CSCI 136
Data Structures & Advanced Programming

Lecture 7

Spring 2018

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Administrative Details

- Lab 3 Wednesday!
  - You ***may*** work with a partner
    - Fill out “Lab 3 Partners” Google form either way!
  - Come to lab with a plan! (no design doc needed)
  - Try to answer warmup questions before lab
    - Subset Sum is challenging but important

Last Time

- Where did I go?
- What did I miss?
- Tell me about Lab 2!
- Should we expect from here?

Today

- Measuring Growth
  - Big-O
- Introduction to Recursion

Measuring Computational Cost

Consider these two code fragments…

```
for (int i=0; i < arr.length; i++)
   if (arr[i] == x) return "Found it!";
```

…and…

```
for (int i=0; i < arr.length; i++)
   for (int j=0; j < arr.length; j++)
     if( i !=j && arr[i] == arr[j]) return "Match!";
```

(What do they do?)

How long does it take to execute each block?

Measuring Computational Cost

- How can we measure the amount of work needed by a computation?
  - Get out a stopwatch (aka wall-clock time)?
    - Problems?
      - Different machines have different clocks
      - Too much other stuff happening (network, OS, etc)
      - Not consistent. Need lots of tests to predict future behavior

Measuring Computational Cost

- A better way: Counting computations
  - Count *all* computational steps?
  - Count how many “expensive” operations were performed?
  - Count number of times “x” happens?
    - For a specific event or action “x”
    - i.e., How many times a certain variable changes
- Question: How accurate do we need to be?
  - 64 vs 65? 100 vs 105? Does it really matter??

An Example

```
// Pre: array length n > 0
public static int findPosOfMax(int[] arr) {
       int maxPos = 0 // A wild guess
       for(int i = 1; i < arr.length; i++)
              if (arr[maxPos] < arr[i]) maxPos = i;
       return maxPos;
}
```

- Can we count steps exactly?
  - "if" makes it hard

- Idea: **Overcount**: assume "if" block always runs
  - Overcounting gives *upper bound* on run time
  - Can also undercount for lower bound

Measuring Computational Cost

- Rather than keeping exact counts, we want to know the *order of magnitude* of occurrences
    - 60 vs 600 vs 6000, *not* 65 vs 68
    - `n`, *not* `4(n-1) + 4`
- We want to make comparisons without looking at details and without running tests
- Avoid using specific numbers or values
- Look for overall trends

Measuring Computational Cost

- **How does work scale with problem size?**
  - E.g.: If I double the size of the problem instance, how much longer will it take to solve:
    - Find maximum: $n - 1 \rightarrow (2n) - 1$ (twice as long)
    - Bubble sort: $n(n-1)/2 \rightarrow 2n(2n - 1)/2$ (4 times as long)
    - Enumerate all subsets: $2^{n-1} \rightarrow 2^{(2n)-1}$ (2^n times as long!!!)
    - Etc.
- We will also measure amount of space used by an algorithm using the same ideas….

Function Growth

Consider the following functions, for `x >= 1`

- $f(x) = 1$
- $g(x) = \log_2(x)$ // Reminder: if $x=2^n$, $\log_2(x) = n$
- $h(x) = x$
- $m(x) = x \log_2(x)$
- $n(x) = x^2$
- $p(x) = x^3$
- $r(x) = 2^x$

Function Growth & Big-O

- Rule of thumb: ignore multiplicative constants
- Examples:
  - Treat n and n/2 as same order of magnitude
  - $n^2/1000$, $2n^2$, and $1000n^2$ are "pretty much" just n^2

- The key is to find the most *significant* or *dominant* term
- Ex: $\lim_{x \to \infty} (3x^4 - 10x^3 - 1)/x^4 = 3$ (Why?)
  - So $3x^4 - 10x^3 - 1$ grows "like" x^4

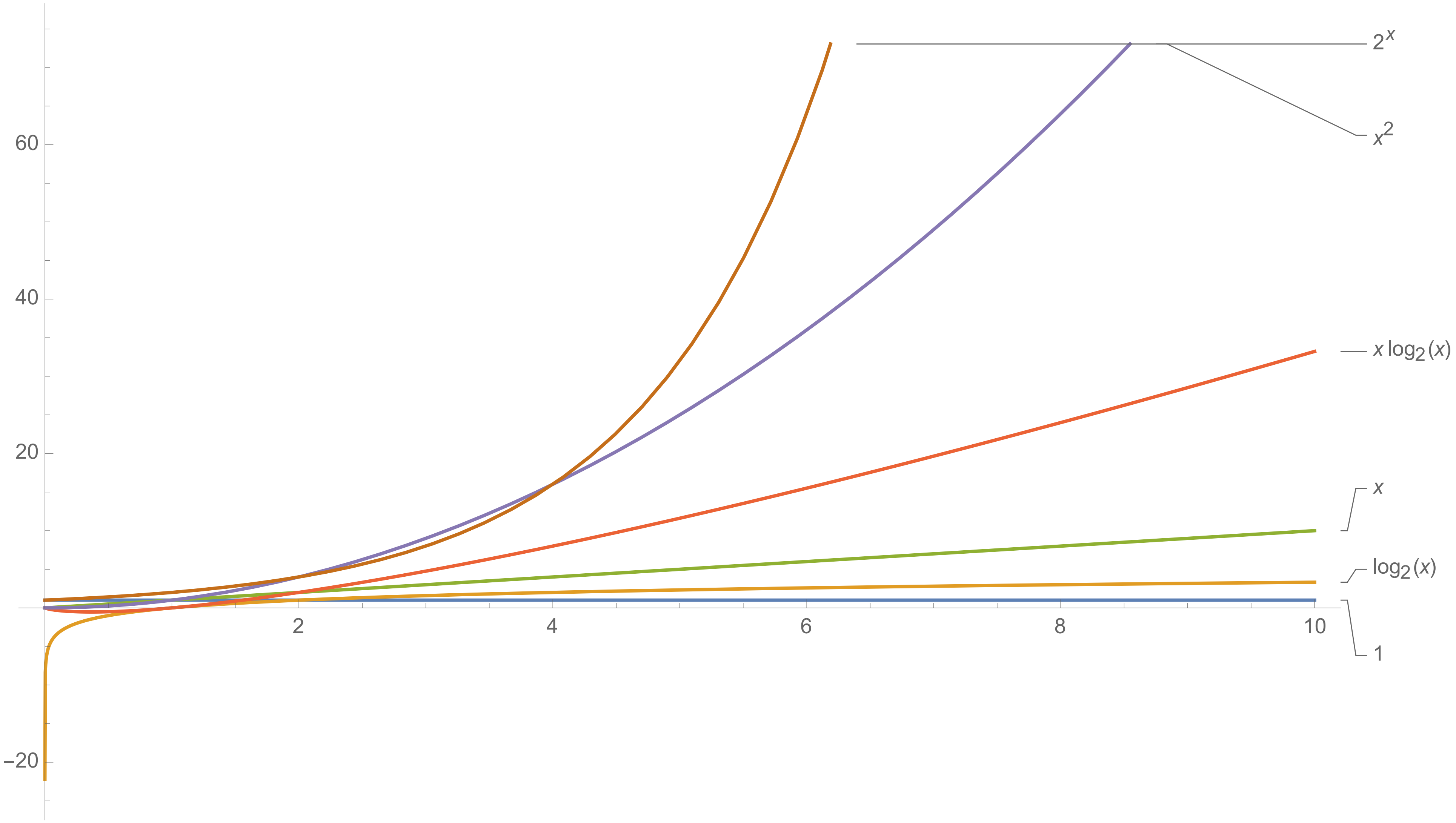
Asymptotic Bounds (Big-O Analysis)

- A function f(n) is *O(g(n))* if and only if there exist positive constants c and n_0 such that

$$|f(n)| \leq c \cdot g(n) \text{ for all } n \geq n_0$$

- g is "at least as big as" f **for large n**
  - Up to a multaplicative constant c!
- Example:
  - $f(n) = n^2/2$ is $O(n^2)$
  - $f(n) = 1000n^3$ is $O(n^3)$
  - $f(n) = n/2$ is $O(n)$

Function Growth



Determining "Best" Upper Bounds

- We typically want the *smallest* upper bound when we estimate running time
- Example: Let $f(n) = 3n^2$
  - $f(n)$ is $O(n^2)$
  - $f(n)$ is $O(n^3)$
  - $f(n)$ is $O(2^n)$
  - $f(n)$ is NOT $O(n)$ (!!)
- "Best" upper bound is $O(n^2)$
- We care about **c** and $\mathbf{n_0}$ in practice, but focus on size of **g** when designing algorithms and data structures

Input-dependent Running Times

- Algorithms may have different running times for different input values
- Best case (typically not useful)
  - Sort already sorted array
  - Find item in first place that we look
- Worst case (generally useful, sometimes misleading)
  - Don't find item in list O(n)
  - Reverse order sort O(n^2)
- Average case (useful, but often hard to compute)
  - Linear search O(n)
  - QuickSort random array O(n log n) ← We'll sort soon

Vector Operations : Worst-Case

For `n` = Vector size (*not* capacity!):

- O(1):
  - `size(), capacity(), isEmpty(), get(i), set(i), firstElement(), lastElement()`
- O(n):
  - `indexOf(), contains(), remove(elt), remove(i)`
- What about add methods?
  - If Vector doesn't need to grow
    - `add(elt)` is `O(1)` but `add(elt, i)` is `O(n)`
  - Otherwise, depends on `ensureCapacity()` time
    - Time to copy array: `O(n)`

Vectors: Add Method Complexity

Suppose we grow the Vector's array by a fixed amount `d`.

How long does it take to add `n` items to an empty Vector?

- The array will be copied each time its capacity needs to exceed a multiple of `d`
  - At sizes `0, d, 2d, … , n/d.`
- Copying an array of size `kd` takes `ckd` steps for some constant `c`, giving a total of

$$\sum_{k=1}^{n/d} ckd = cd \sum_{k=1}^{n/d} k = cd \left(\frac{n}{d}\right)\left(\frac{n}{d} + 1\right)/2 = O(n^2)$$

Vectors: Add Method Complexity

Suppose we grow the Vector's array by doubling.

How long does it take to add n items to an empty Vector?

- The array will be copied each time its capacity needs to exceed a power of 2
  - At sizes `0, 1, 2, 4, 8 …, n/2`
- The total number of elements are copied when n elements are added is:
  - `1 + 2 + 4 + ... + n/2 = n-1 = O(n)`

- Very cool!