

[TAP:IVHZO] Bit-shifting

Given

`int x = 12 >> 3;` *0...01100 \Rightarrow 0...01*

`int y = x << 1;` *\Rightarrow 0...010*

• What is y?

A. 1

B. 2

C. 10

D. None of the above

E. Whatever

*11
2*

Today's Outline

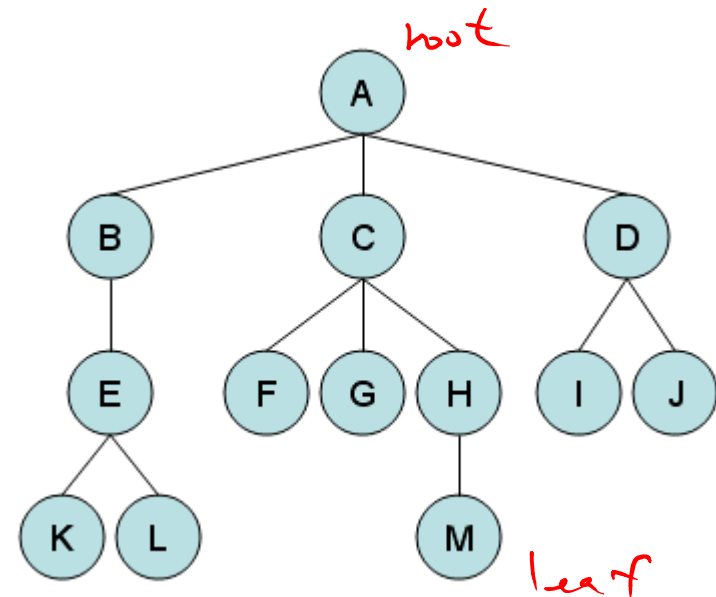
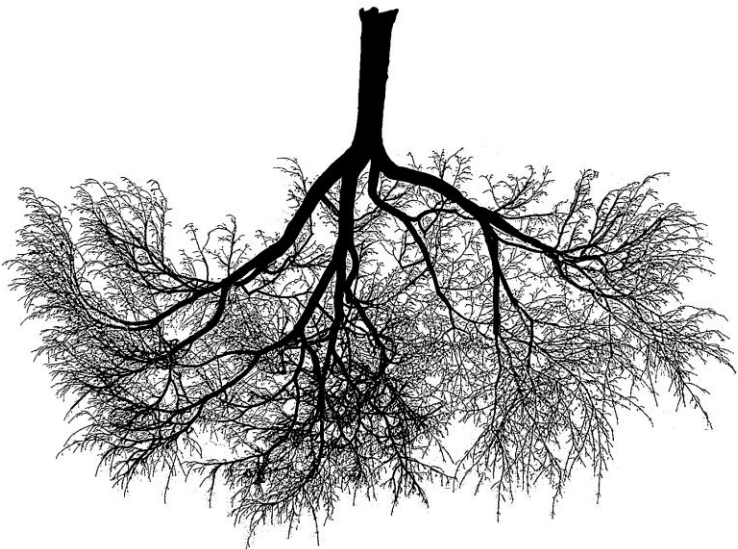
- Tree
-  • Tree
- Binary Tree

Introducing Trees

- We have been studying structures with a linear organization, i.e. each node has at most 1 successor.
 - line at a final exam
 - stacks of plates
- But you may want to allow more than 1 successor!
 - family tree ← ?
 - road network ← not a tree
 - directory hierarchy ← tree!

Tree

- A tree is a data structure where nodes can have:
 - one predecessor (called parent)
 - multiple successors (called children)



Tree

ex

parent

A to C

child

C to A

ancestor

A to E, A to C

descendant

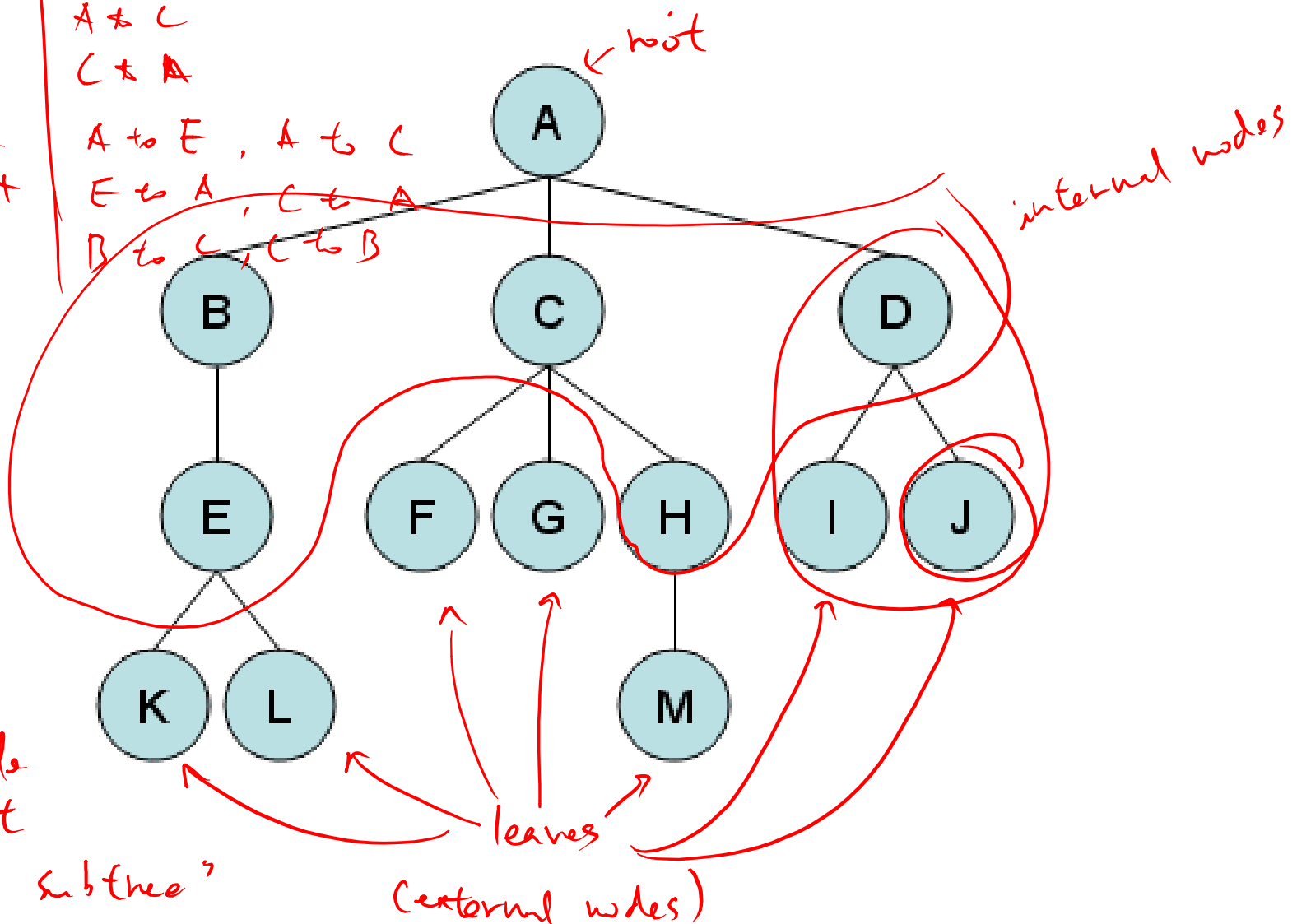
E to A, C to A

sibling

B to C, C to B

subtree:

"every node is the root of its own subtree"

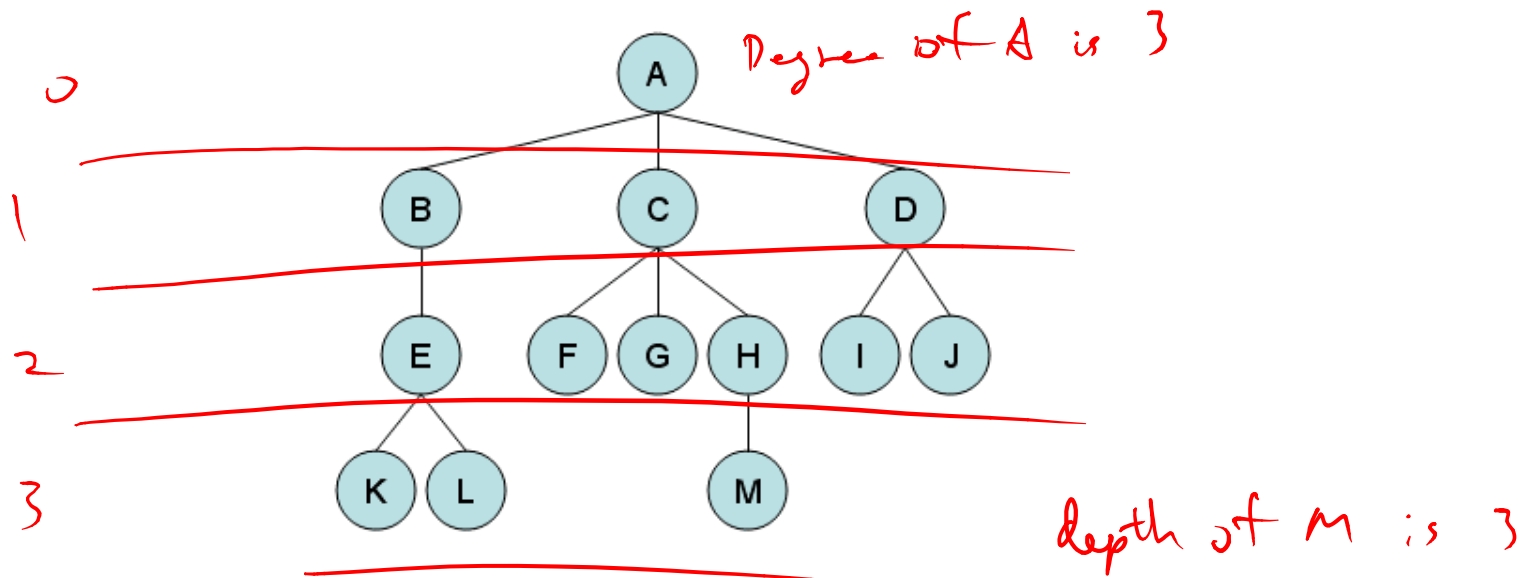




Tree Logic (Natalie Jeremijenko) at Mass MoCA

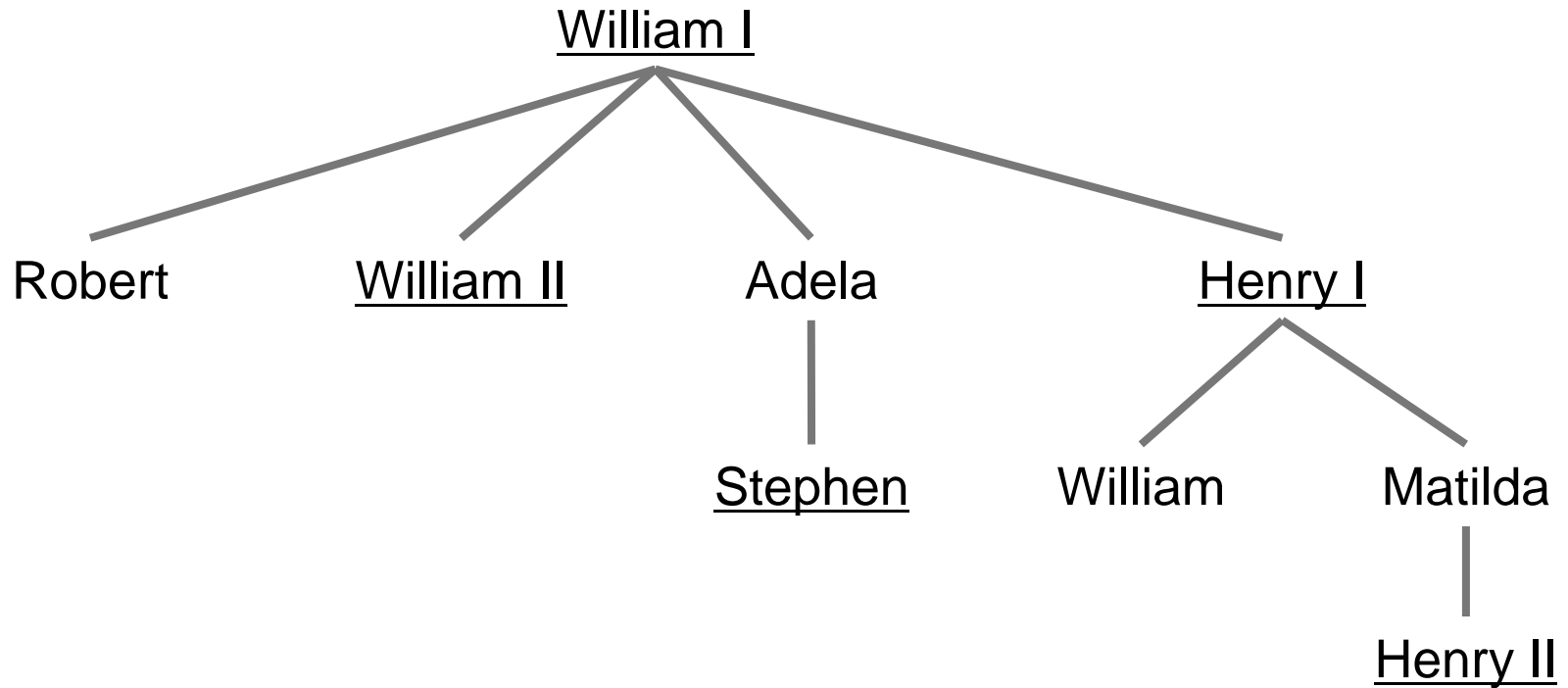
Tree Features

- **Degree (of node)**: number of children of node
- **Degree (of tree)**: maximum degree (across all nodes)
- **Depth** of node: number of *edges* from root to node
- **Height** of tree: maximum depth (across all nodes)



Tree examples

House of Normandy, Battle of Hastings, 1066



Miocene Pliocene Pleistocene Millions of Years Before Present





~jannen

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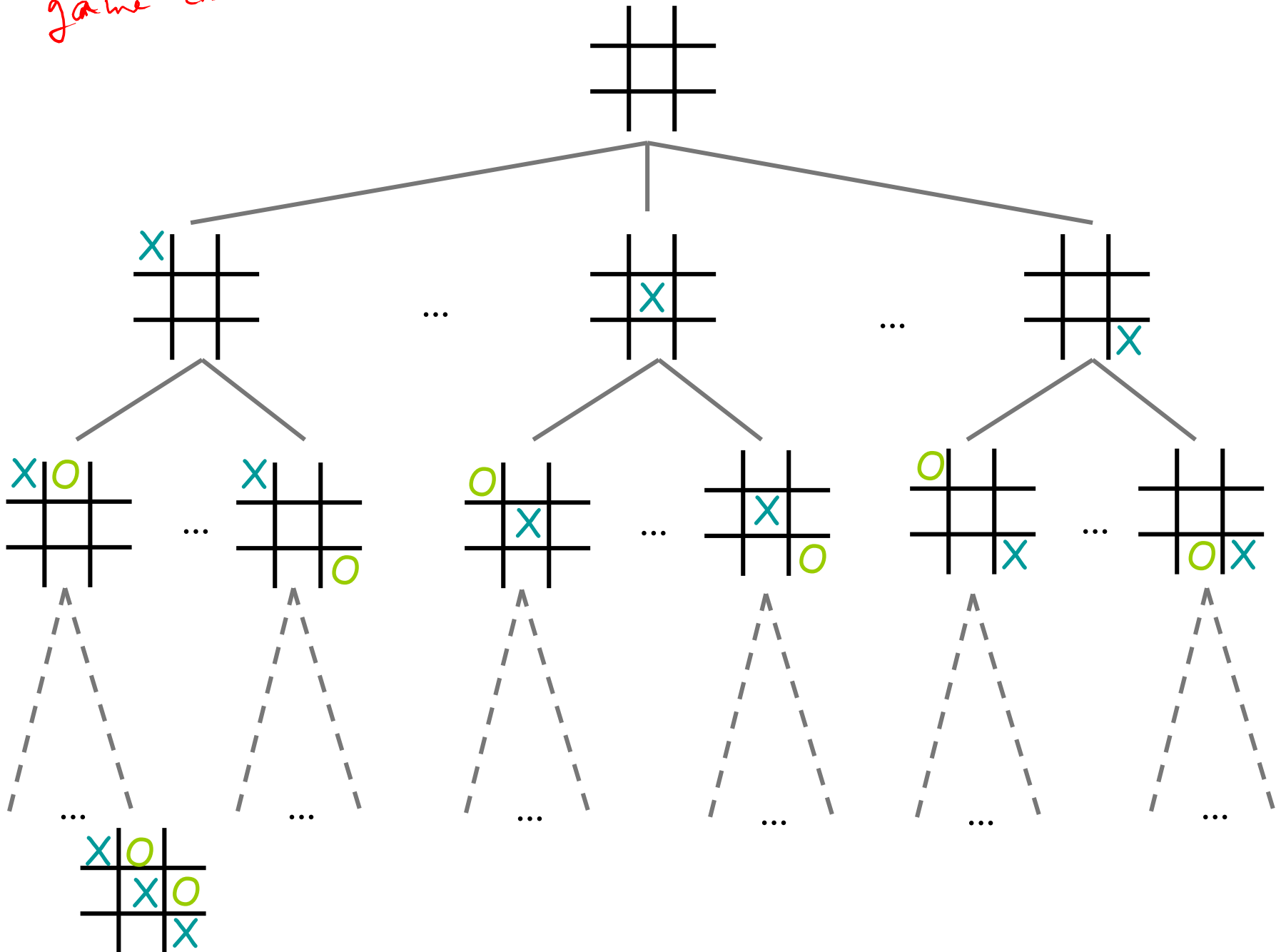
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game tree



Today's Outline

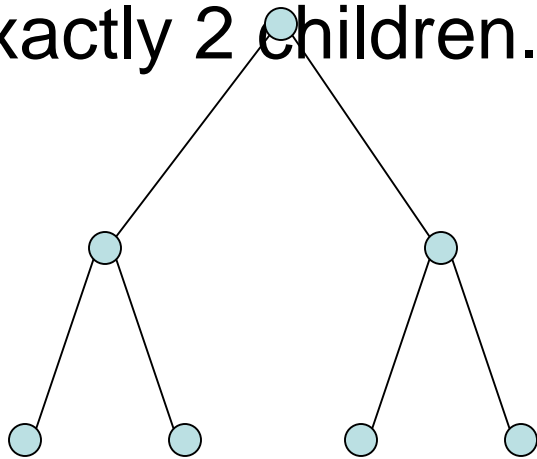
- Tree
 - Tree
 - • Binary Tree

Binary Trees

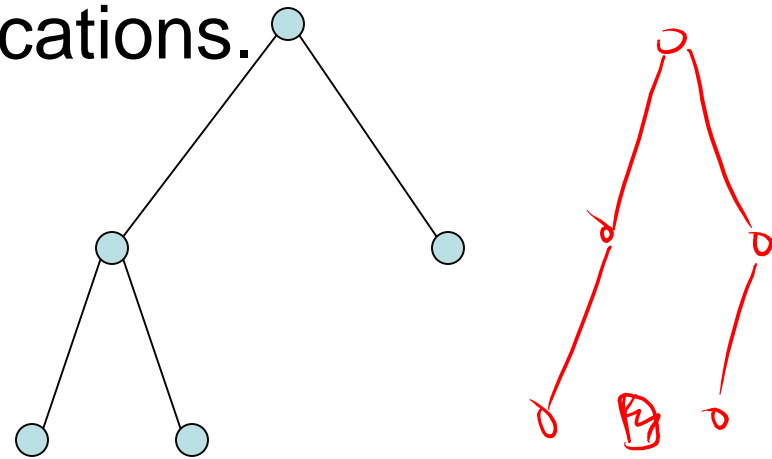
- Binary Tree: Tree with Degree of each node ≤ 2
- Recursively defined. A tree can either be:
 - Empty
 - Root with left and right subtrees

Full vs. Complete

- **Full** tree – A full binary tree of height h has *leaves only* on level h , and each internal node has exactly 2 children.



- **Complete** tree – A *complete* binary tree of height h is *full* to height $h-1$ and has all leaves at level h in leftmost locations.



All full trees are complete, but not all complete trees are full!

Example: Expression Trees

4 * 2 + 3

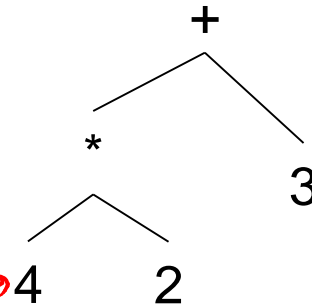
SLL: head $\square \rightarrow$

SLLN:

value \square
next $\square \rightarrow$

BT:

value \square
left $\square \rightarrow$ 4
right $\square \rightarrow$ 2



Build using constructor

```
new BinaryTree<E>(value, leftSubTree, rightSubTree)
```

BinaryTree<String> fourTimesTwo =

```
new BinaryTree<String>("*",  
new BinaryTree<String>("4"),  
new BinaryTree<String>("2"));
```

BinaryTree<String> fourTimesTwoPlusThree =

```
new BinaryTree<String>("+",  
fourTimesTwo,  
new BinaryTree<String>("3"));
```


Evaluating Expression Trees

- Starting at the root,
 - Evaluate left subtree
 - Evaluate right subtree
 - Perform operation (+, -, *, /) with left and right

```
int evaluate(BinaryTree<String> tree) {
```

```
// base case
```

```
if (tree.height() == 0)
```

```
    return Integer.parseInt(tree.value());
```

```
// recursive case
```

```
int left = evaluate(tree.left());
```

```
int right = evaluate(tree.right());
```

```
switch (tree.value()) {
```

```
    case "+":
```

```
        return left + right;
```

```
    case "-":
```

```
        return left - right;
```

```
    case "*":
```

```
        return left * right;
```

```
    case "/":
```

```
        return left / right;
```

```
    }  
    return ERROR_CODE;
```