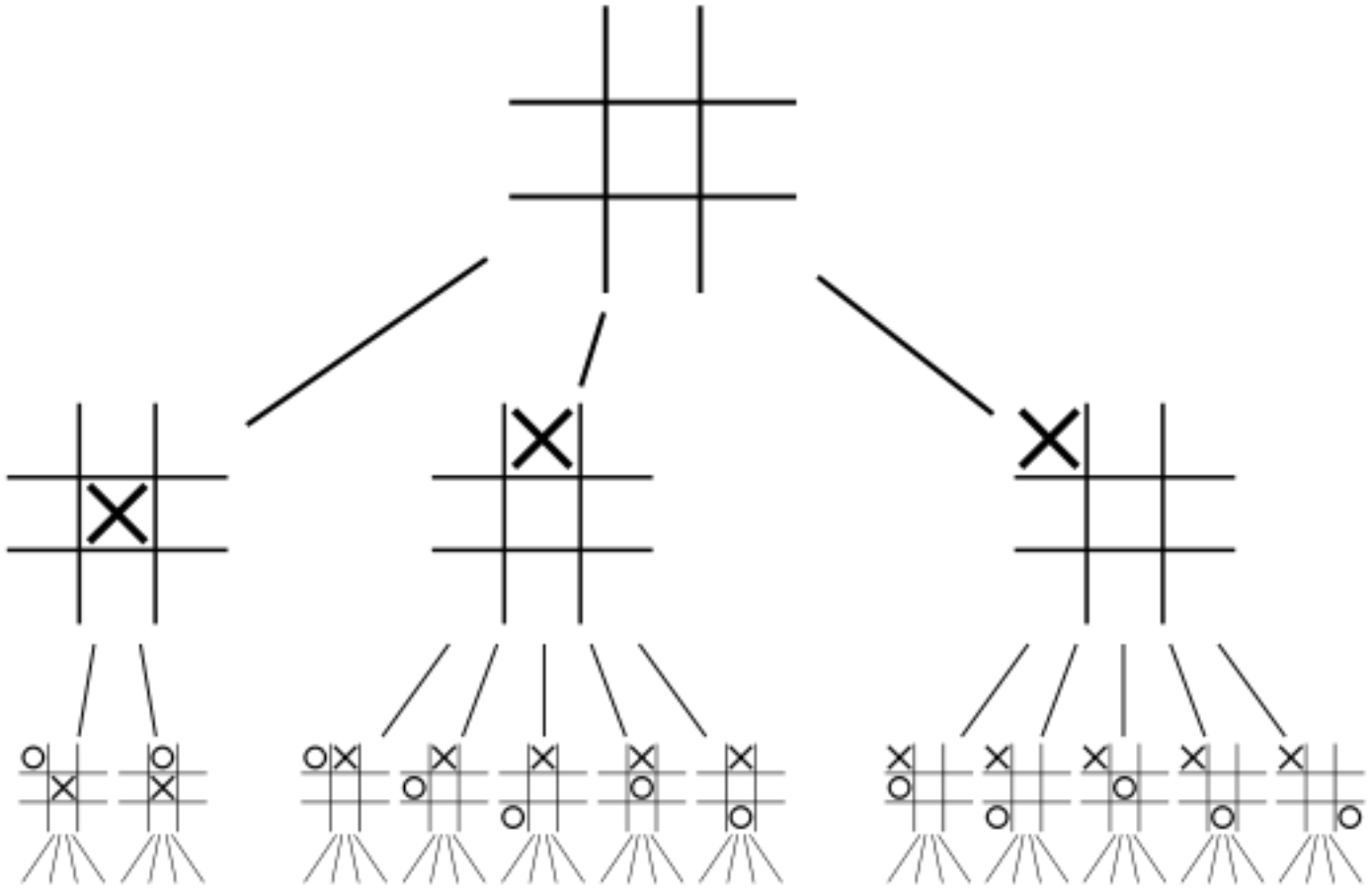
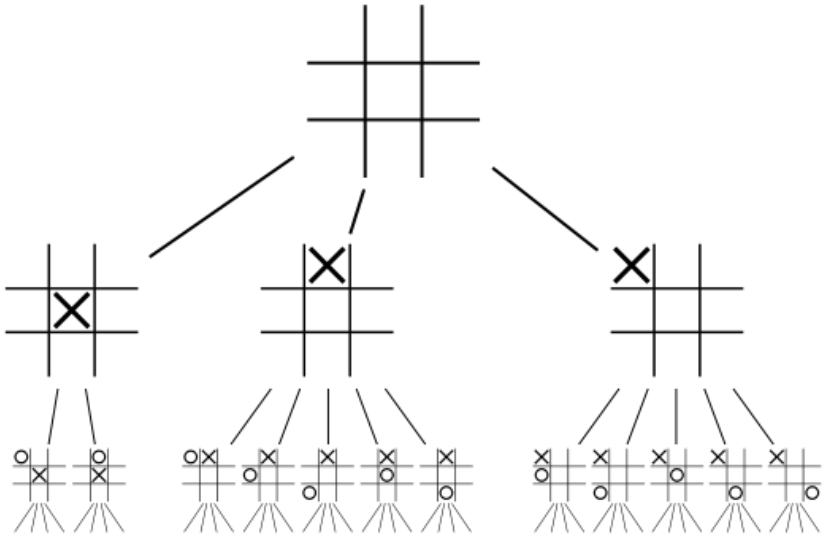
Game Trees



Game Trees

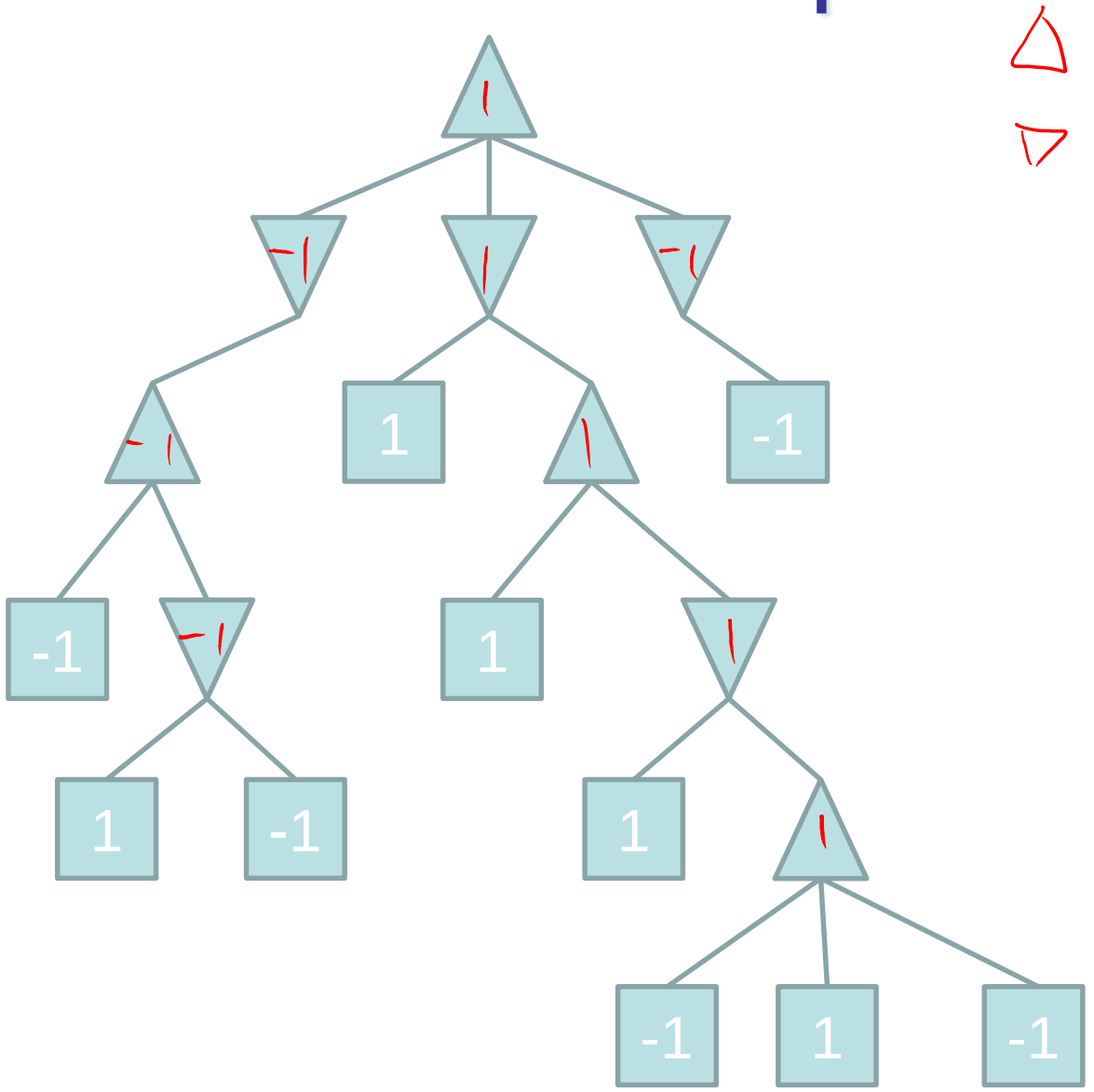
- Nodes are positions in a game (game state)
- Edges are moves (transition from one game state to another)
  - All nodes at a given level represent moves by the same player
- Leaf nodes represent ending board states (winner or tie)
  - # of leaf nodes = # of ways a game can be played



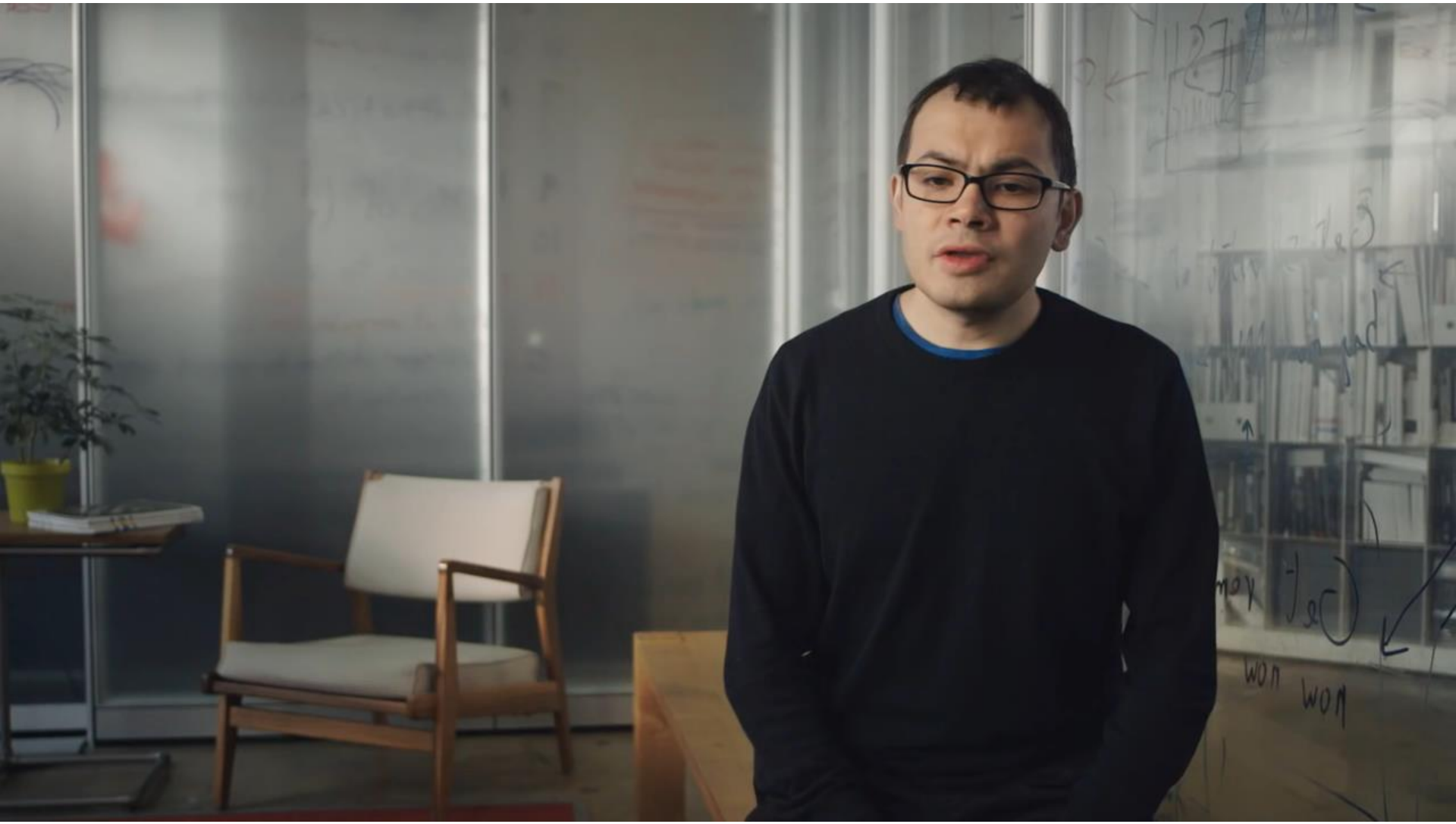
Game Trees

- In AI, often search the game tree and use an algorithm like **minimax** to choose the next “best move”
  - Chess, checkers, Go, etc.

Minimax Example



Modern Game AI



Hexapawn