CSCI 136
Data Structures &
Advanced Programming

Lecture 8

Spring 2018

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Administrative Details

- Lab 3 Today
  - Declare your partner (or independence) by 10am
    - One repository where both people have access
    - Beware of merge conflicts!
  - Questions about warm-up problems?
    - We'll go over at start of lab, but does anyone feel like they have a good solution?

Last Time

- Measuring Growth
  - Big-O
    - We care about trends
    - Goal: determine how performance scales with input size.
    - Best, worst, and average cases

Today

- Applying O() to Compute Complexity
  - Finish `Vector` growing examples
- Recursion
- Mathematical Induction

Vector Operations : Worst-Case

Let `n` = Vector size (*not* capacity!):

- O(1) operations (cost is same regardless of size):
  - `size(), capacity(), isEmpty(), get(i), set(i), firstElement(), lastElement()`
- O(n) operations (cost grows proportionally to size):
  - `indexOf(), contains(), remove(elt), remove(i)`
- What about add methods?
  - If Vector doesn't need to grow
    - `add(elt)` is `O(1)` but `add(elt, i)` is `O(n)`
  - Otherwise, depends on `ensureCapacity()` time
    - Time to copy array: `O(n)`

Vectors: Add Method Complexity

Suppose we grow the Vector's array by a fixed amount `d`.

How long does it take to add `n` items to an empty Vector?

- The array will be copied each time its capacity needs to exceed a multiple of `d`
  - At sizes `0, d, 2d, … , n/d.`
- Copying an array of size `kd` takes `ckd` steps for some constant `c`, giving a total of

$$\sum_{k=1}^{n/d} ckd \quad = cd \sum_{k=1}^{n/d} k \ = cd \left(\frac{n}{d}\right)\left(\frac{n}{d}+1\right)/2 \ = O(n^2)$$

Vectors: Add Method Complexity

Suppose we grow the Vector's array by doubling.

How long does it take to add `n` items to an empty Vector?

- The array will be copied each time its capacity needs to exceed a power of 2
  - At sizes `0, 1, 2, 4, 8 …, n/2`
- The total number of elements are copied when `n` elements are added is:
  - `1 + 2 + 4 + ... + n/2 = n-1 = O(n)`

- Very cool! (So cool that we'll prove it later)

Common Complexities

For `n` = measure of problem size:

- $O(1)$: constant time and space
- $O(\log n)$: divide and conquer algorithms, binary search
- $O(n)$: linear scan (e.g., list lookup)
- $O(n \log n)$: divide and conquer sorting algorithms
- $O(n^2)$: matrix addition, selection sort
- $O(n^3)$: matrix multiplication
- $O(n^k)$: cell phone switching algorithms
- $O(2^n)$: subset sum, graph 3-coloring, satisfiability, ...
- $O(n!)$: traveling salesman problem (in fact $O(n^2 2^n)$)

Recursion

- General problem solving strategy
  - Break problem into sub-problems of same type
  - Solve sub-problems
  - Combine sub-problem solutions into solution for original problem
    - Recursive leap of faith!

Recursion

- Many algorithms are recursive
  - Can be easier to understand (and prove correctness/state efficiency of) than iterative versions
  - They feel *elegant*
- Today we will review recursion and then talk about techniques for reasoning about recursive algorithms

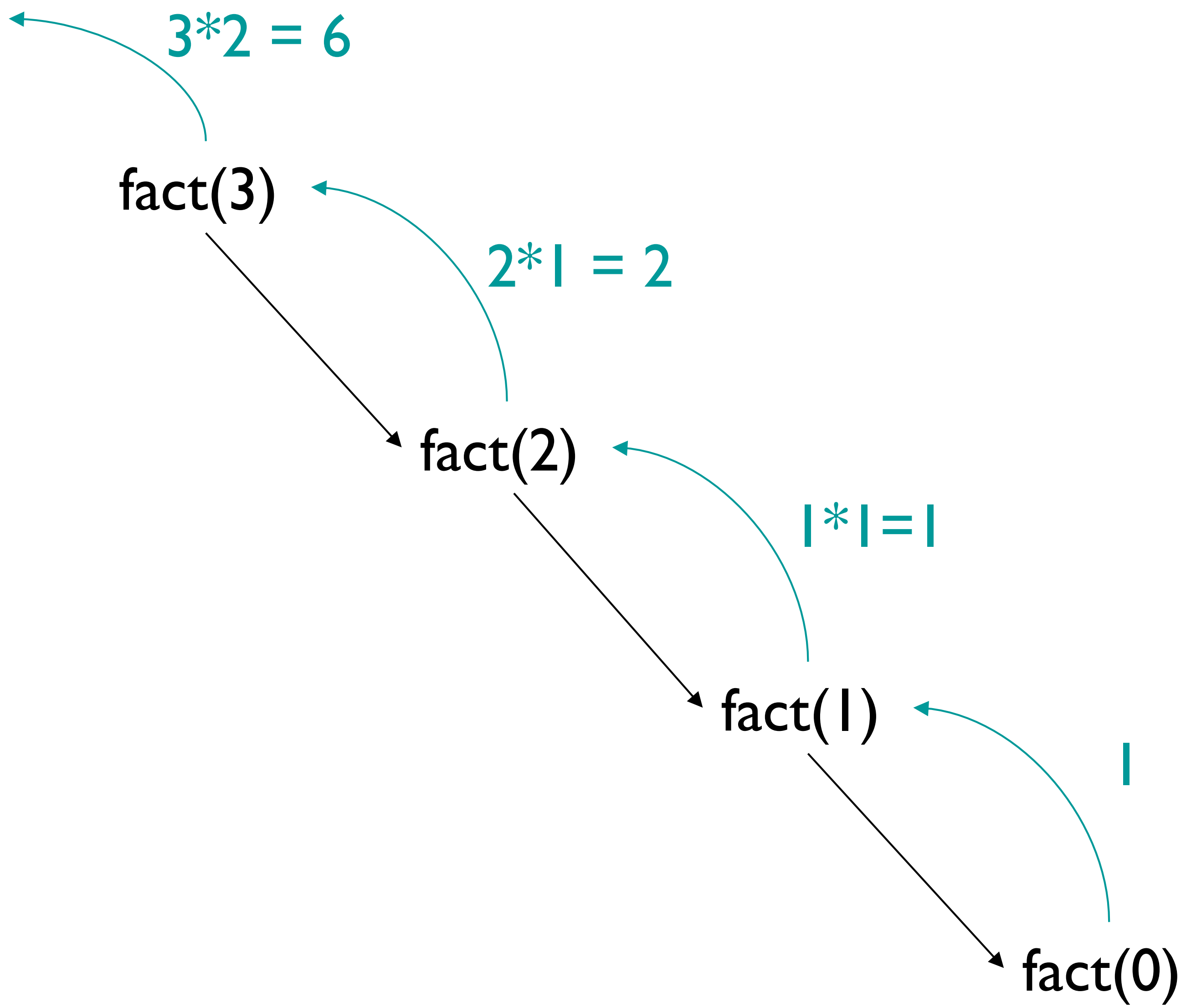
Think Recursively

- In recursion, we always use the same basic approach
- What's our base case? [Sometimes "cases"]
  - n=0? list.isEmpty()?
- What' s the recursive relationship?
  - How can we use the solution to a smaller version of the problem to answer the question?

Factorial

- $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1$
- How can we implement this?
  - We could use a for loop…
- But we could also write it recursively
  - $n! = n \cdot (n-1)!$
  - $0! = 1$

Factorial



Fact.java

```
public class Fact{

    // Pre: n >= 0
    public static int fact(int n) {
       // base case
       if (n==0) {
          return 1;
       }
       // recursive leap of faith
       else {
          return n*fact(n-1);
       }
    }

    public static void main(String args[]) {
       System.out.println(fact(Integer.valueOf(args[0]).intValue()));
    }

}
```

Fibonacci Numbers

- 1, 1, 2, 3, 5, 8, 13, ...
- Definition
  - $F_0 = 1$, $F_1 = 1$
  - For `n > 1`, $F_n = F_{n-1} + F_{n-2}$
- Inherently recursive!
- It appears almost everywhere
  - Growth: Populations, plant features
  - Architecture
  - Data Structures!

Fib.java

```
public class Fib{

    // pre: n is non-negative
    public static int fib(int n) {
        // base case
        if (n==0 || n == 1) {
            return 1;
        }
        // recursive leap of faith
        else {
            return fib(n – 1) + fib(n – 2);
        }
    }

    public static void main(String args[]) {
        System.out.println(fib(Integer.valueOf(args[0]).intValue()));
    }

}
```

Recursion Tradeoffs

- Advantages
  - Often easier to construct recursive solution
  - Code is usually cleaner (so *elegant*!)
  - Some problems do not have obvious non-recursive solutions
- Disadvantages
  - Overhead of recursive calls
  - Can use lots of memory (need to store state for each recursive call until base case is reached)
    - E.g. recursive fibonacci method

Alternate contains() for Vector

```
// Helper method: returns true if elt has index in range from..to
public boolean contains(E elt, int from, int to) {
   if (from > to) // Base case: empty range
        return false;
   else
        return elt.equals(elementData[from]) ||
               contains(elt, from+1, to);
}

public boolean contains(E elt) {
   return contains(elt, 0, size()-1);
}
```

- What's the time complexity of contains?
  - O(to – from + 1) = O(n) (n is the portion of the array searched)
  - Why?
    - Bootstrapping argument! True for: to – from = 0, to – from = 1, …
- Let's formalize this bootstrapping idea....