# [TAP:SDHGX] ADT vs data structure

- Which of the following is true?

A. Tree is an ADT just like list.

B. Tree is a data structure just like linked list, vector, and array.

C. A and B

D. None of the above

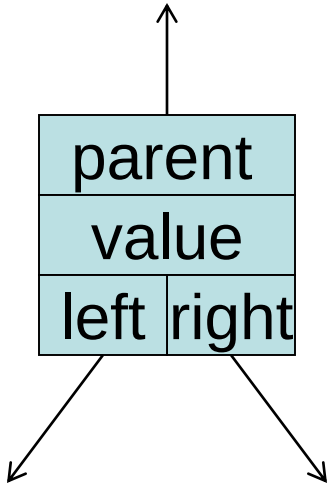
E. Whatever

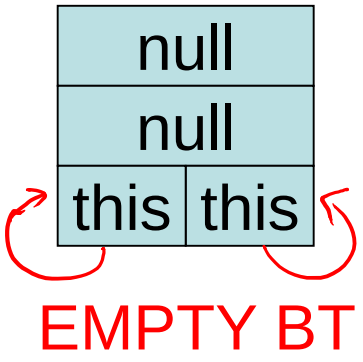
# Today's Outline

- Binary Tree
  - Internal Structure
  - Properties
  - Traversals

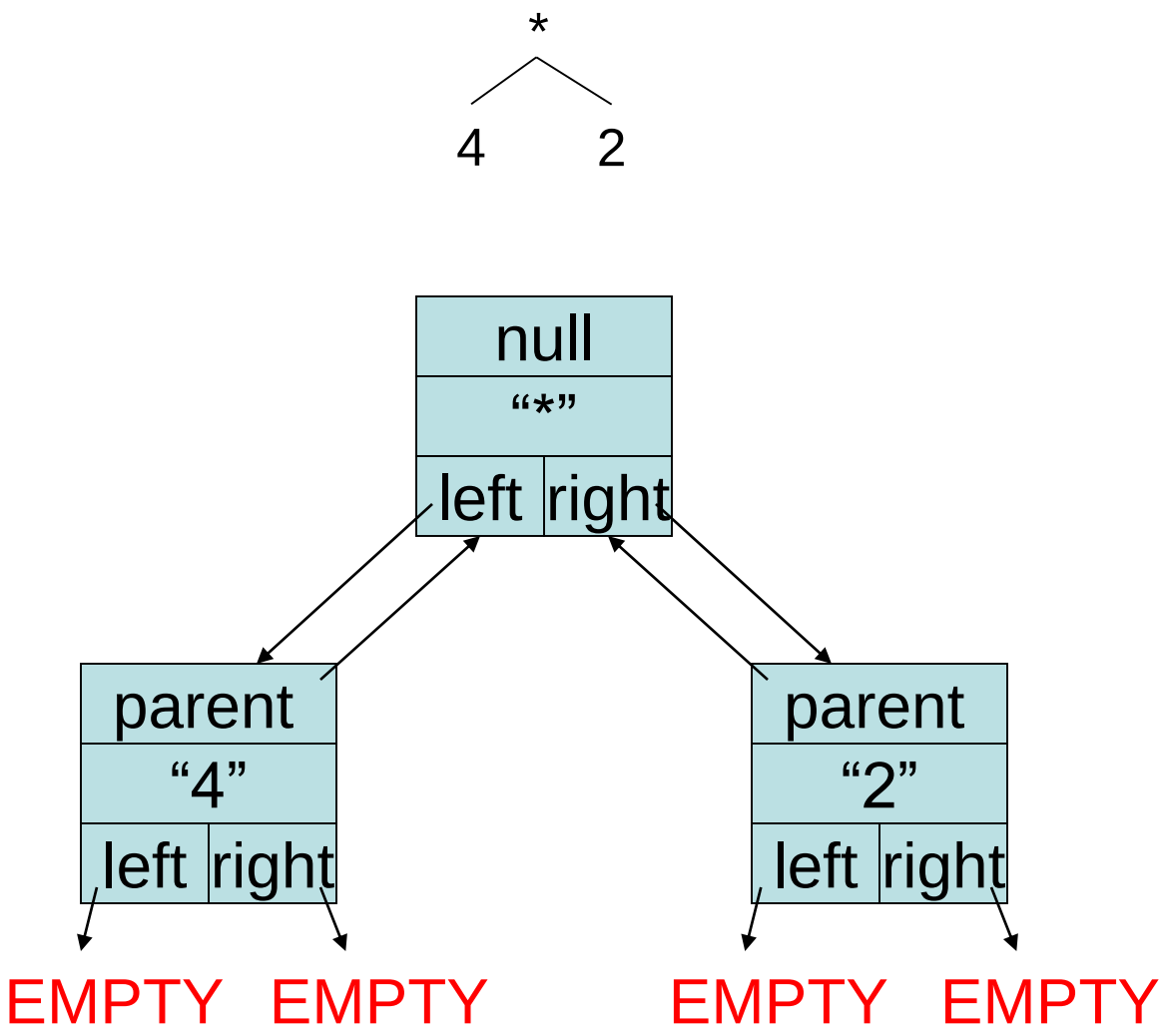
# Implementing BinaryTree

- BinaryTree<E> class
  - Instance variables
    - BinaryTree: parent, left, right
    - E: value
- left and right are never null
  - If no child, they point to an "empty" tree
    - Empty tree T has value null, parent null, left = right = T
  - Only empty tree nodes have null value



# Example


*
4
2
null
“*”
left
right
parent
“4”
left
right
parent
“2”
left
right
EMPTY
EMPTY
EMPTY
EMPTY

# Representing Arbitrary Trees

- What if nodes can have many children?
  - Example: Game trees
- Replace left/right node references with a list of children (Vector, SLL, etc)

getLeft() → getChild(i)

# Representing Knowledge

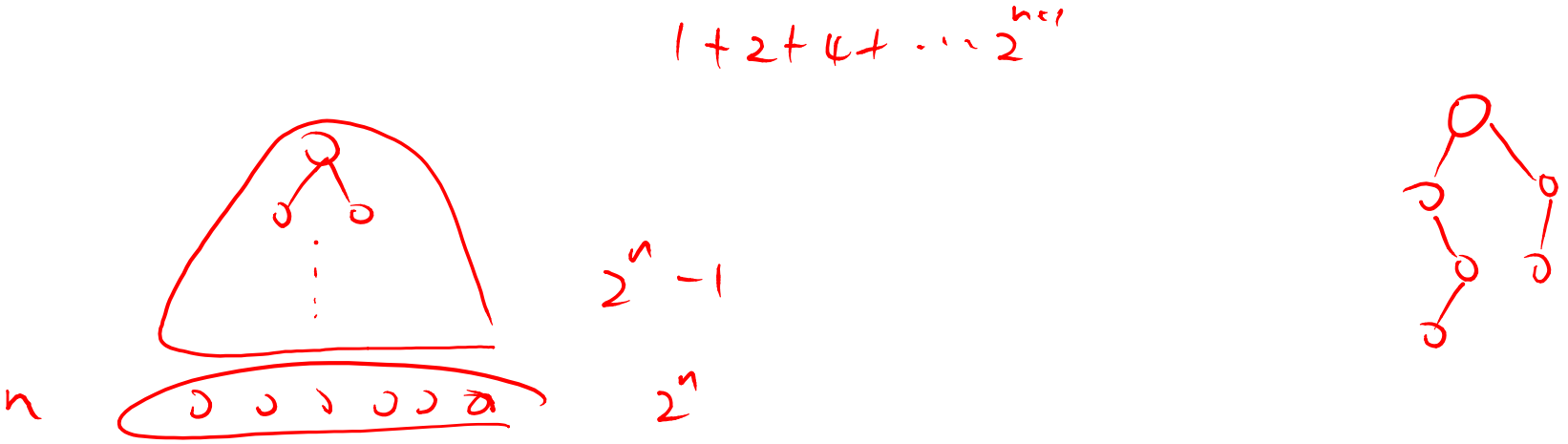
- Trees can be used to represent knowledge
- We often call these decision trees
  - Leaf: object
  - Internal node: question to distinguish objects
  - Move down decision tree until we reach a leaf node

# Today's Outline

- Binary Tree
  - Internal Structure
  - Properties
  - Traversals

# BT Questions/Proofs

- Some of the properties
  - The number of nodes at depth n is at most $2^n$.
  - A tree with n nodes has exactly n-1 edges
  - The number of nodes in tree of height n is at most $2^{(n+1)}$-1.



# BT Questions/Proofs

Prove: Number of nodes at depth $n \geq 0$ is at most $2^n$.

Base case: $n = 0$ $\quad 2^0 = 1$ ✓

Assume at depth $n$, there are at most $2^n$ nodes

- Each of the nodes at depth $n$ has at most 2 children

$\Rightarrow 2^n$ nodes by IH.

- Thus, at depth $n+1$, there are at most $2 \cdot 2^n = 2^{n+1}$ nodes

□

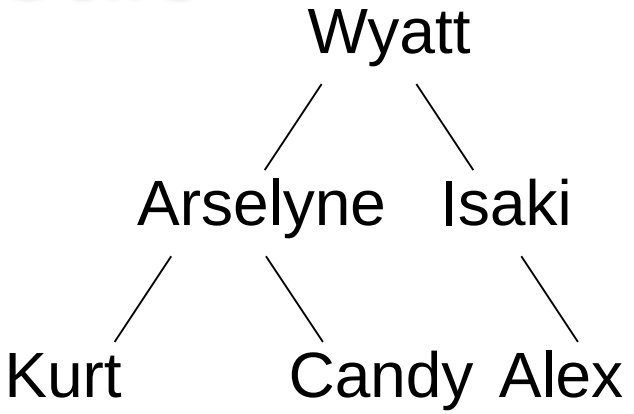
# Today's Outline

- Binary Tree
  - Internal Structure
  - Properties
  - Traversals

# Tree Traversals

- In linear structures, there are only a few basic ways to traverse the data structure
  - Start at one end and visit each element
  - Start at the other end and visit each element
- How do we traverse binary trees?
  - (At least) four reasonable mechanisms

# Tree Traversals




In-order: “left, node, right”

Kurt, Arselyne, Candy, Wyatt, Isaki, Alex

Pre-order: “node, left, right”

Wyatt, Arselyne, Kurt, Candy, Isaki, Alex

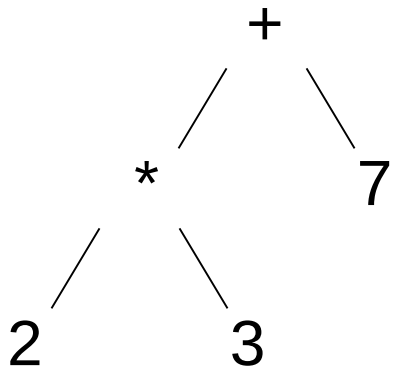
Post-order: “left, right, node”

Kurt, Candy, Arselyn, Alex, Isaki, Wyatt

Level-order: visit all nodes at depth i before depth i+1

Wyatt, Arselyne, Isaki, Kurt, Candy, Alex

# Tree Traversals




In-order:

2 * 3 + 7

Pre-order:

+ * 2 3 7

Post-order:

2 3 * 7 +

Level-order:

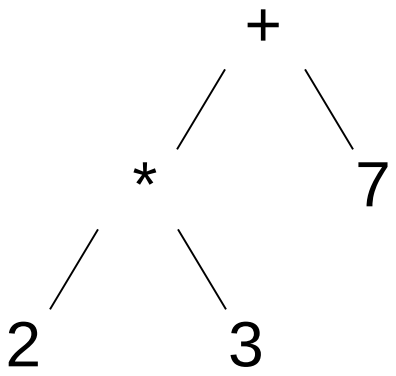
+ * 7 2 3

# Tree Traversals

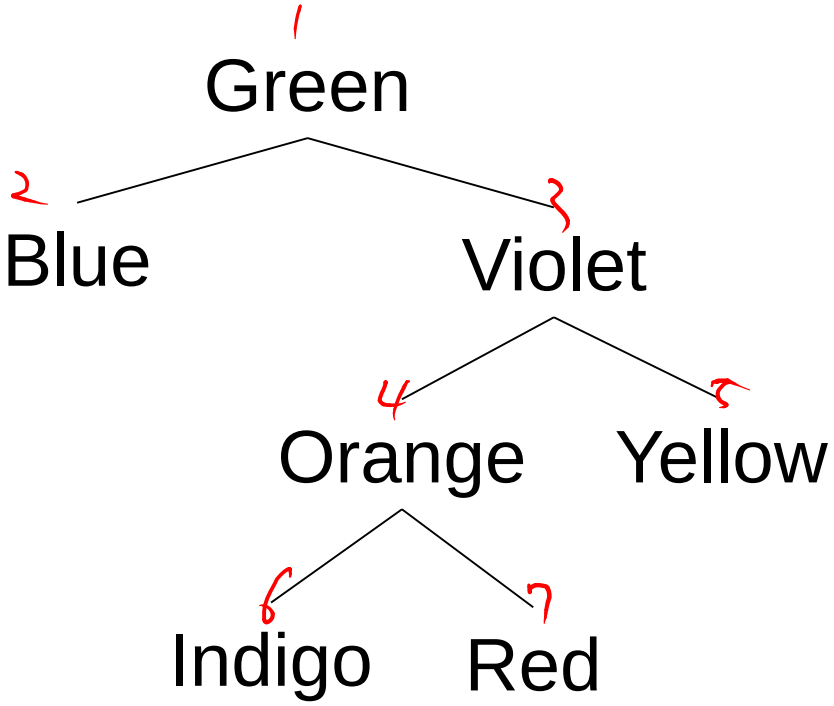
postOrder
inOrder

```
public void preOrder(BinaryTree t) {
    if (t.isEmpty())
        return;

    doSomething(t);
    preOrder(t.left());
    preOrder(t.right());
}
```



# Level-Order Traversal



# Level-Order Tree Traversal

```
public static <E> void levelOrder(BinaryTree<E> root) {
   if (t.isEmpty()) return;

   // The queue holds nodes for in-order processing
   Queue<BinaryTree<E>> q = new QueueList<BinaryTree<E>>();
   q.enqueue(root); // put root of tree in queue

   while(!q.isEmpty()) {
      BinaryTree<E> next = q.dequeue();
      doSomething(next);
      if(!next.left().isEmpty())  q.enqueue(next.left());
      if(!next.right().isEmpty()) q.enqueue(next.right());
   }
}
```