CSCI 136
Data Structures & Advanced Programming

Lecture 14

Spring 2018

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Announcements

- Lab 5 Today
  - Submit partners!
  - Challenging, but shorter and a partner lab – more time for exam prep!
- Mid-term exam is Wednesday, March 14
  - During your normal lab session
  - You'll have approximately 1 hour & 45 minutes (if you come on time!)
  - Closed-book: Covers Chapters 1-7 & 9, handouts, and all topics up through Sorting
  - A "sample" mid-term **and** study sheet will be available online

Last Time

- Basic Sorting Summary
- Comparator interfaces for flexible sorting
- More Efficient Sorting Algorithms
  - MergeSort

Today

- Sorting Wrap-Up (Merge and Quick)
- Linear Structures
  - The Linear Interface (LIFO & FIFO)
  - The AbstractLinear and AbstractStack classes
- Stack Implementations
  - StackArray, StackVector, StackList,
- Stack applications
  - Expression Evaluation
  - PostScript: Page Description & Programming
  - Mazerunning (Depth-First-Search)

Merge Sort

- A divide and conquer algorithm
- Merge sort works as follows:
  - Base case:
    - If the list is of length 0 or 1, then it is already sorted. Return the sorted list.
  - Divide the unsorted list into two sublists of about half the size of original list.
  - Recursive call:
    - Sort each sublist by re-applying merge sort.
  - Merge the two sublists back into one sorted list.

Merge Sort

- [8 14 29 1 17 39 16 9]
- [8 14 29 1] [17 39 16 9] split
- [8 14] [29 1] [17 39] [16 9] split
- [8] [14] [29] [1] [17] [39] [16] [9] split
- [8 14] [1 29] [17 39] [9 16] merge
- [1 8 14 29] [9 16 17 39] merge
- [1 8 9 14 16 17 29 39] merge

Transylvanian Merge Sort Folk Dance

Merge Sort

- How would we implement it?
- Pseudocode:

```
//recursively mergesorts A[from..To] "in place"
void recMergeSortHelper(A[], int from, int to)
  if ( from < to )
      // find midpoint
      mid = (from + to)/2
      //sort each half
      recMergeSortHelper(A, from, mid)
      recMergeSortHelper(A, mid+1, to)
      // merge sorted lists
      merge(A, from, to)
```

But `merge` hides a number of important details....

Merge Sort

- How would we implement it?
  - Review `MergeSort.java`
  - Note carefully how temp array is used to reduce copying
  - Make sure the data is in the correct array!
- Time Complexity?
  - Takes at most `2k` comparisons to merge two lists of size `k`
  - Number of splits/merges for list of size `n` is `log n`
  - Claim: At most time `O(n log n)`…We'll see soon...
- Space Complexity?
  - `O(n)`?
  - "Clever" implementation "ping-pongs" between 2 arrays
    - Need an extra array, so really `O(2n)`!
    - But `O(2n) = O(n)`

Merge Sort = O(n log n)

- [8 14 29 1 17 39 16 9]
- [8 14 29 1] [17 39 16 9] split
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- [8] [14] [29] [1] [17] [39] [16] [9] split
- [8 14] [1 29] [17 39] [9 16] merge
- [1 8 14 29] [9 16 17 39] merge
- [1 8 9 14 16 17 29 39] merge

log n (split steps)

log n (merge steps)

merge takes at most n comparisons per line

Merge Sort

- Unlike Bubble, Insertion, and Selection sort, Merge sort is a divide and conquer algorithm
  - Bubble, Insertion, Selection sort: `O(n`^{2}`)`
  - Merge sort: `O(n log n)`
- Are there any problems or limitations with Merge sort?
- Why would we ever use any other algorithm for sorting?

Problems with Merge Sort

- Need extra temporary array
  - If data set is large, this could be a problem
- Waste time copying values back and forth between original array and temporary array
- Can we avoid this?

Quick Sort

- Quick sort is designed to behave much like Merge sort, without requiring extra storage space

Merge Sort	Quick Sort
Divide list in half	Partition* list into 2 parts
Sort halves	Sort parts
Merge halves	Join* sorted parts

Recall Merge Sort

```
private static void mergeSortRecursive(Comparable data[],
                              Comparable temp[], int low, int high) {
   int n = high-low+1;
   int middle = low + n/2;

   if (n < 2) return; // already sorted

   // move lower half of data into temporary storage
   for (int i = low; i < middle; i++)
       temp[i] = data[i];

   // sort lower half of array
   mergeSortRecursive(temp, data, low,middle-1);
   // sort upper half of array
   mergeSortRecursive(data, temp, middle, high);
   // merge halves together
   merge(data, temp, low, middle, high);
}
```

Quick Sort

```
// pre: low <= high
// post: data[low..high] in ascending order
public static void quickSortRecursive(Comparable data[],
                                      int low, int high) {
        int pivot;

        // base case: low and high coincide
        if (low >= high) return;

        // step 1: split using pivot
        pivot = partition(data, low, high);
        // step 2: sort small
        quickSortRecursive(data, low, pivot-1);
        // step 3: sort large
        quickSortRecursive(data, pivot+1, high);
}
```

Partition

1. Put first element (pivot) into sorted position
2. When done, all to the left of pivot are smaller and all to the right are larger
3. Return index of pivot

Partition by Hungarian Folk Dance

Partition

```
int partition(int data[], int left, int right) {
  while (true) {
    while (left < right && data[left] < data[right])
      right--;

    if (left < right)
      swap(data, left++, right);
    else
      return left;

    while (left < right && data[left] < data[right])
      left++;

    if (left < right)
      swap(data, left, right--);
    else
      return right;
  }
}
```

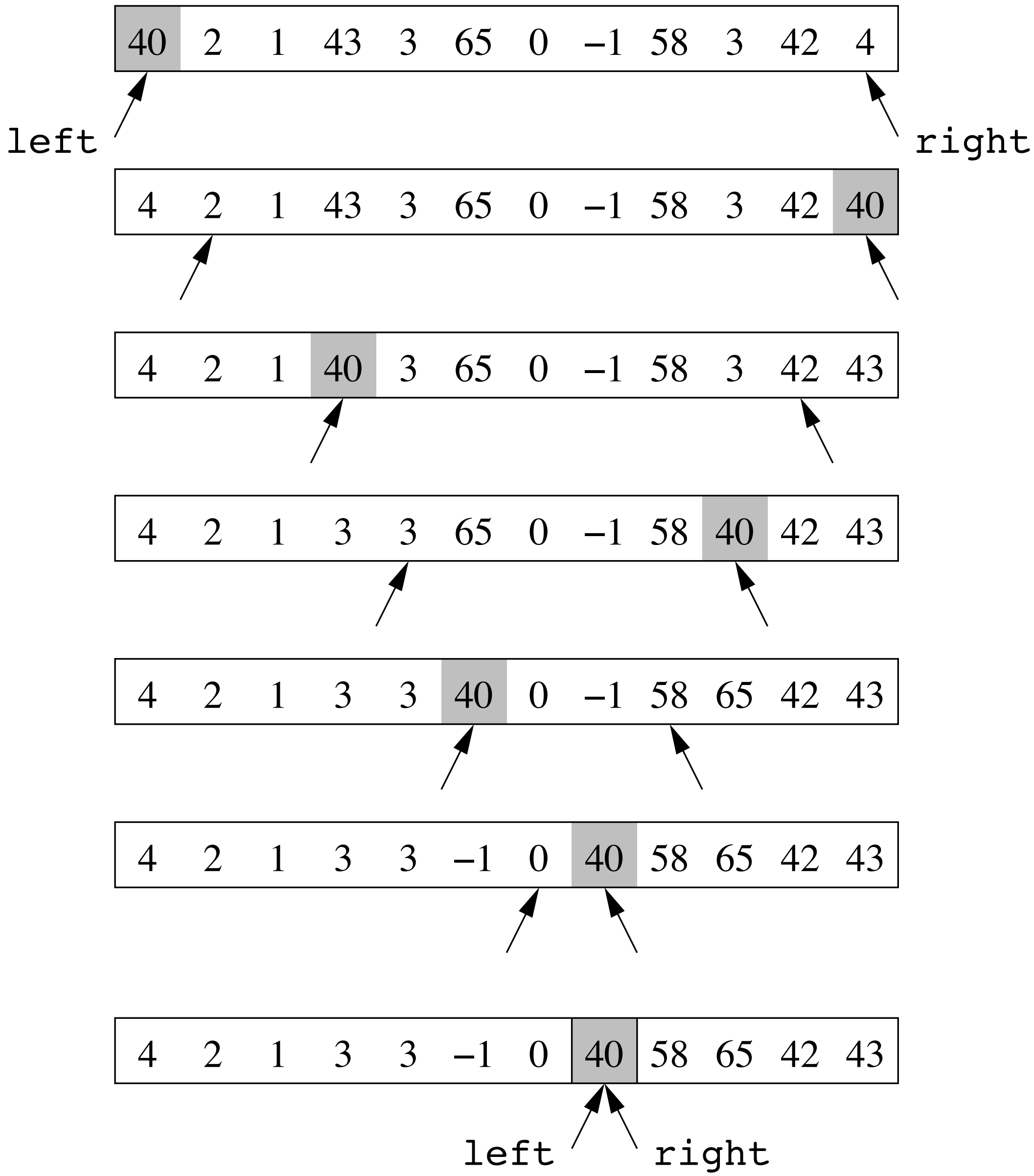
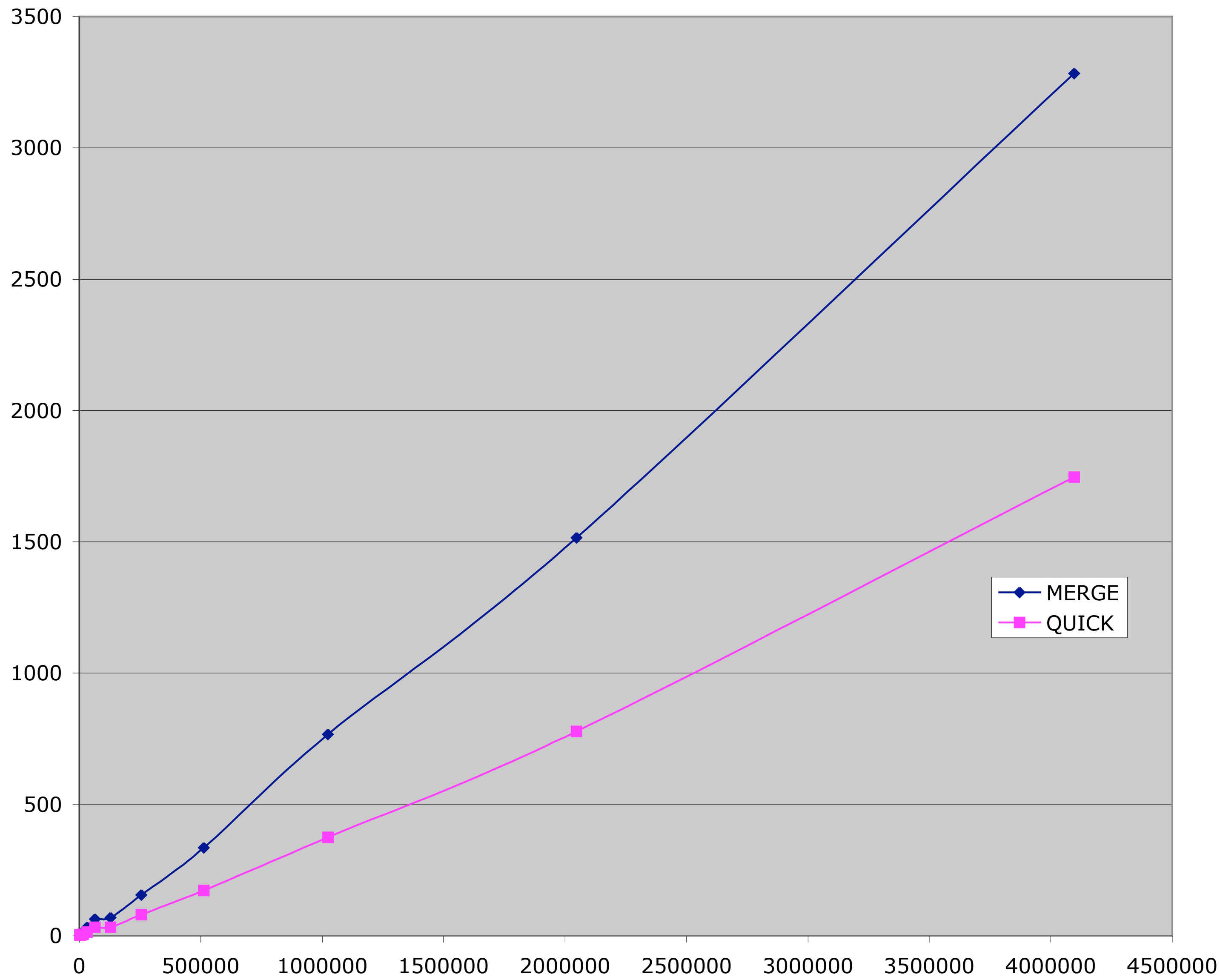


Figure 6.7
Bailey pg 132

Complexity

- Time:
  - Partition is O(n)
  - If partition breaks list exactly in half, same as merge sort, so O(n log n)
  - If data is already sorted, partition splits list into groups of 1 and n-1, so $O(n^2)$
- Space:
  - O(n) (so is MergSort)
    - In fact, it's n + c compared to 2n + c for MergeSort

Merge vs. Quick



Food for Thought…

- How to avoid picking a bad pivot value?
  - Pick median of 3 elements for pivot
    - Heuristic! No guarantees!
- Combine selection sort with quick sort
  - For small n, selection sort is faster
  - Switch to selection sort when elements is <=7
  - Switch to selection/insertion sort when the list is almost sorted (partitions are very unbalanced)
    - Heuristic! No guarantees!

Sorting Wrapup

	Time	Space
Bubble	Worst: $O(n^2)$ Best: $O(n^2)$ as written, but can be "optimized" to $O(n)$	$O(n) : n + c$
Insertion	Worst: $O(n^2)$ Best: $O(n)$	$O(n) : n + c$
Selection	Worst = Best: $O(n^2)$	$O(n) : n + c$
Merge	Worst = Best: $O(n \log n)$	$O(n) : 2n + c$
Quick	Average = Best: $O(n \log n)$ Worst: $O(n^2)$	$O(n) : n + c$