CSCI 136
Data Structures & Advanced Programming

Lecture 22

Spring 2018

Profs Bill & Jon

Administrative Details

- CS Colloquium?!?!
  - Meets (almost) every Friday at 2:30pm
  - Guest speaker presents their research
  - Next Friday (4/20) we will have an information session instead of a normal speaker
    - Discussion of courses offered next semester
    - Advising about majoring in CS
      - We can sign major declaration sheets there
  - Food!

Last Time

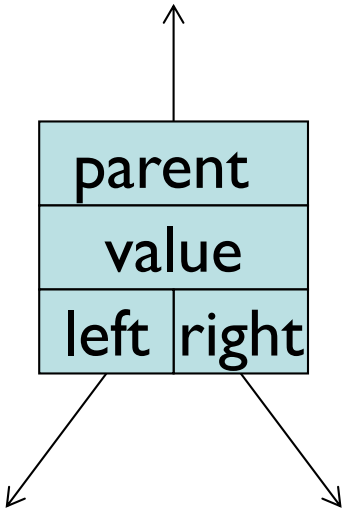
- The `structure5 BinaryTree` class
  - implementation details
- Quick proofs and theory
  - Number of nodes at a given depth d is at most 2^d
  - The total number of nodes in a tree of height n is at most $2^{(n+1)}-1$
  - A tree with n nodes has exactly n-1 edges
- Implications: if a tree is balanced (full or complete), the height is $\log_2 n$

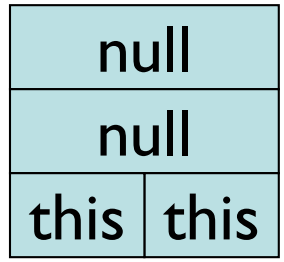
Today

- Traversing trees
  - Pre/post/in/level-order traversals
  - Iterators for each traversal strategy
- Alternative Tree Representations
  - Array-based binary trees

Implementing structure5 BinaryTree

- BinaryTree<E> class
  - Instance variables
    - `BinaryTree`: `parent`, `left`, `right`
    - `E`: `value`





EMPTY BT

- `public BinaryTree()`
- `public BinaryTree(E value)`
- `public BinaryTree(E value,`
  `BinaryTree<E> left,`
  `BinaryTree<E> right)`

Implementing BinaryTree

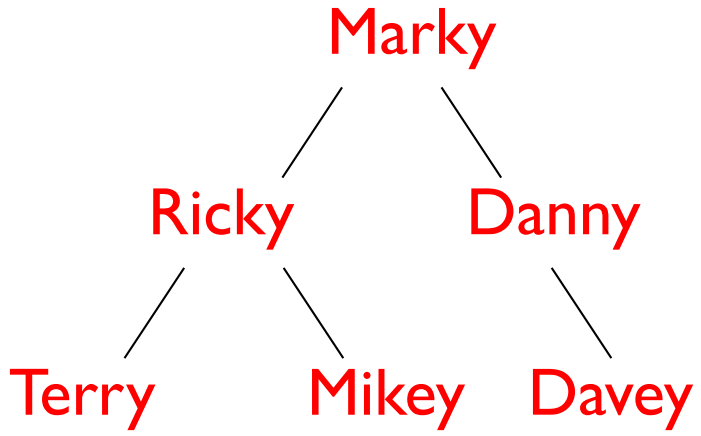
- Connect BinaryTree nodes:
  - public void setLeft(BinaryTree<E> newLeft)
    public void setRight(BinaryTree<E> newLeft)
    protected void setParent(BinaryTree<E> newParent)
- Navigate edges:
  - public BinaryTree<E> left()
    public BinaryTree<E> right()
    public BinaryTree<E> parent()
- Interact with BinaryTree data:
  - public E value()
    public void setValue(E value)
- Get an Iterator:
  - public Iterator<E> iterator()
  - public Iterator<E> preorderIterator()
    public Iterator<E> postorderIterator()
    public Iterator<E> levelorderIteratorterator()

???

Tree Traversals

- In linear structures, there are only a few basic ways to traverse the data structure
  - Start at one end and visit each element
  - Start at the other end and visit each element
- How do we traverse binary trees?
  - (At least) four reasonable mechanisms

Tree Traversals



In-order: "left, node, right"

Terry, Ricky, Mikey, Marky, Danny, Davey

Pre-order: "node, left, right"

Marky, Ricky, Terry, Mikey, Danny, Davey

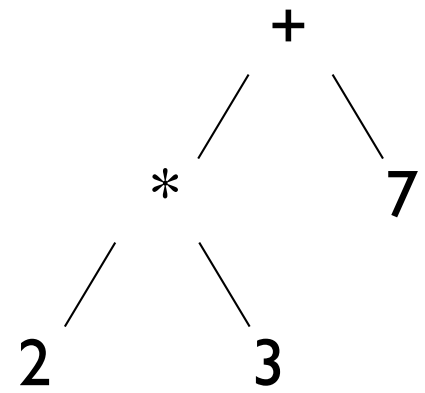
Post-order: "left, right, node"

Terry, Mikey, Ricky, Davey, Danny, Marky,

Level-order: visit all nodes at depth `i` before depth `i+1`

Marky, Ricky, Danny, Terry, Mikey, Davey

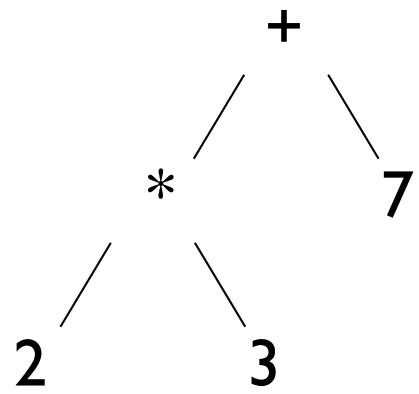
Tree Traversals



- Pre-order
  - Each node is visited before any children. Visit node, then each node in left subtree, then each node in right subtree. (node, left, right)
    - +*237
- In-order
  - Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree. (left, node, right)
    - 2*3+7

("pseudocode")

Tree Traversals

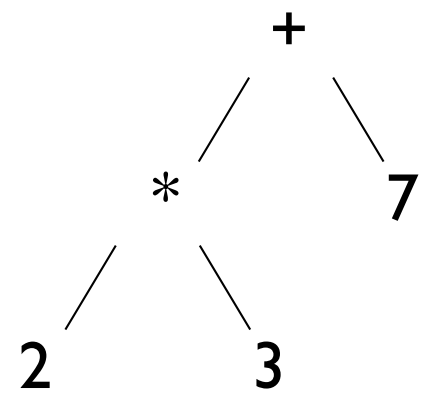


- Post-order
  - Each node is visited after its children are visited. Visit all nodes in left subtree, then all nodes in right subtree, then node itself. (left, right, node)
    - 23*7+
- Level-order (not obviously recursive!)
  - All nodes of level `i` are visited before nodes of level `i+1`. (visit nodes left to right on each level)
    - +*723

("pseudocode")

Tree Traversals

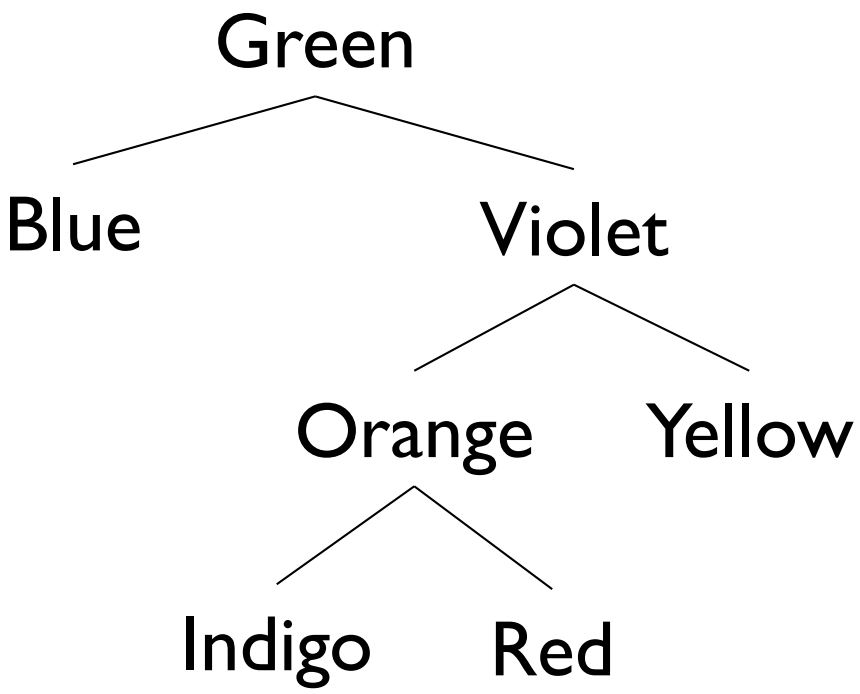
```
public void preOrder(BinaryTree t) {
      if(t.isEmpty())
            return;
      visit(t); // some method
      preOrder(t.left());
      preOrder(t.right());
}
```



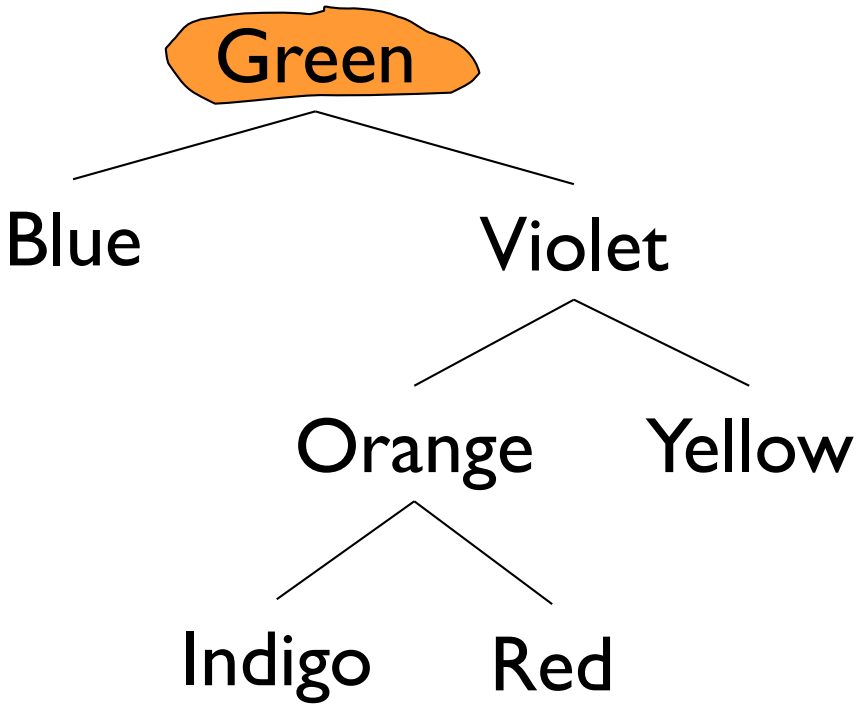
For in-order and post-order: just move `visit(t)`!

But what about level-order???

Level-Order Traversal



Level-Order Traversal



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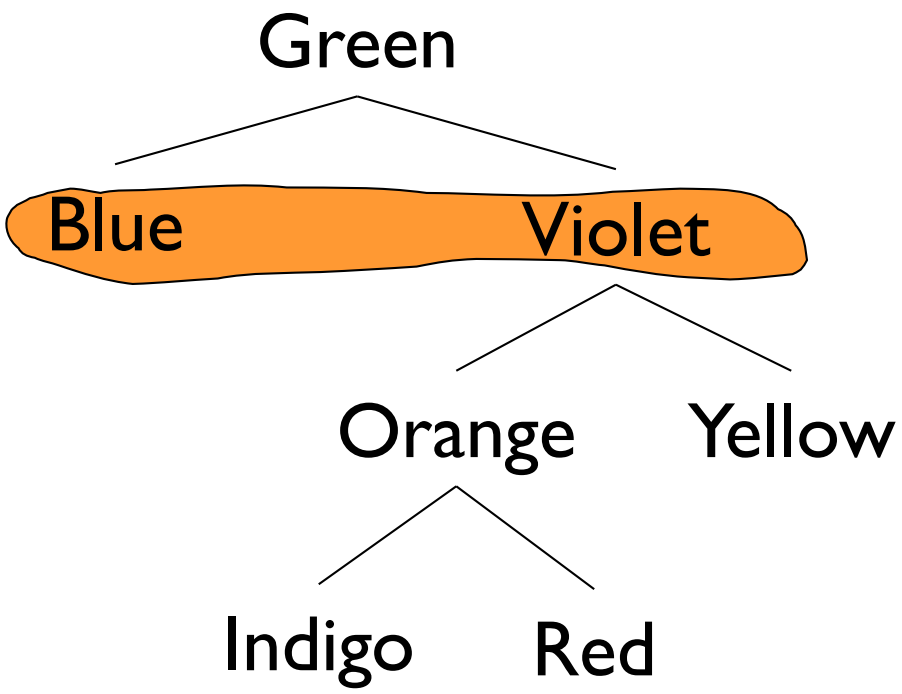
Level-Order Traversal



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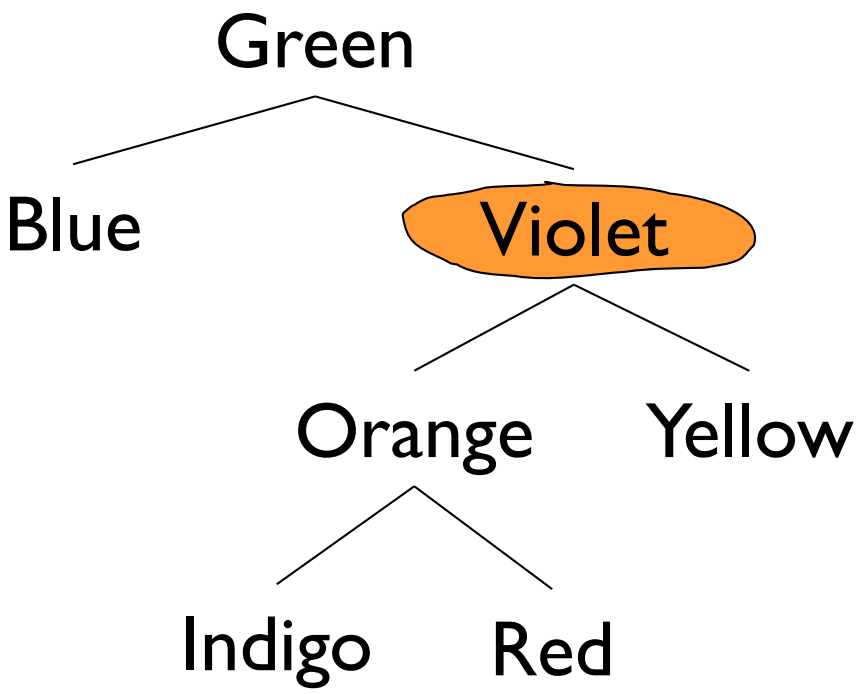

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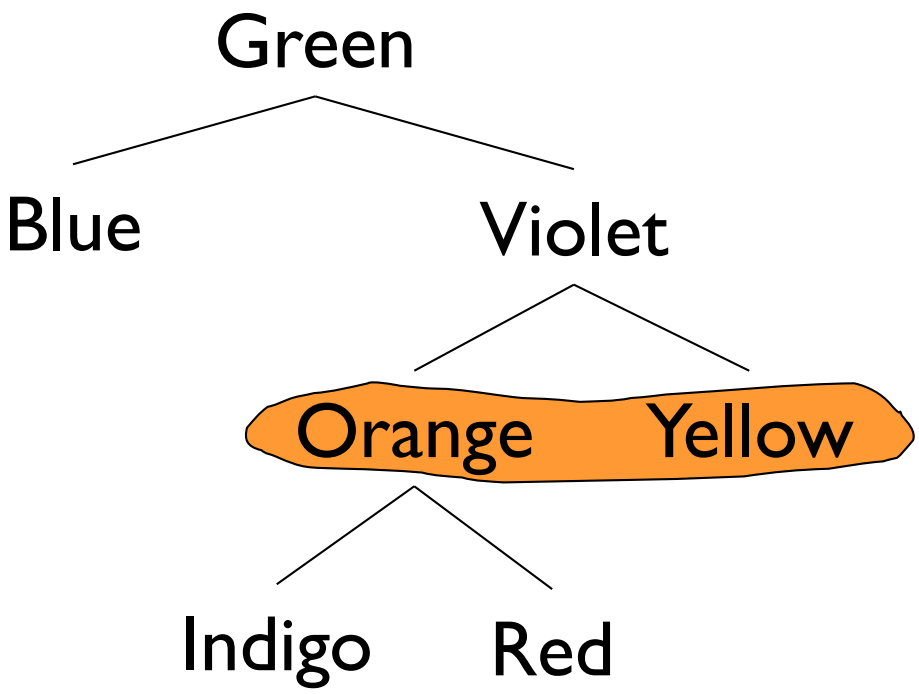
Level-Order Traversal



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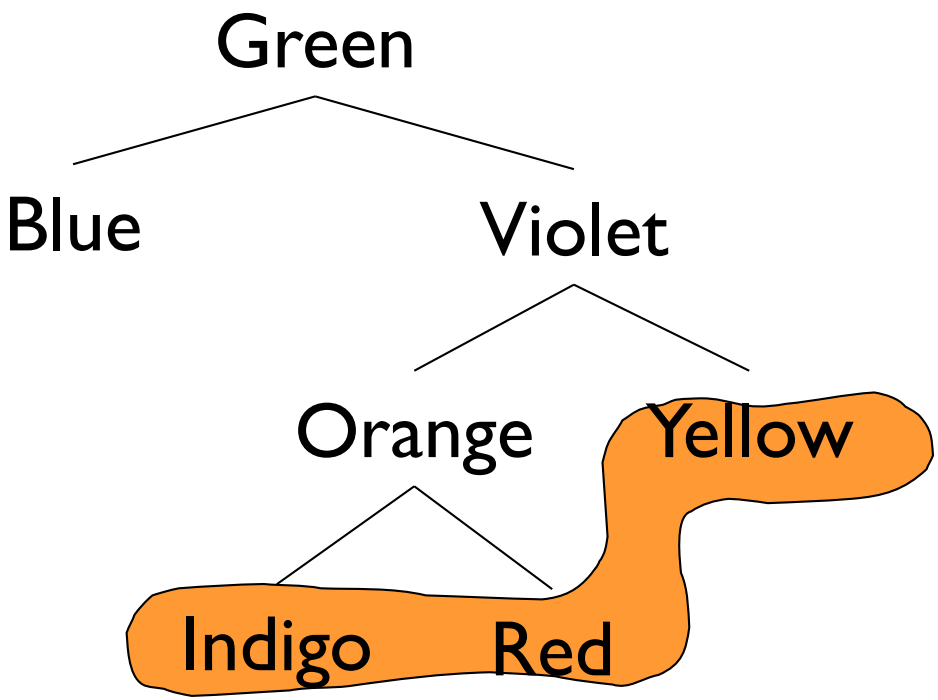
Level-Order Traversal



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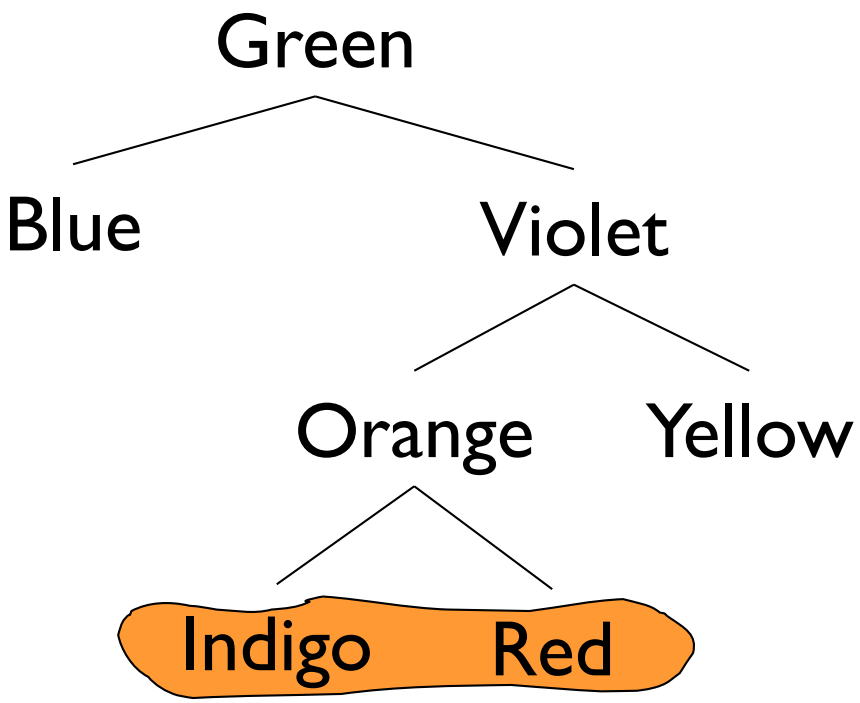
Level-Order Traversal



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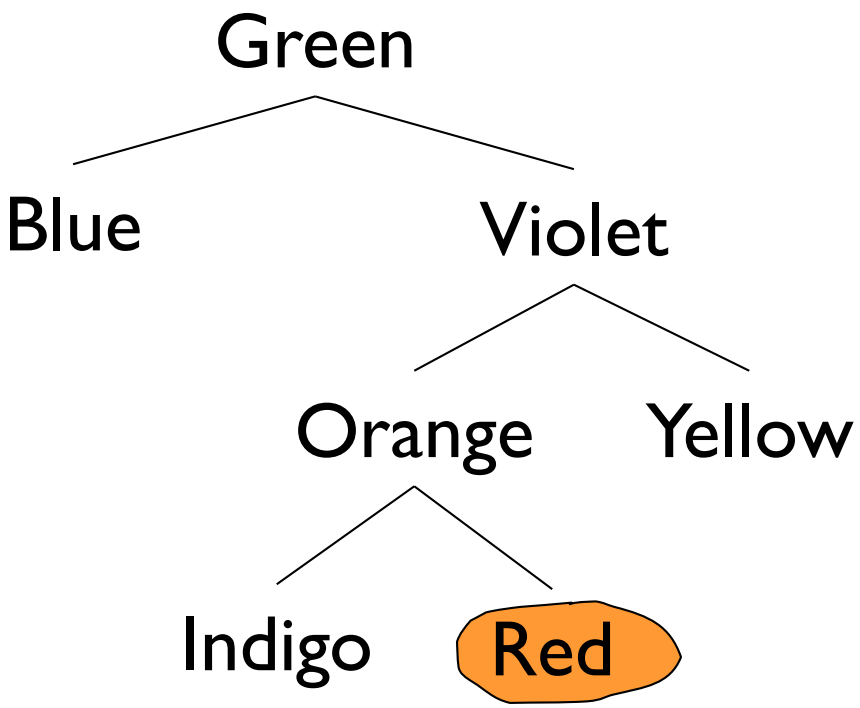
Level-Order Traversal



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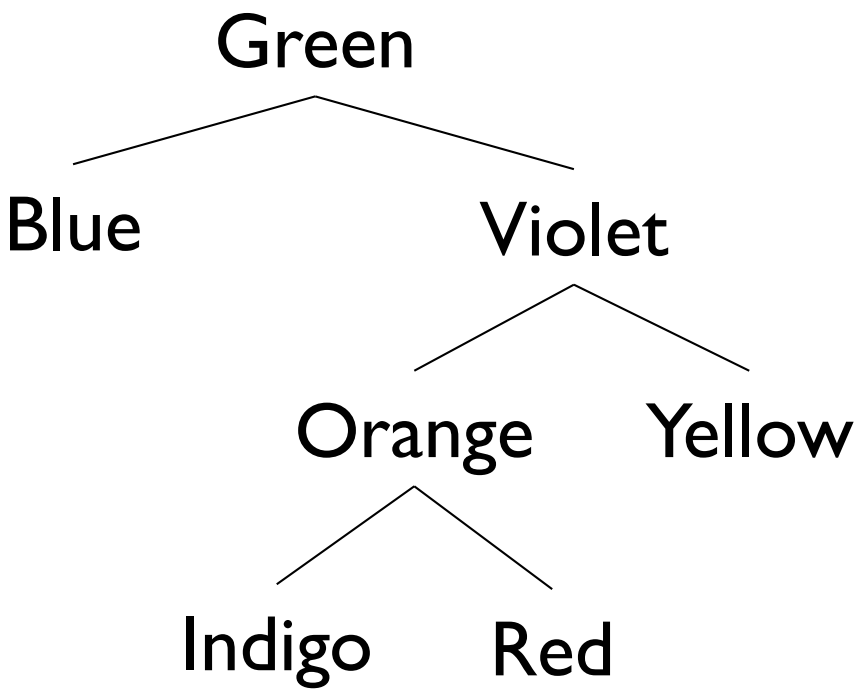
Level-Order Traversal

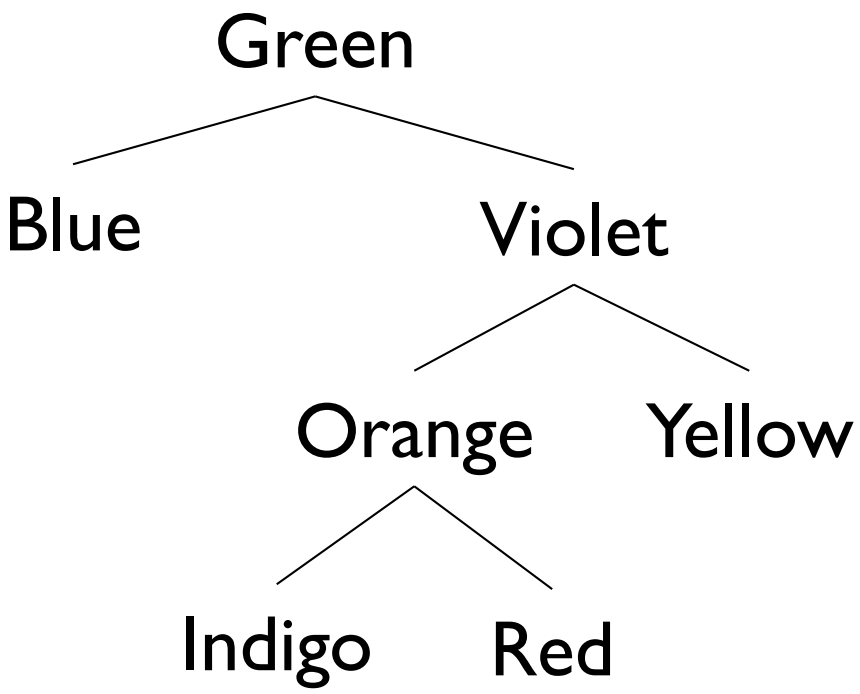


G B V O Y I R

Level-Order Traversal

- How does level-order work?
  - We visit the nodes level by level, left to right
- Hint: we will use a linear structure…

Level-Order Traversal



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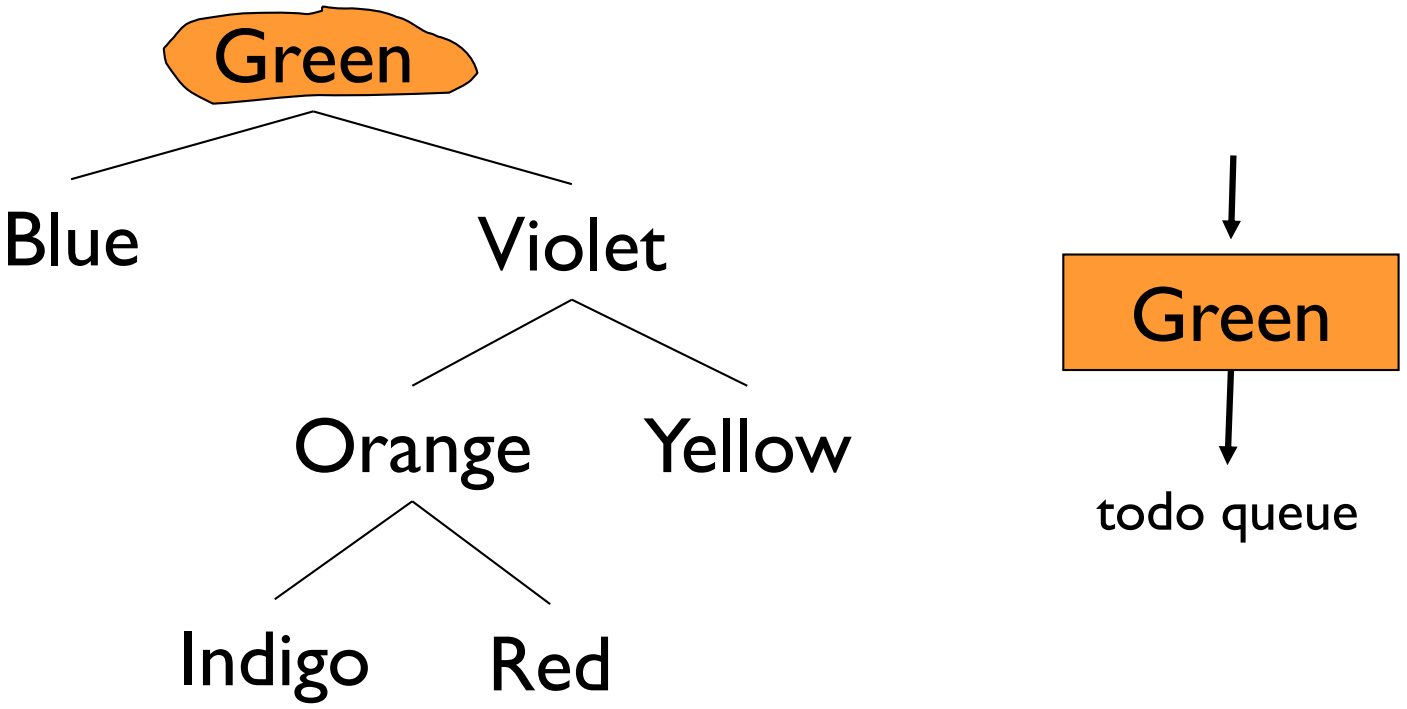
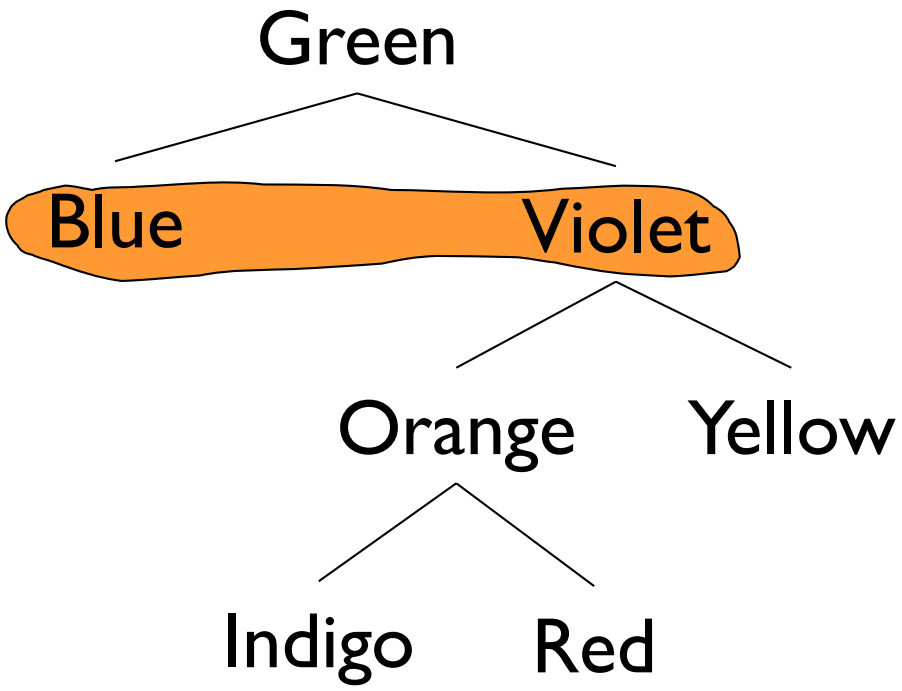
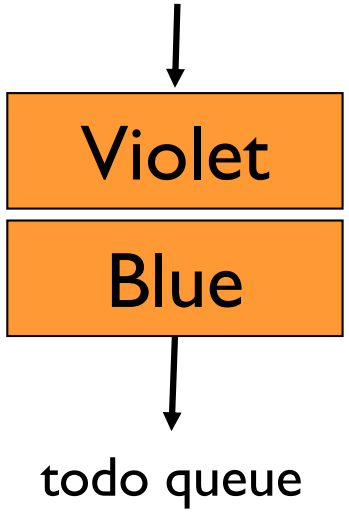


Level-Order Traversal





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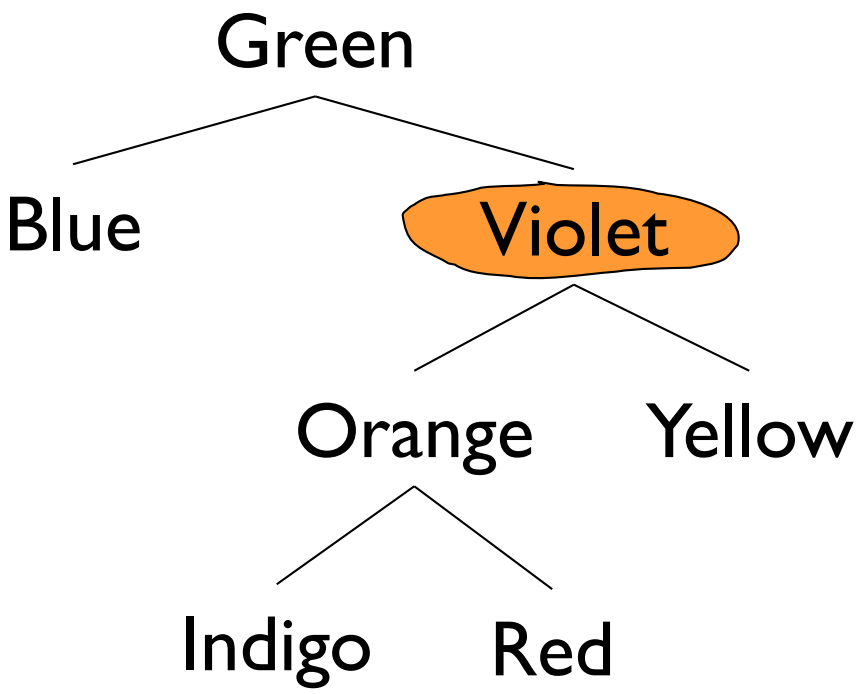
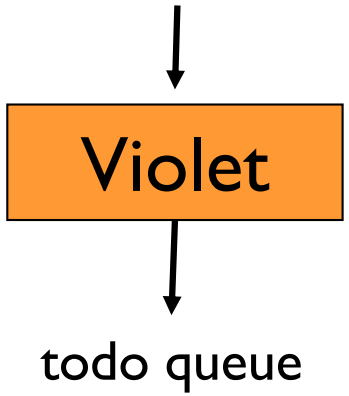
Level-Order Traversal





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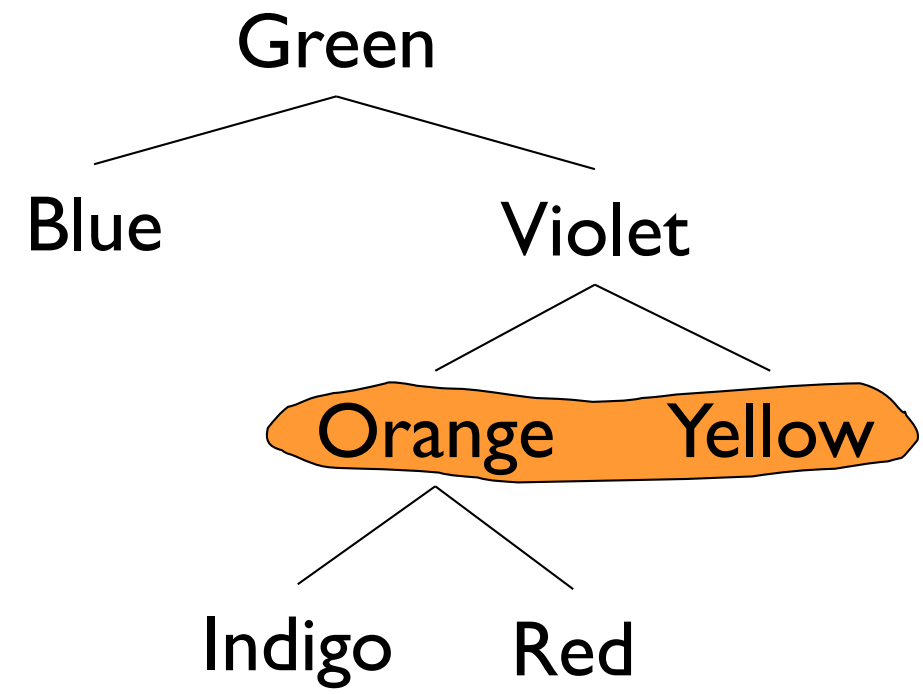
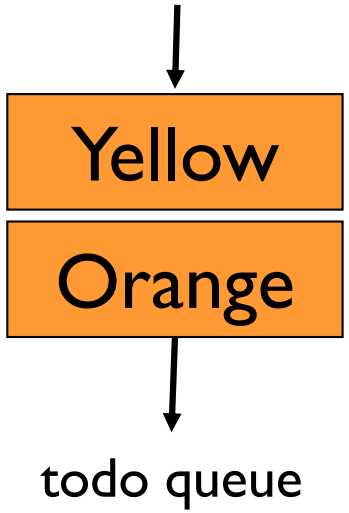
Level-Order Traversal





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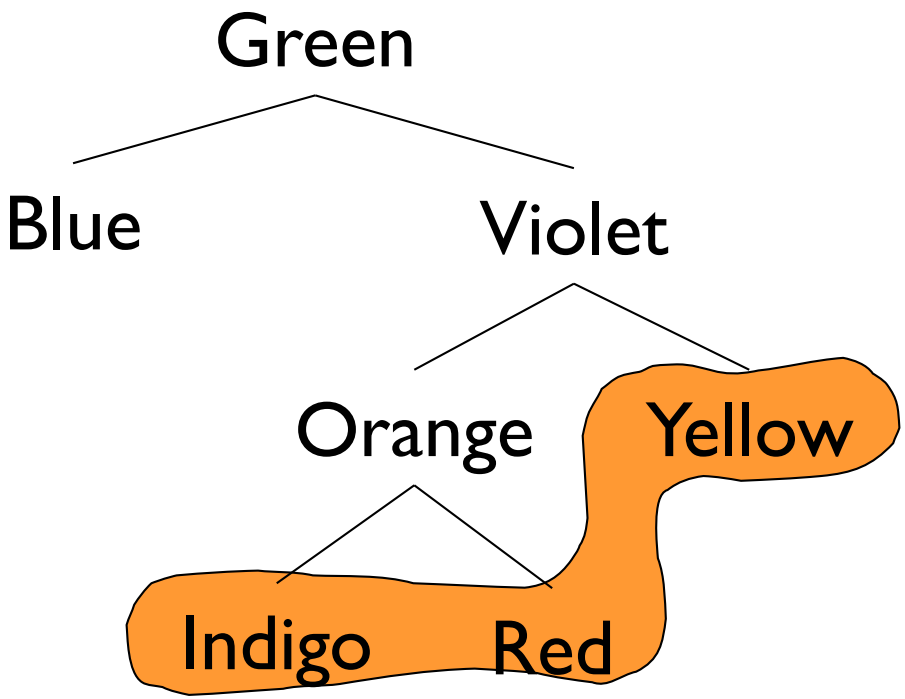
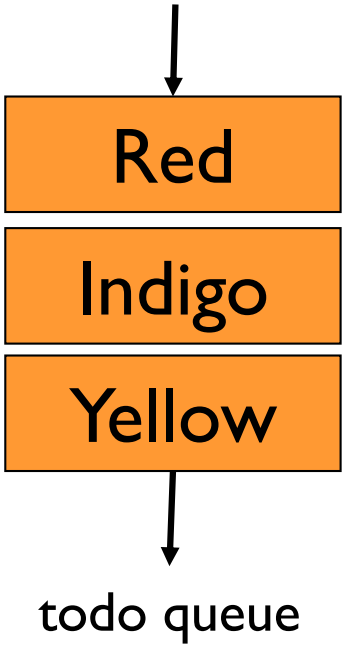
Level-Order Traversal





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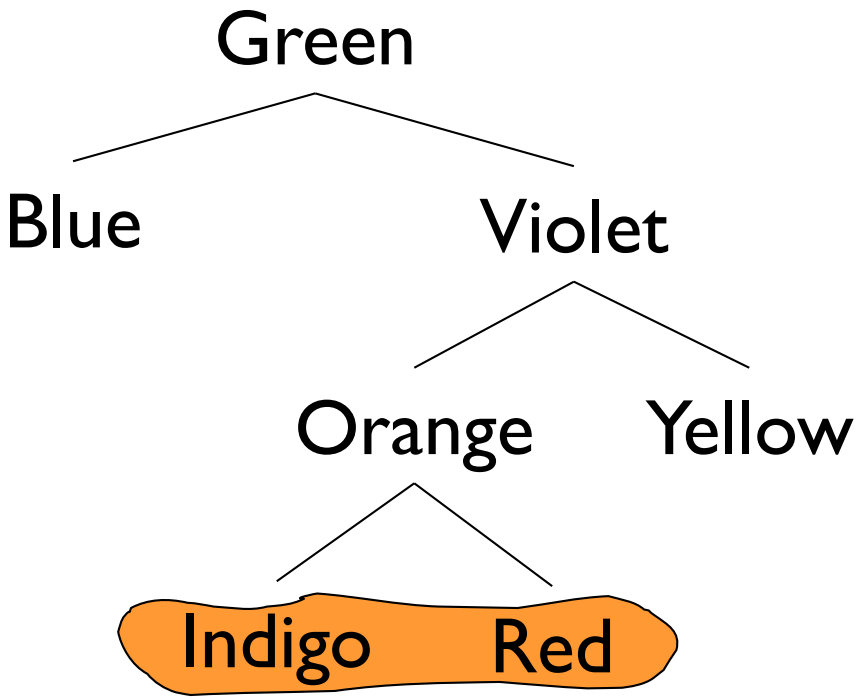
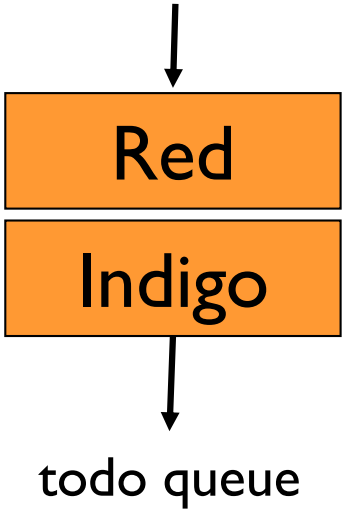
Level-Order Traversal





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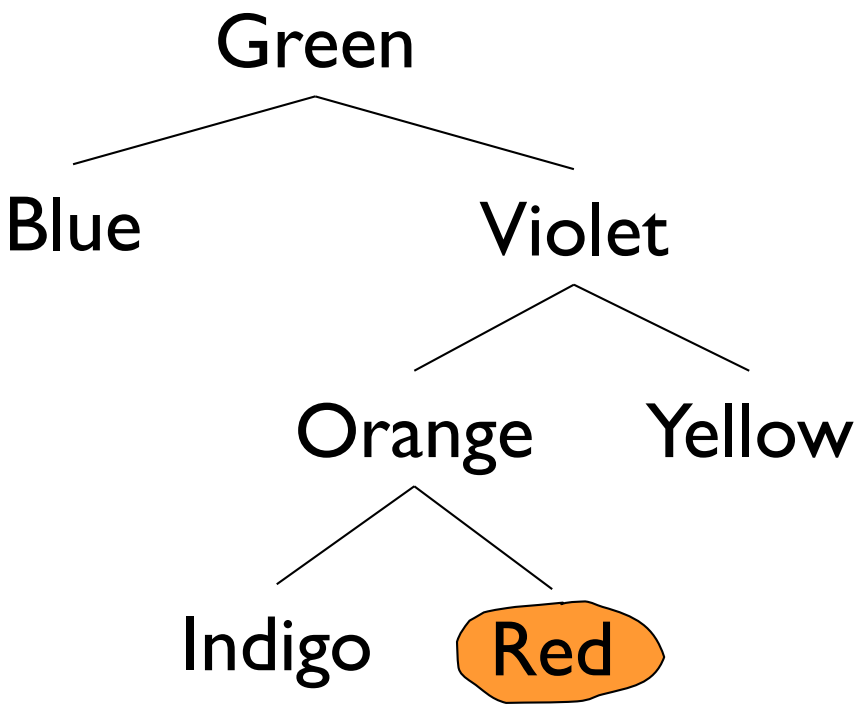
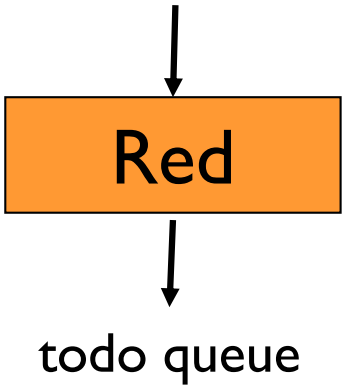
Level-Order Traversal





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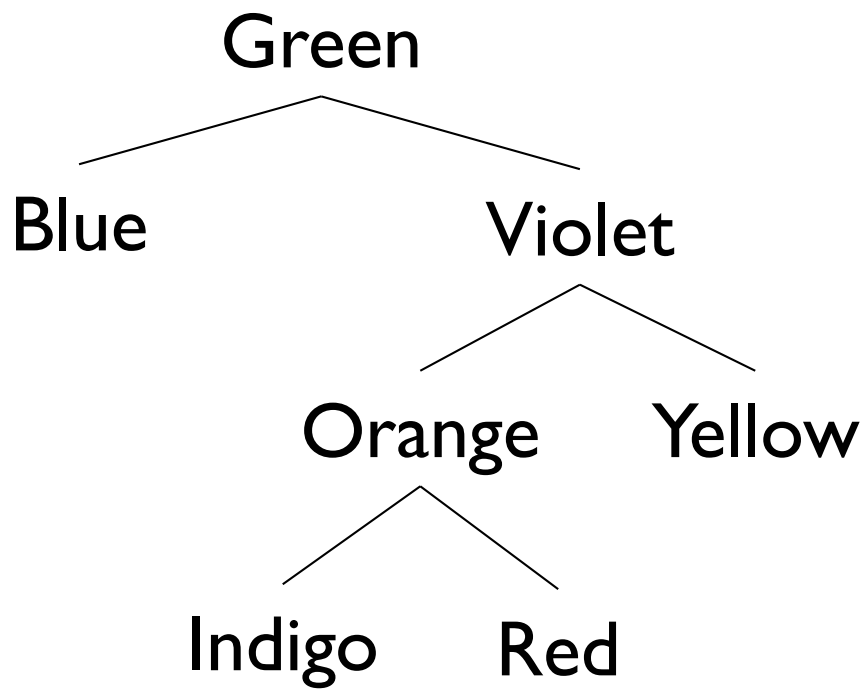
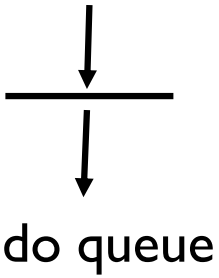
Level-Order Traversal





G B V O Y I R

Level-Order Tree Traversal

```
public static <E> void levelOrder(BinaryTree<E> t) {
   if (t.isEmpty()) return;

   // The queue holds nodes for in-order processing
   Queue<BinaryTree<E>> q = new QueueList<BinaryTree<E>>();
   q.enqueue(t); // put root of tree in queue

   while(!q.isEmpty()) {
      BinaryTree<E> next = q.dequeue();
      visit(next);
      if(!next.left().isEmpty())  q.enqueue(next.left());
      if(!next.right().isEmpty()) q.enqueue(next.right());
   }
}
```

Iterators

- Provide iterators that implement the different tree traversal algorithms

- Methods provided by BinaryTree class:
  - preorderIterator()
  - inorderIterator()
  - postorderIterator()
  - levelorderIterator()

Implementing the Iterators

- Basic idea
  - Should return elements in same order as corresponding traversal method shown
  - Recursive methods don't convert as easily: must phrase in terms of next() and hasNext()
    - Similar to how we implemented SkipIterator: do some prep work before returning from next()
  - So, let's start with levelOrder!

Level-Order Iterator

```
public BTLevelorderIterator(BinaryTree<E> root) {
        todo = new QueueList<BinaryTree<E>>();
        this.root = root; // needed for reset
        reset();
}

public void reset() {
        todo.clear();
        // empty queue, add root
        if (!root.isEmpty()) todo.enqueue(root);
}
```

Level-Order Iterator

```
public boolean hasNext() {
        return !todo.isEmpty();
}

public E next() {
        BinaryTree<E> current = todo.dequeue();
        E result = current.value();
        if (!current.left().isEmpty())
            todo.enqueue(current.left());
        if (!current.right().isEmpty())
            todo.enqueue(current.right());
        return result;
}
```

Pre-Order Iterator

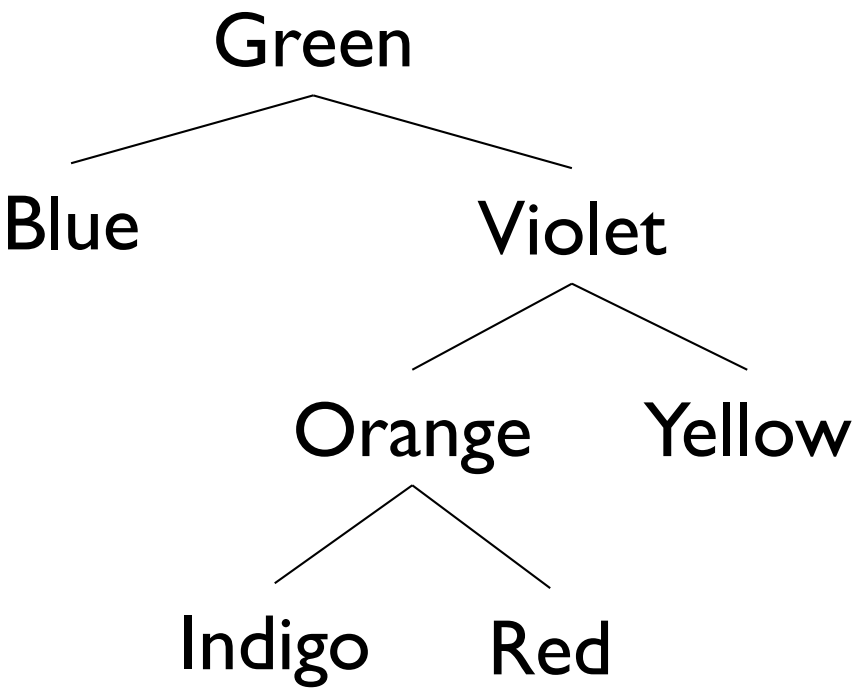
- Basic idea
  - Should return elements in same order as processed by pre-order traversal method
  - Must phrase in terms of next() and hasNext()
  - We "simulate recursion" with stack
    - The stack holds "partially processed" nodes

Pre-Order Iterator

- Outline: node - left tree – right tree
  1. Constructor: Push root onto todo stack
  2. On call to next():
     - Pop node from stack
     - Push right and then left children of popped node onto stack
     - Return node's value
  3. On call to hasNext():
     - return !stack.isEmpty()

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



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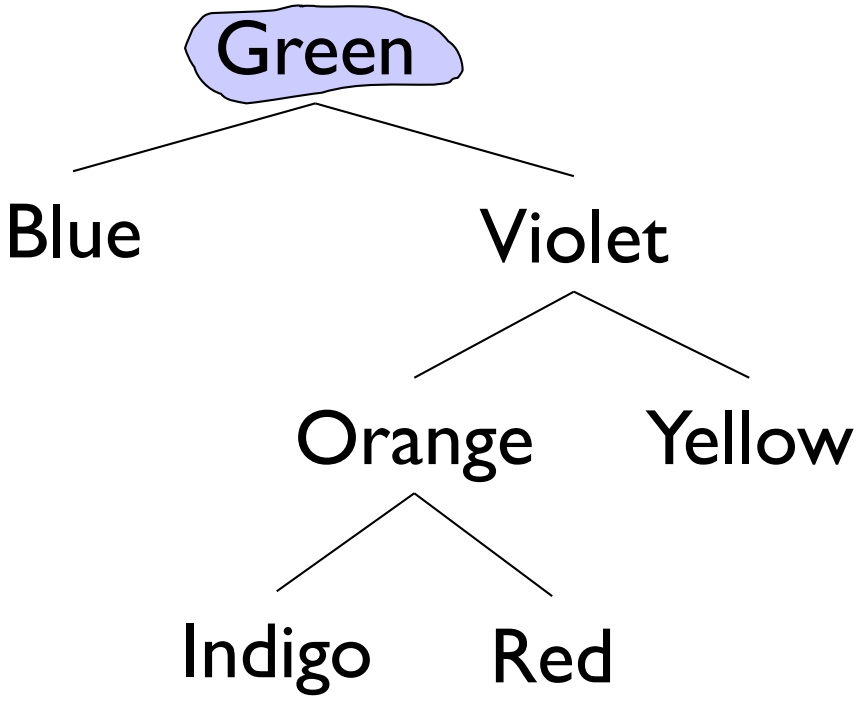
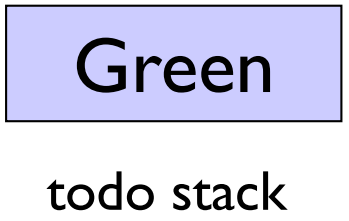
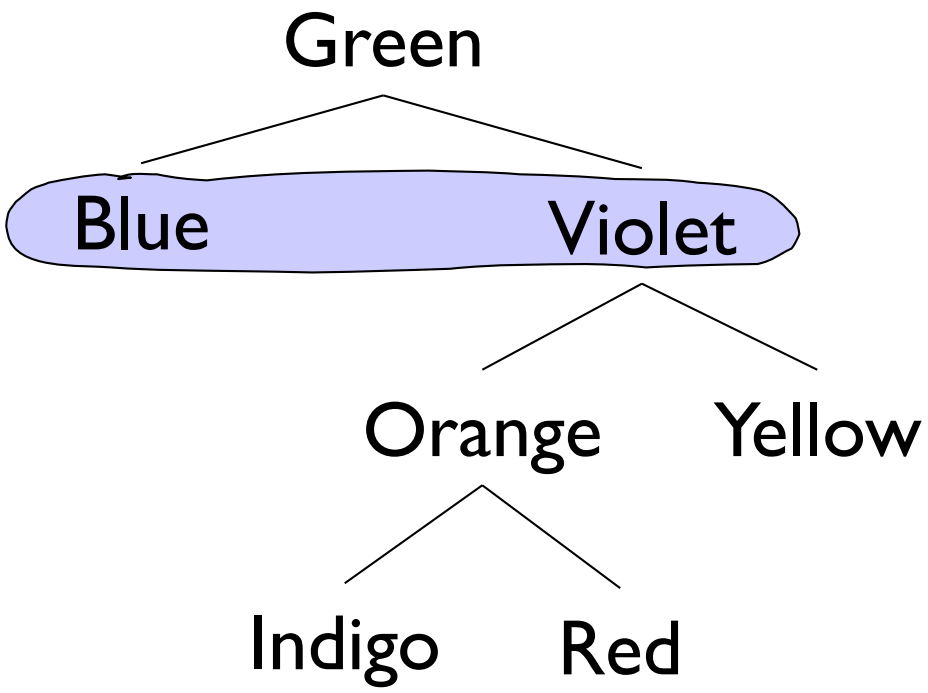


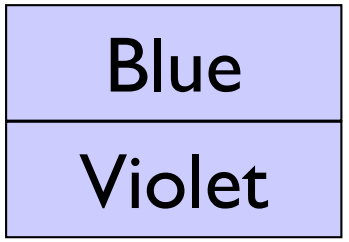


todo stack

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.





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Pre-Order Iterator

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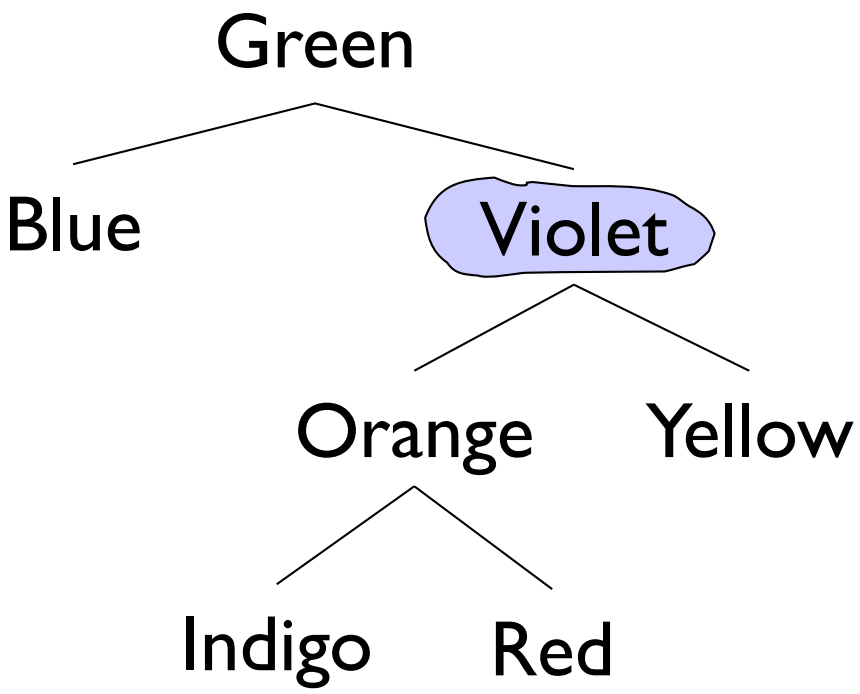

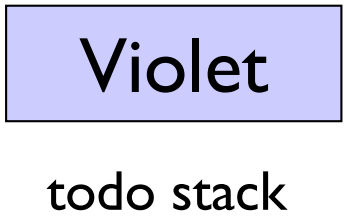

todo stack

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Pre-Order Iterator

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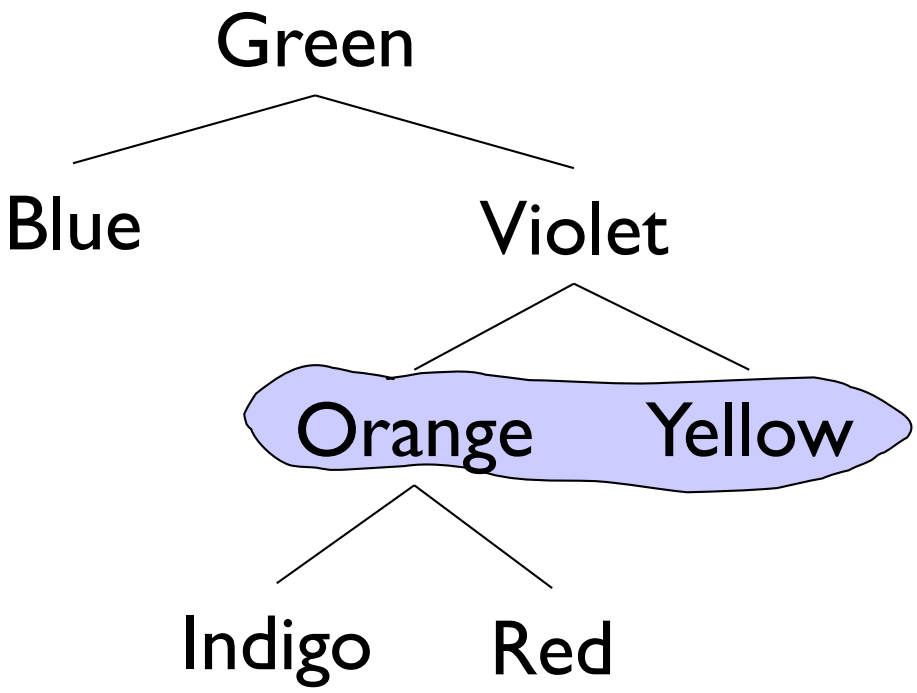

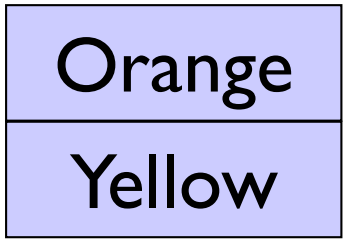

todo stack

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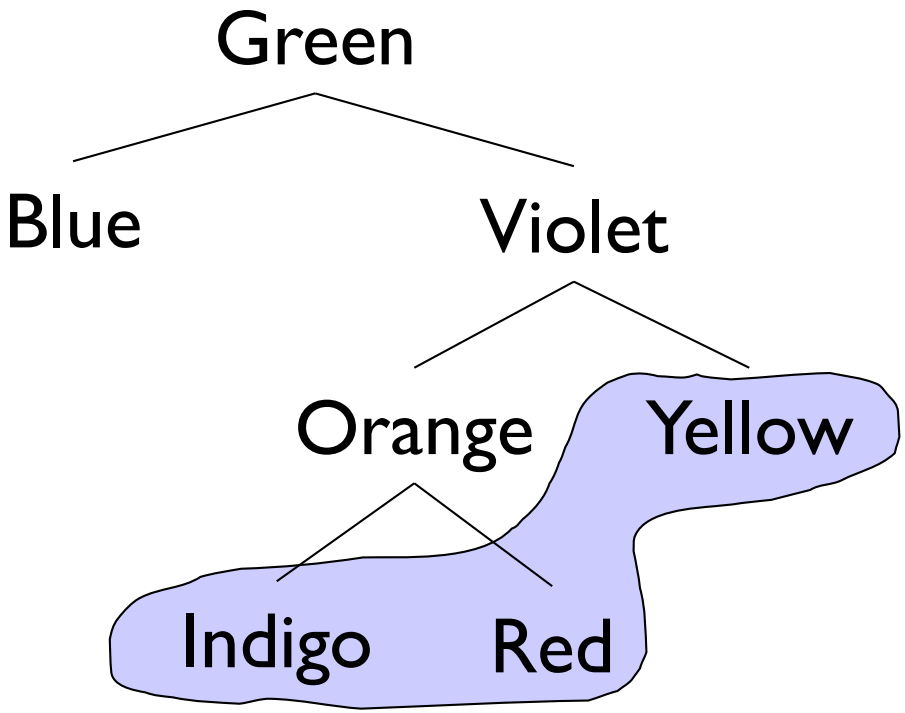

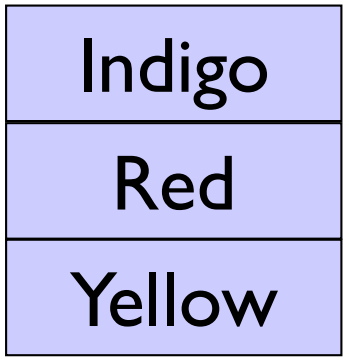

todo stack

G B V O

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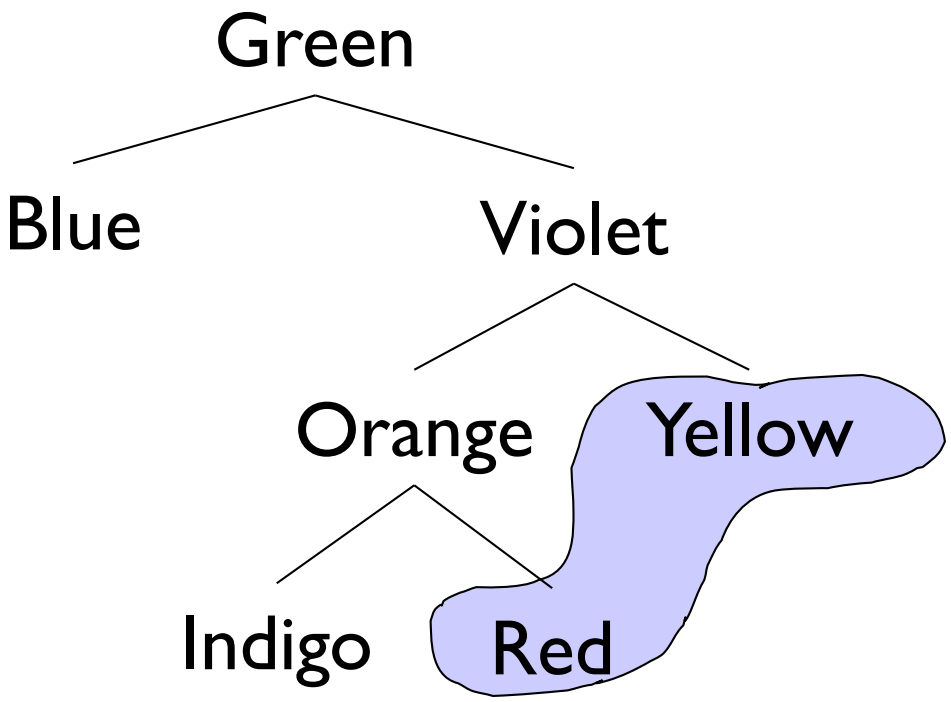

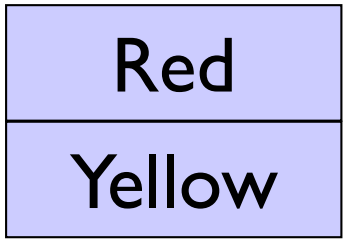

todo stack

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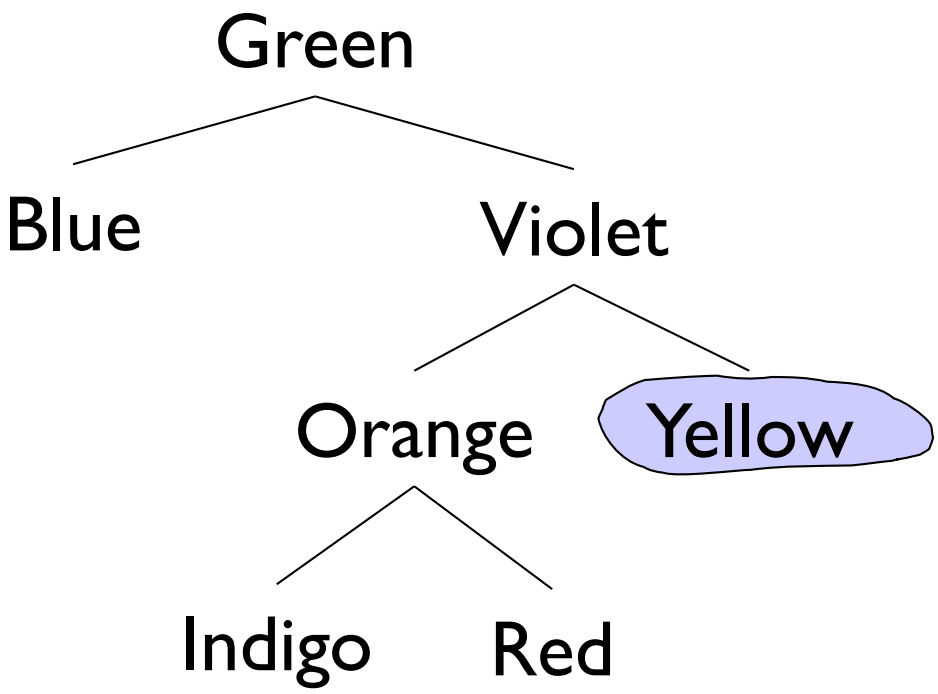
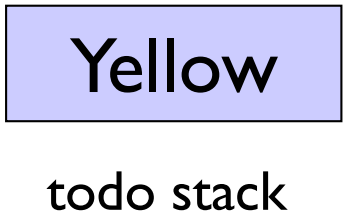
Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.

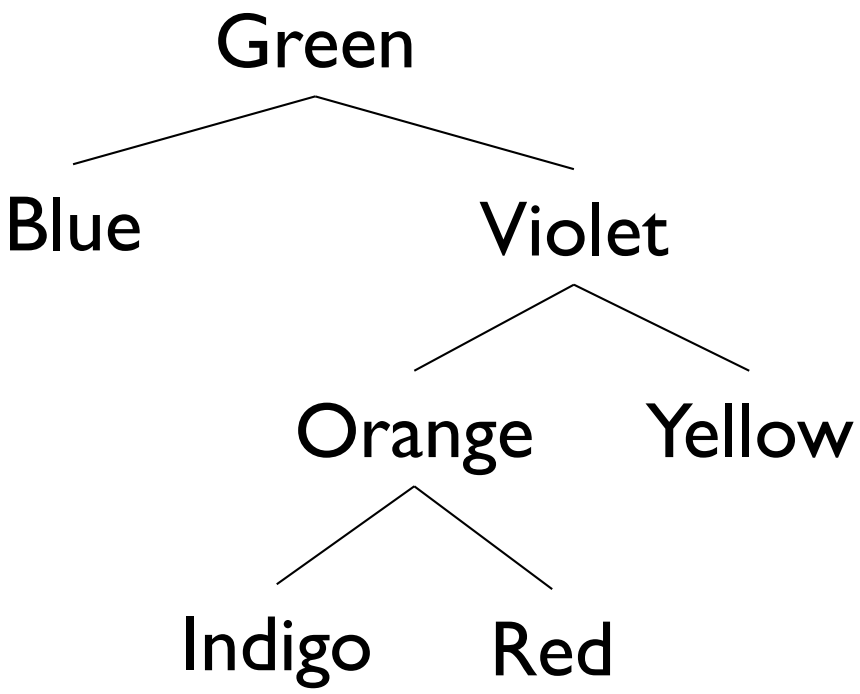




G B V O I R

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



todo stack

G B V O I R Y

Pre-Order Iterator

```
public BTPreorderIterator(BinaryTree<E> root) {
        todo = new StackList<BinaryTree<E>>();
        this.root = root;
        reset();
}

public void reset() {
        todo.clear(); // stack is empty; push on root
        if ((!root.isEmpty()) todo.push(root);
}
```

Pre-Order Iterator

```
public boolean hasNext() {
        return !todo.isEmpty();
}

public E next() {
     BinaryTree<E> old = todo.pop();
     E result = old.value();

     if (!old.right().isEmpty())
           todo.push(old.right());
     if (!old.left().isEmpty())
           todo.push(old.left());

    return result;
}
```

Tree Traversal (Practice) Problems

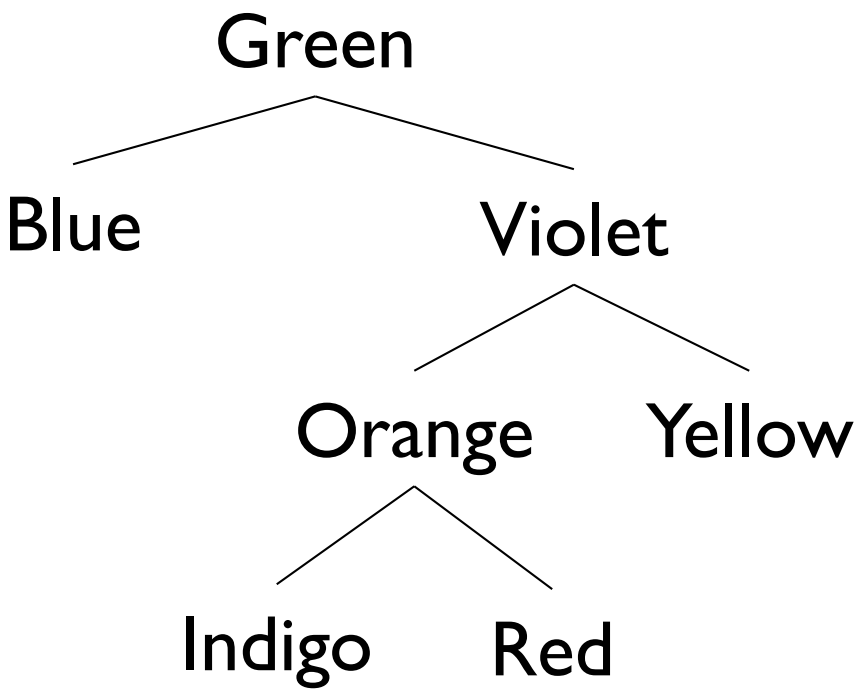
- Prove that levelOrder() is correct: that is, that it touches the nodes of the tree in the correct order (Hint: induction by level)
- Prove that levelOrder() takes O(n) time, where n is the size of the tree
- Prove that the PreOrder (LevelOrder) Iterator visits the nodes in the same order as the PreOrder (LevelOrder) traversal method

In-Order Iterator

- Outline: left - node - right
  1. Push left children (as far as possible) onto stack
  2. On call to next():
     - Pop node from stack
     - Push right child and follow left children as far as possible
     - Return node's value
  3. On call to hasNext():
     - return !stack.isEmpty()

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



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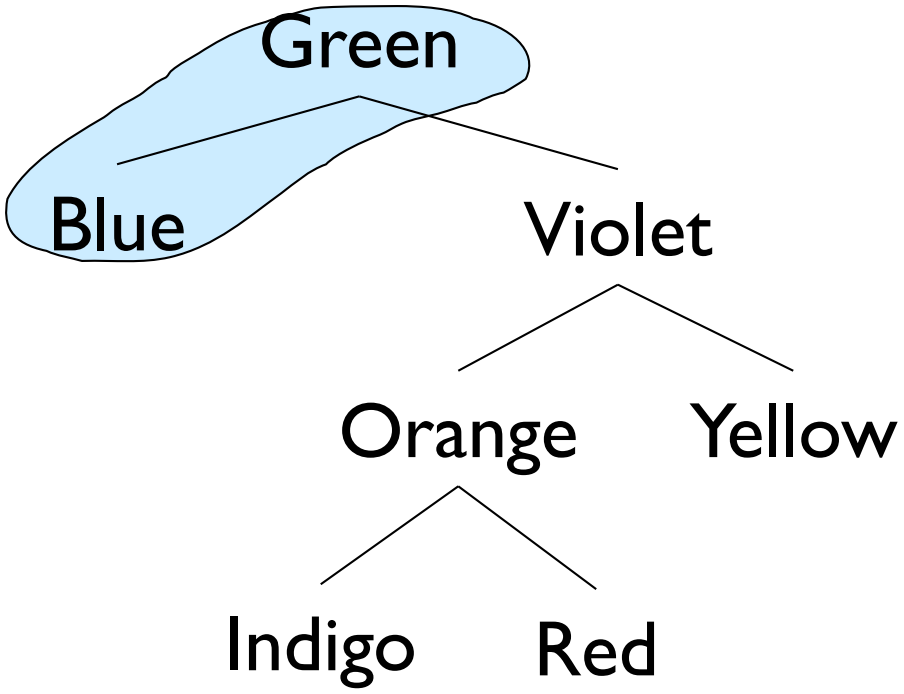

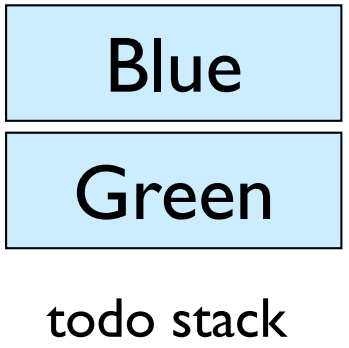

todo stack

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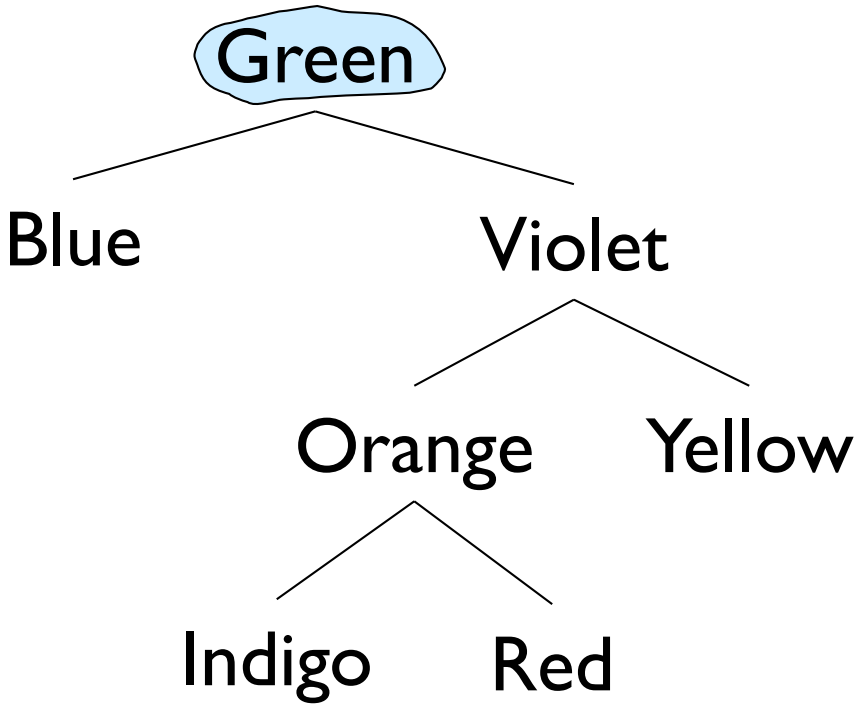

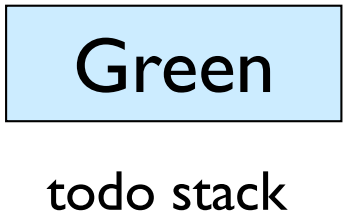

todo stack

B

In-Order Iterator

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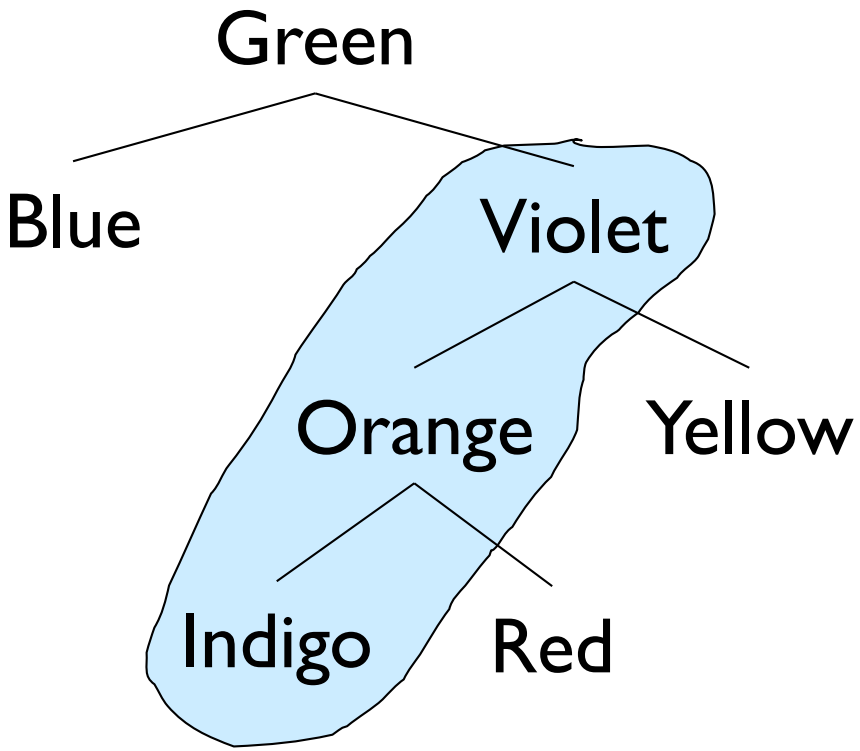

todo stack

B G

In-Order Iterator

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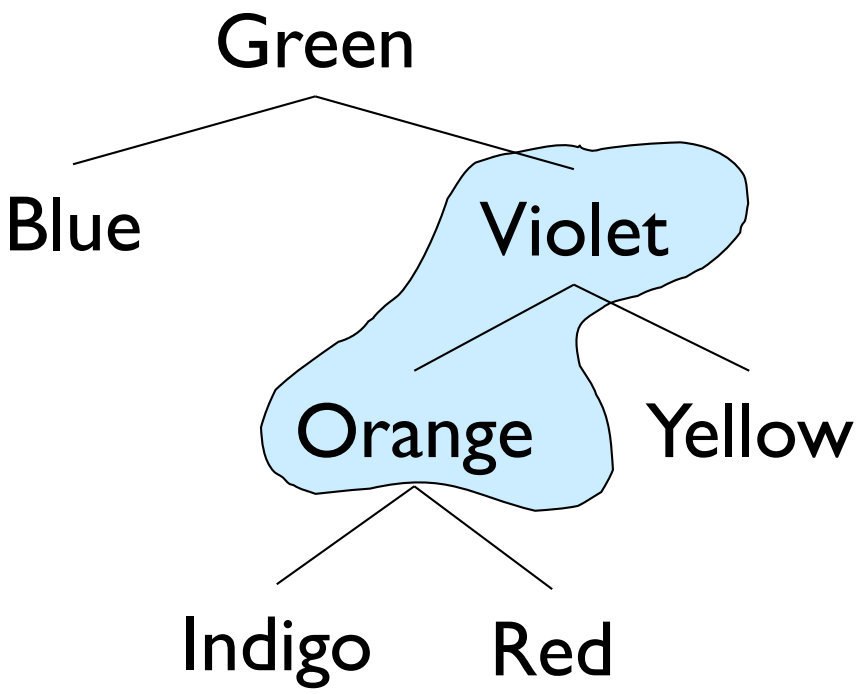

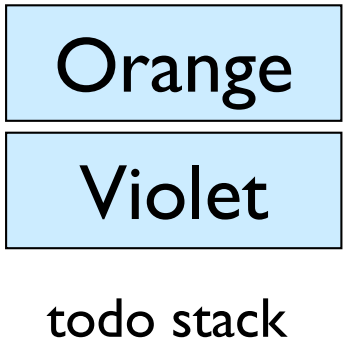

B G I

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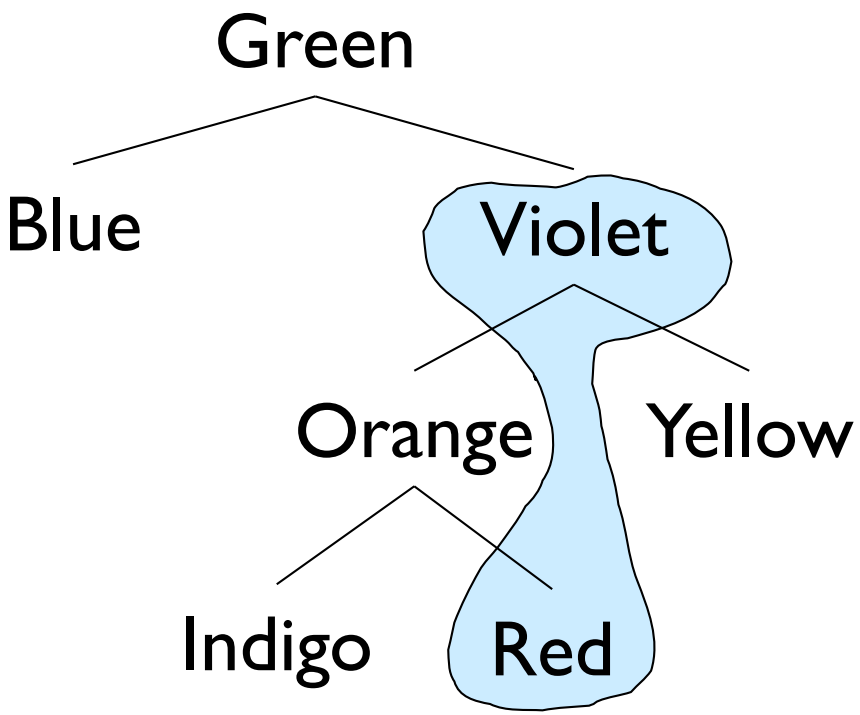

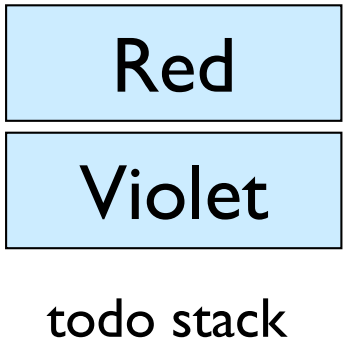

B G I O

In-Order Iterator

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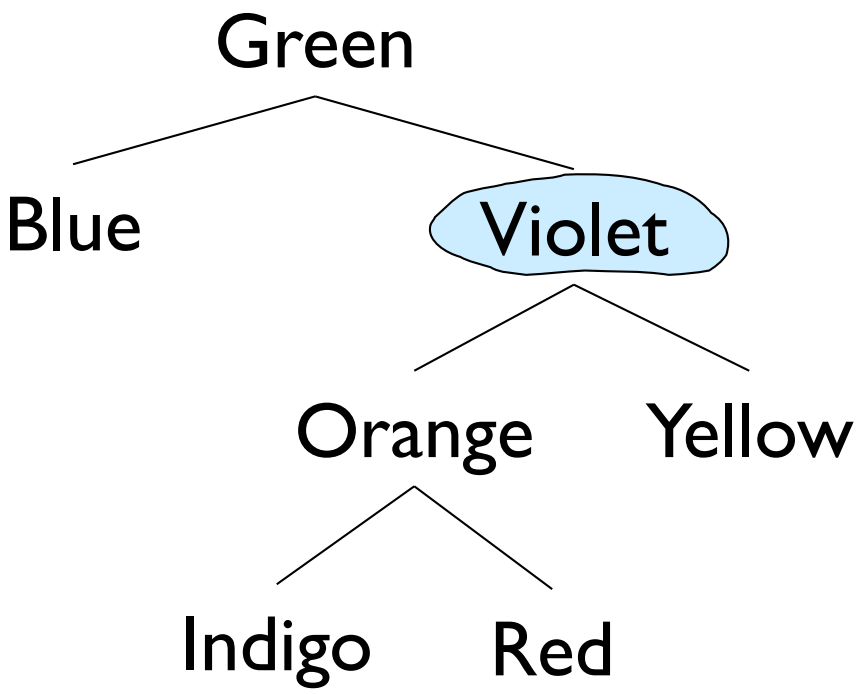

todo stack

B G I O R

In-Order Iterator

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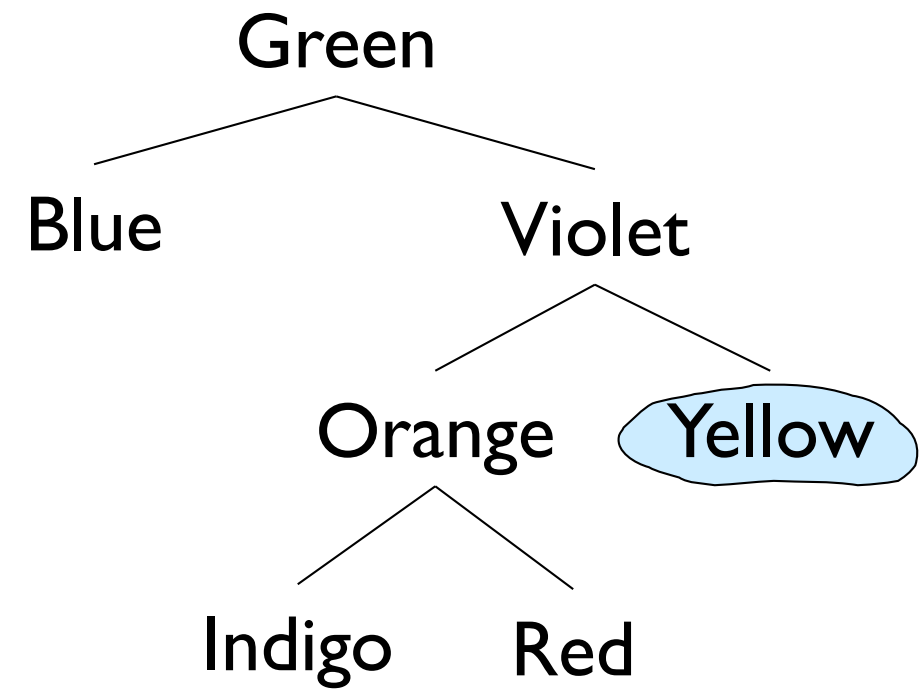

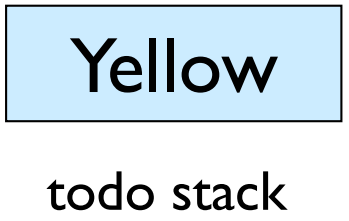

todo stack

B G I O R V

In-Order Iterator

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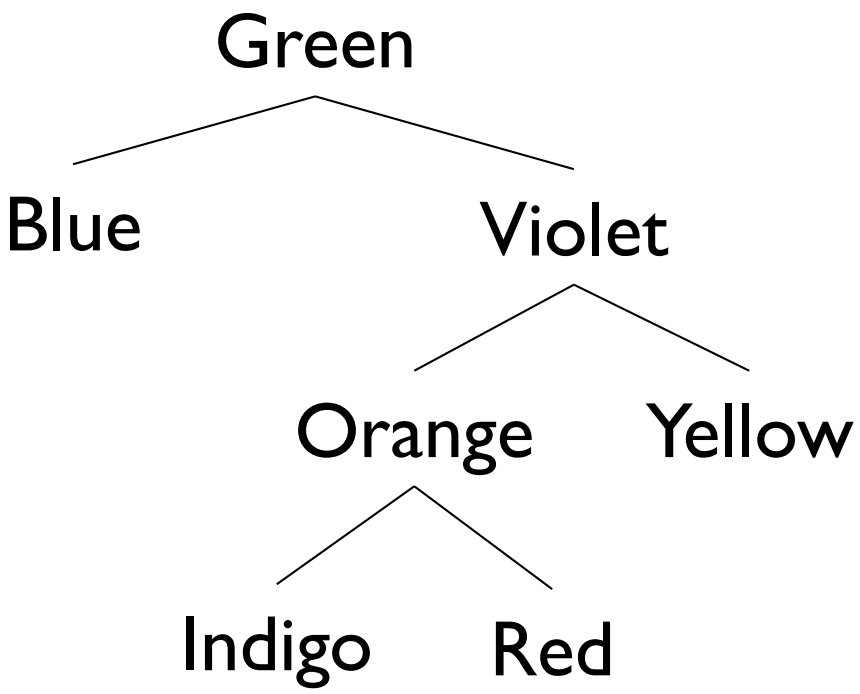


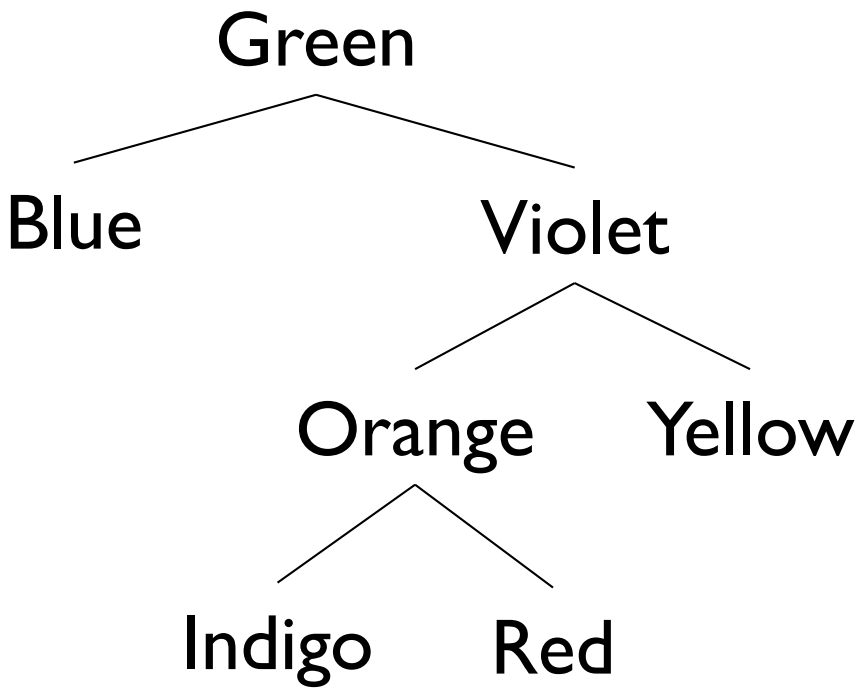
todo stack

B G I O R V Y

Post-Order Iterator

- Left as an exercise…

Alternative Tree Representations



- Total # "slots" = 4n
  - Since each BinaryTree maintains a reference to left, right, parent, value
- 2-4x more overhead than vector, SLL, array, …
- But trees capture successor and predecessor relationships that other data structures don't…

Array-Based Binary Trees

- Encode structure of tree in array indexes
  - Put root at index 0
- Where are children of node i?
  - Children of node i are at 2i+1 and 2i+2
  - Look at example
- Where is parent of node j?
  - Parent of node j is at (j-1)/2

ArrayTree Tradeoffs

- Why are ArrayTrees good?
  - Save space for links
  - No need for additional memory allocated/garbage collected
  - Works well for full or complete trees
    - Complete: All levels except last are full and all gaps are at right
    - "A *complete* binary tree of height h is a full binary tree with 0 or more of the rightmost leaves of level h removed"
- Why bad?
  - Could waste a lot of space
  - Tree of height of n requires 2^{n+1}-1 array slots even if only O(n) elements