CSCI 136
Data Structures & Advanced Programming

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Lecture 33

May 5, 2017

Administrative Details

- Lab 10 - Hexapawn
  - Any questions?
- Study Groups
  - A handful of people expressed interest
  - Let me know by 5pm and I'll send a group email

Last Time

- Began Graphs
- Core concepts:
  - Adjacency Matrix
    - |V|x|V| matrix, each (i,j) represents a possible edge
    - matrix[i][j] = T if edge between v_i , v_j, F otherwise
    - Good for dense graphs
  - Adjacency List
    - Vector of length |V|
    - Each index *i* stores a list of vertex *i*'s outgoing edges
    - Good for sparse graphs

Today's Outline

- Graph Implementations (details in book)
  - Graph Interface
  - Intuition for abstract graph classes
- Traversing graphs
  - BFS
  - DFS

Graph Implementation Approach

- Interface specifies general graph requirements
  - Graph<V,E>
    - Edge<V,E> (connects two vertices, public)
    - Vertex<E> (labeled vertex, non-public)
- Abstract class**es** implement graph interface, provide common code
  - Strategy: provide all the code that can be written without assuming a graph's "directedness"

Vertex Class

- Graph *vertices* are defined in a non-public class
  - `Vertex<E> (E label)`
- Just a label and a marker to track "visits" during traversals
- Methods

  `label()`,
  `visit()`, `isVisited()`,
  `reset()`,
  `equals()`

Edge Class

- Graph *edges* are defined in their own public class
  - `Edge<V, E> (V vtx1, V vtx2, E label, boolean directed)`
- Construct a (possibly directed) edge between two labeled vertices (vtx1 ➡ vtx2)
- Methods:

  `label()`, `setLabel()`,
  `here()`, `there()`
  `visit()`, `isVisited()`,
  `reset()`,
  `isDirected()`

Graph<V, E> Interface Methods

- void add(V label), V remove(V label)
  - Add/remove vertex to graph
- void addEdge(V vtx1Label, V vtx2Label, E edgeLabel), E removeEdge(V vtx1Label, V vtx2Label)
  - Add/remove edge between vtx1 and vtx2
- boolean containsEdge(V vtx1Label, V vtx2Label)
  - Returns true iff there is an edge between vtx1 and vtx2
- Edge getEdge(V vtx1Label, V vtx2Label)
  - Returns edge between vtx1 and vtx2
- void clear()
  - Remove all nodes (and edges) from graph

Graph<V, E> Interface Methods

- boolean visit(V vertexLabel)
  - Mark vertex as "visited" and return *previous* value of visited flag
- boolean visitEdge(Edge<V, E> edge)
  - Mark edge as "visited"
- boolean isVisited(V vtxLabel), boolean isVisitedEdge(Edge<V,E> e)
  - Returns true iff vertex/edge has been visited
- Iterator<V> neighbors(V vertexLabel)
  - Get iterator for all neighbors to vertex
  - For directed graphs, out-edges only
- Iterator<V> iterator()
  - Get vertex iterator (over all vertices in graph)
- void reset()
  - Remove visited flags for all nodes/edges

Abstract Graph Classes

- **GraphMatrix<V, E>**
  - GraphMatrixDirected<V,E>
  - GraphMatrixUndirected<V,E>
- **GraphList<V, E>**
  - GraphListDirected<V,E>
  - GraphListUnDirected<V,E>

GraphMatrix Intuition

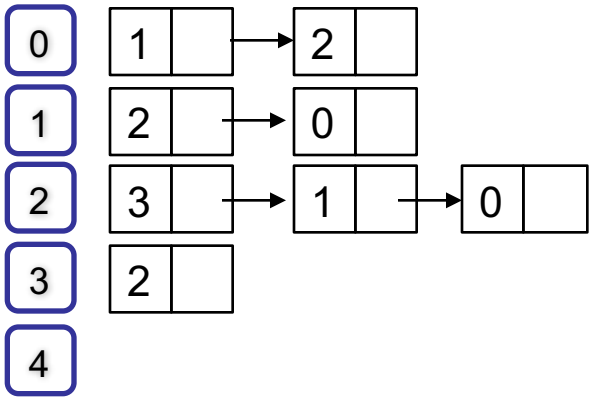
- NxN array of Edge objects
- Dictionary to map Vertex label to array index
- add(vLabel)
  - Add mapping from label to index to dictionary
- addEdge(v1Label, v2Label, edgeLabel)
  - Check dictionary for v1, v2 indexes (*i*, *j*)
  - Create edge and add it at matrix[*i*][*j*]
- getEdge(v1Label, v2Label)
  - Check dictionary for v1, v2 indexes (*i*, *j*)
  - Check matrix[*i*][*j*] for edge

	0	1	2	3	4
0	_	_	_	_	_
1	_	_	_	_	_
2	_	_	_	_	_
3	_	_	_	_	_
4	_	_	_	_	_

{ }

GraphList Intuition

- Each Vertex stores a list of Edge objects
- Dictionary to map Vertex label to Vertex
- add(vLabel)
  - Create a new Vertex with an empty list
  - Add label to vertex mapping to dictionary
- addEdge(v1Label, v2Label, edgeLabel)
  - Check dictionary for v1 and v2 vertexes
  - Create edge, add to appropriate vertex lists
- getEdge(v1Label, v2Label)
  - Check dictionary for v1 and v2 vertexes
  - Search v1 vertex list for target edge



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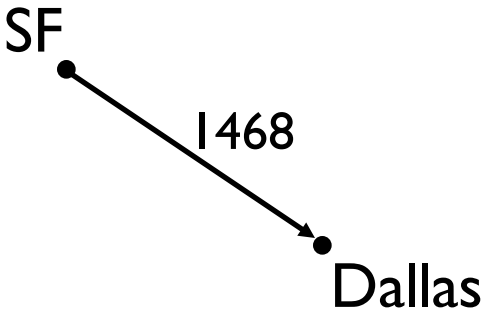
Example Graph/Edge Usage

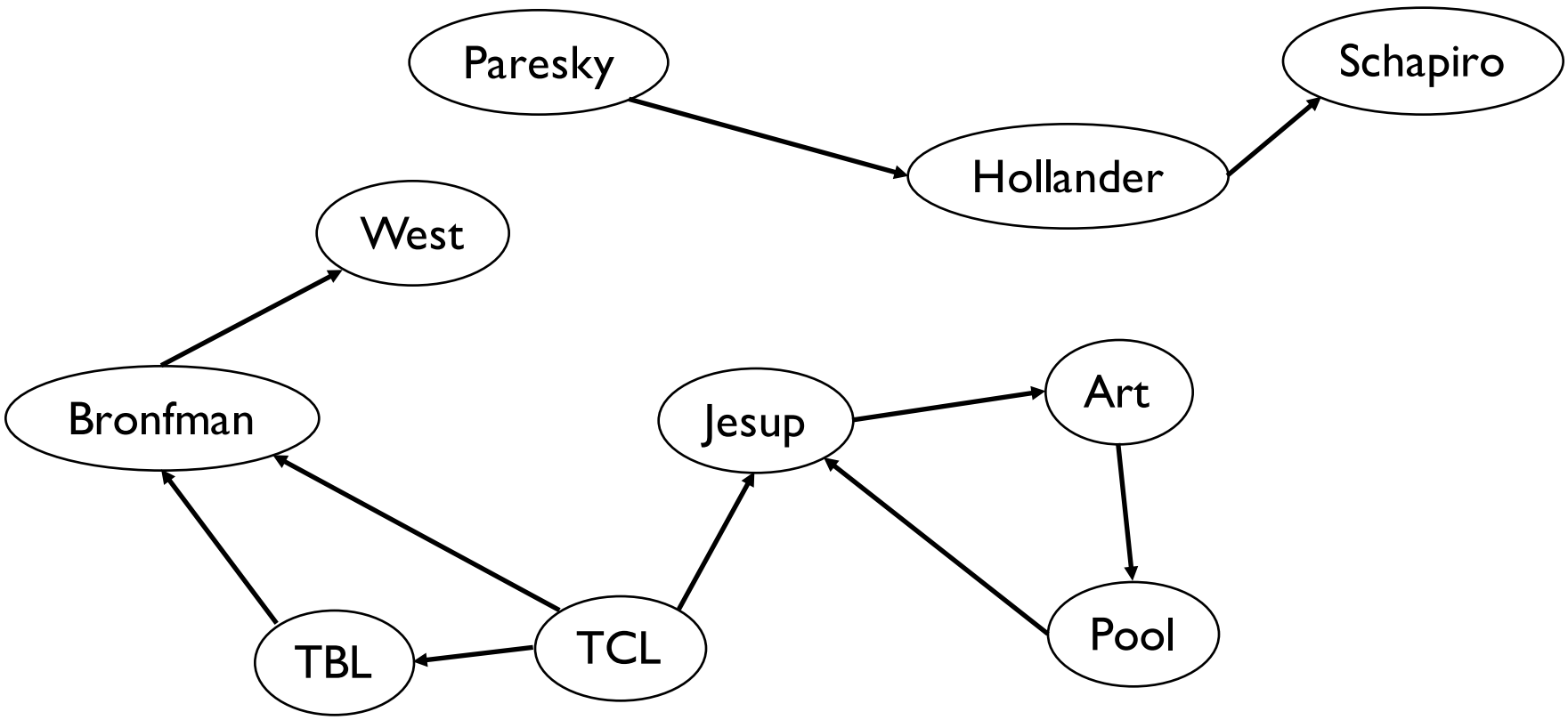
- For example map from last week with edge labels:

```
Graph<String,Integer> g =
          new GraphMatrixDirected<String, Integer>(10);
g.add("SF");
g.add("Dallas");
g.addEdge("SF", "Dallas", new Integer(1468));
…
Edge<String, Integer> SFtoDallas =
                              g.getEdge("SF", "Dallas");
int dist = SFtoDallas.label();
```



Graph Exploration

Suppose we applied a direction to the edges in our campus map…

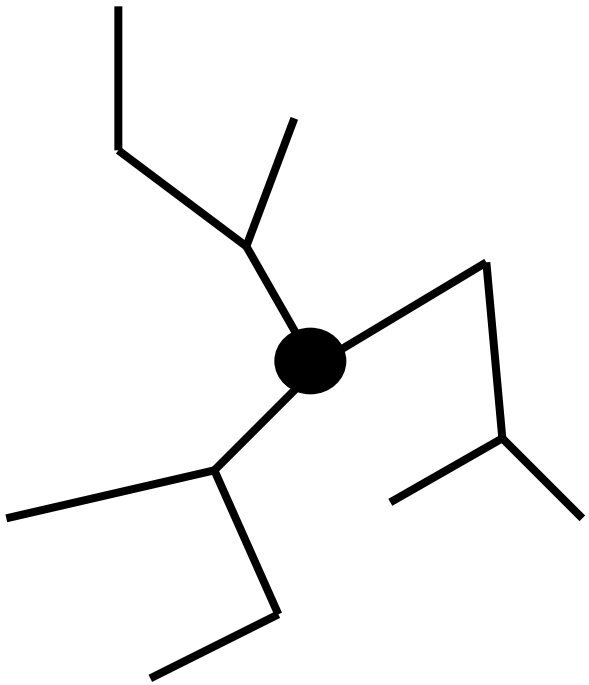


Reachability

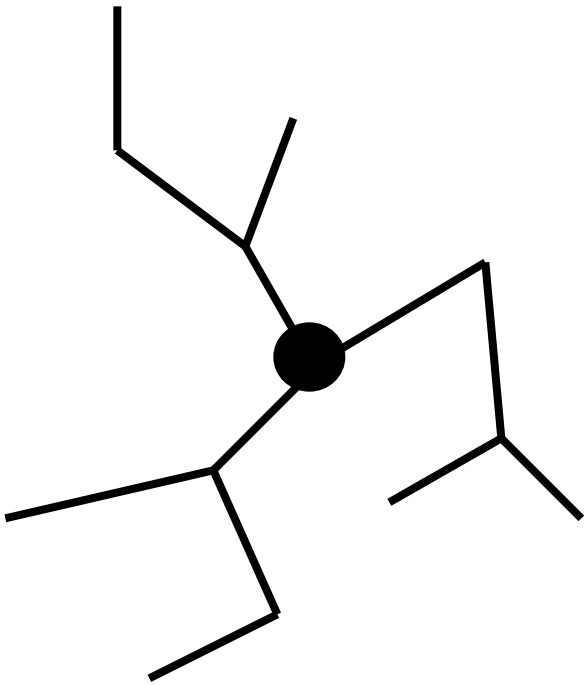
- There are two ways to measure reachability in our graph
  - Depth-first search and breadth-first search
- How did we do DFS and BFS in trees?
- DFS uses a stack
  - Stack records path from src to current node
  - Like pre-order tree traversal with visited flags to only visit nodes once
  - Runtime: O(|E|)
- BFS uses a queue
  - Queue records nodes whose out edges have not been explored
  - Like level-order tree traversal
  - Runtime: O(|E|)

DFS vs. BFS

DFS:

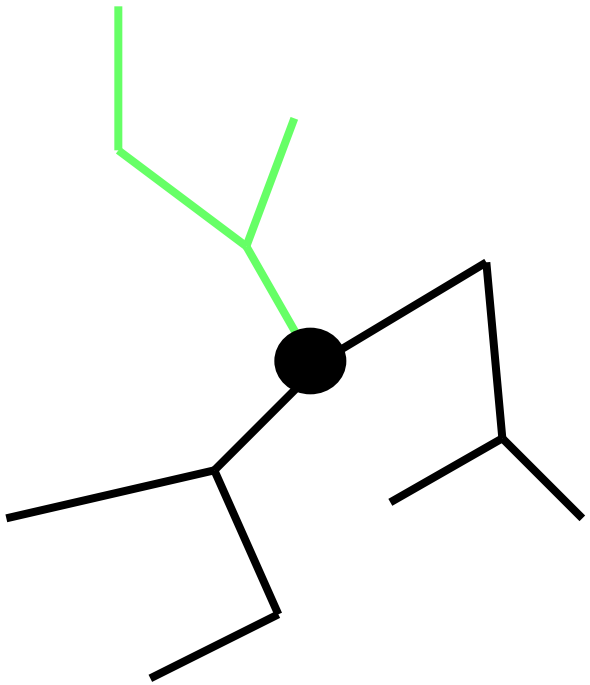


BFS:

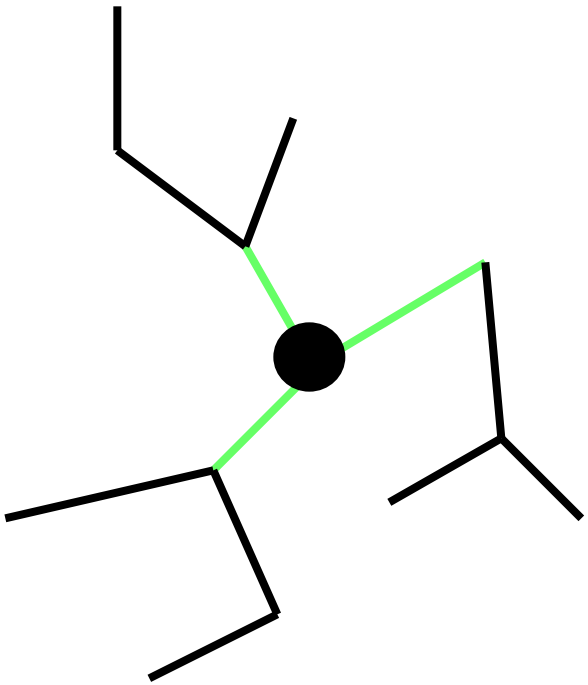


DFS vs. BFS

DFS:

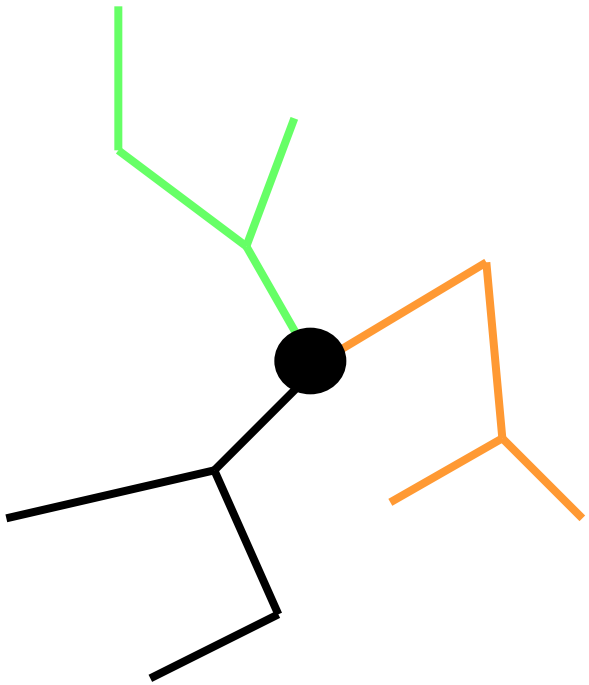


BFS:

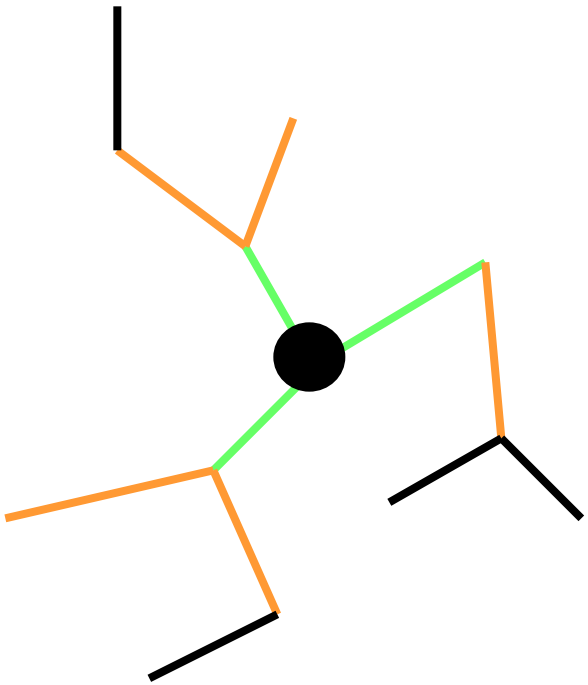


DFS vs. BFS

DFS:

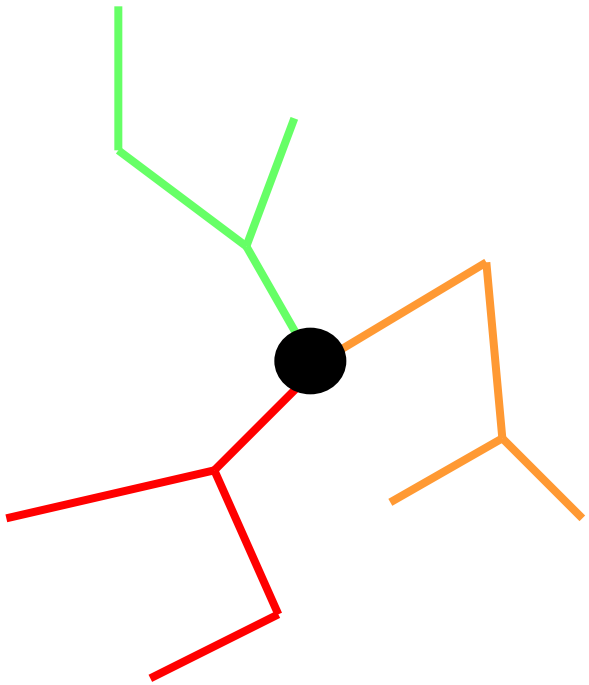


BFS:

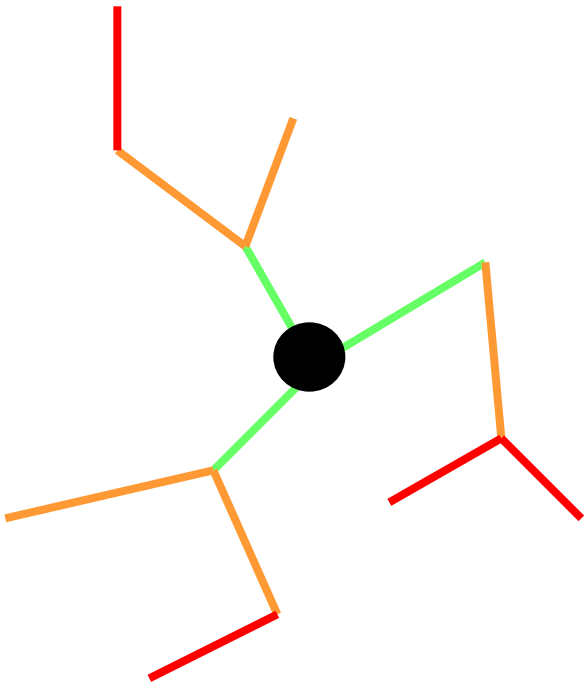


DFS vs. BFS

DFS:



BFS:



Depth-First Search

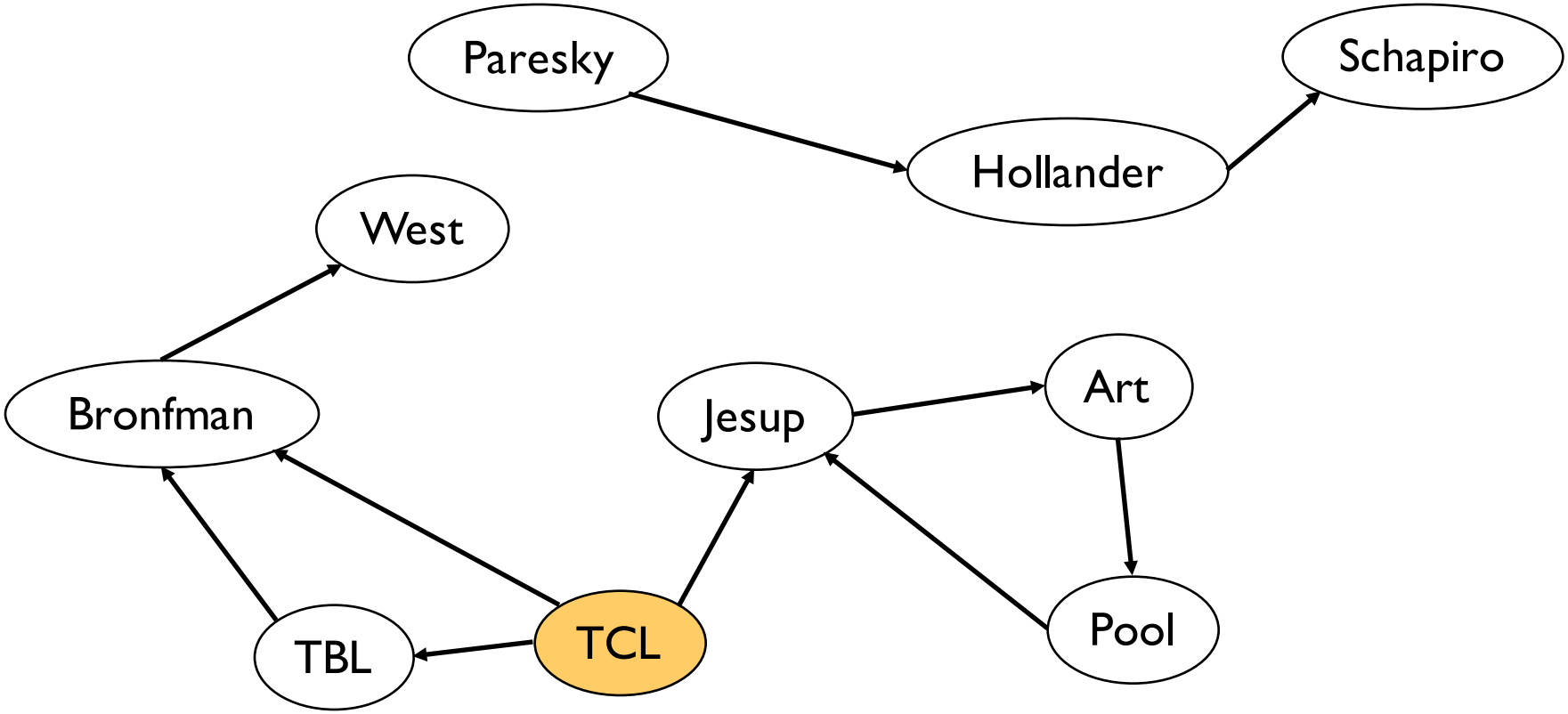
```
void reachableFrom(Graph<V,E> g, V src) {

  if (!g.visited(src)) {
    g.visit(src);

     Iterator<V> neighbors = g.neighbors(src);
     while (neighbors.hasNext()) {
       V next = neighbors.next();
        if (!g.visited(next))
           reachableFrom(g, next);
    }
  }

}
```

Depth-First Search

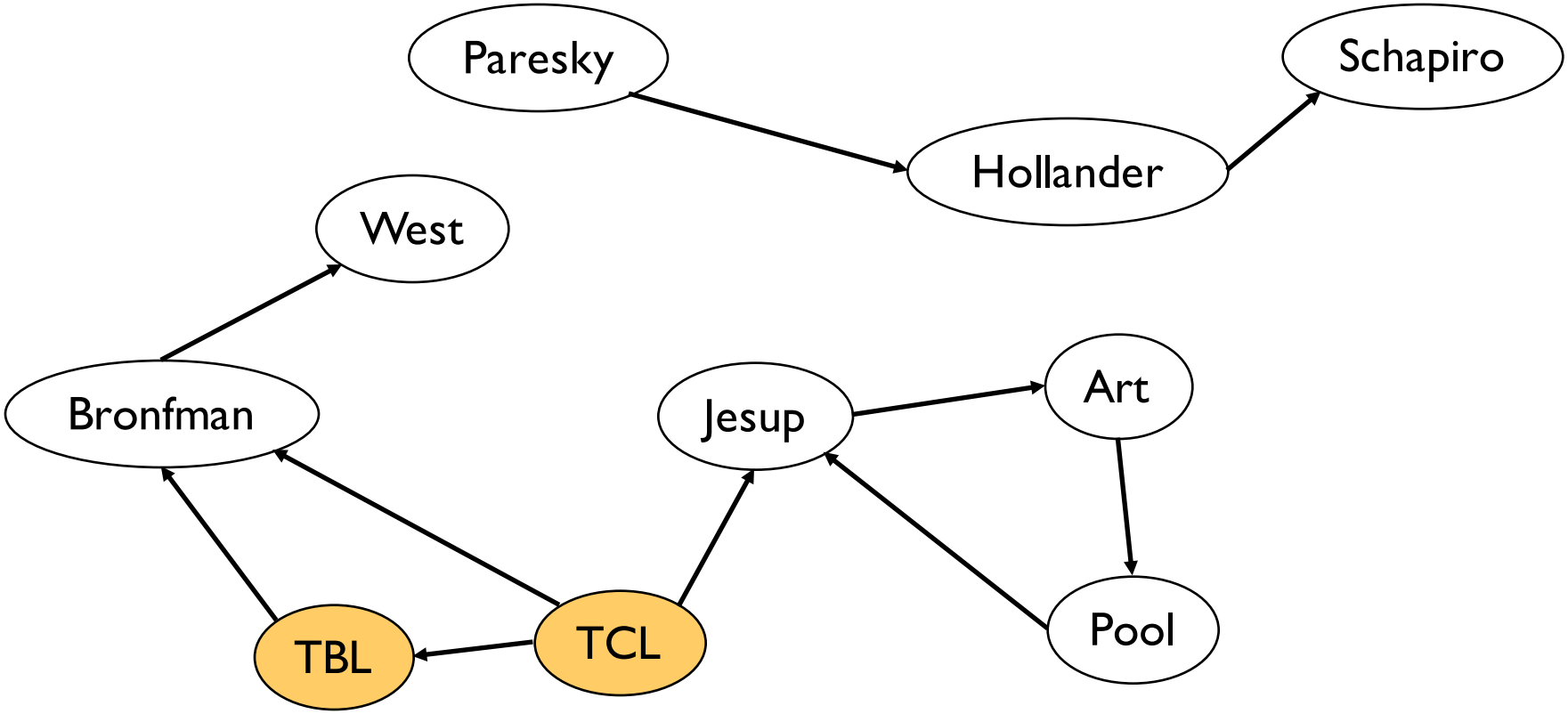


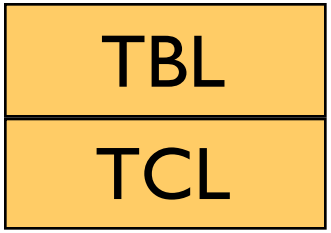
TCL

Find first unvisited neighbor of TCL…

todo stack

Depth-First Search

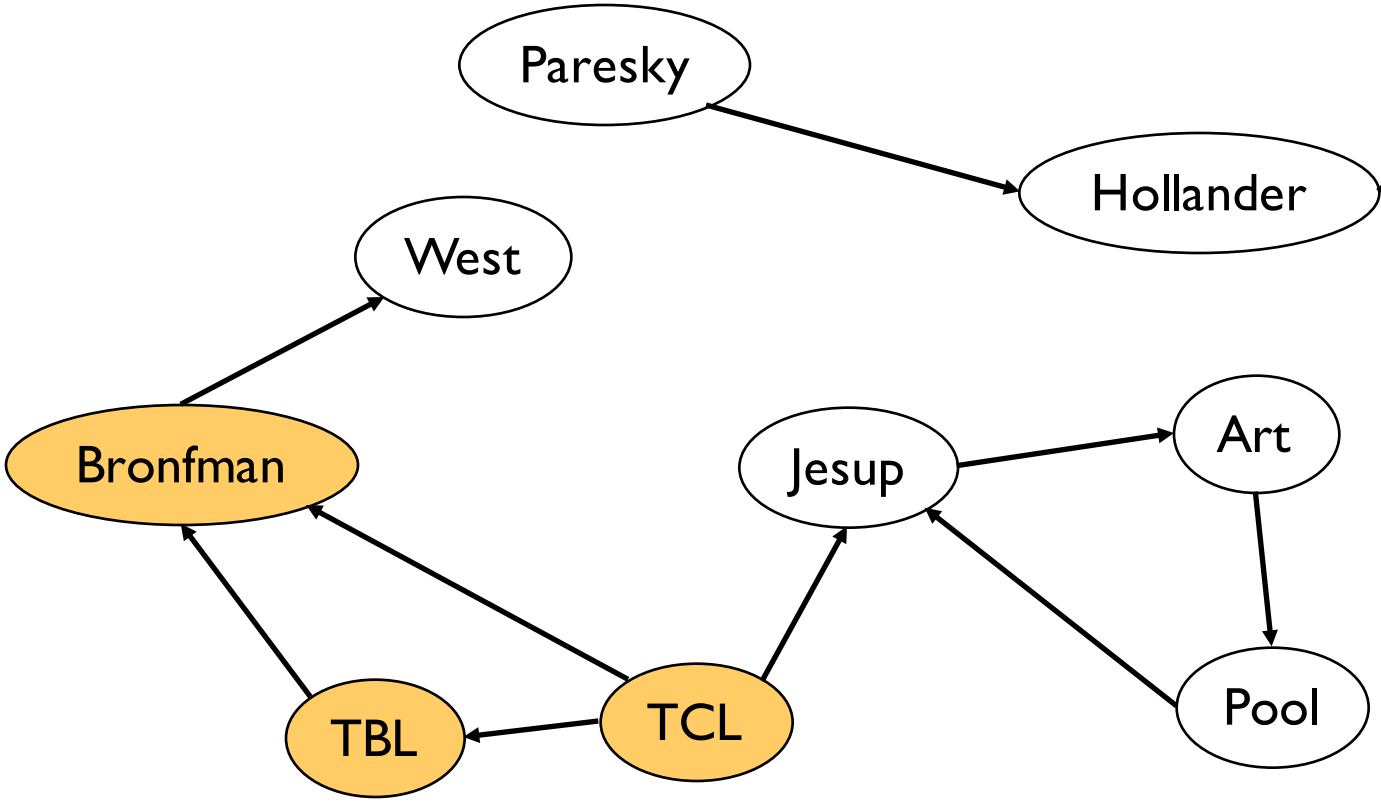


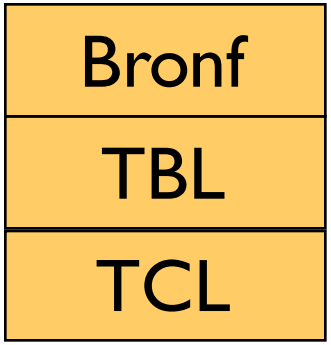


Find first neighbor of TBL…

todo stack

Depth-First Search

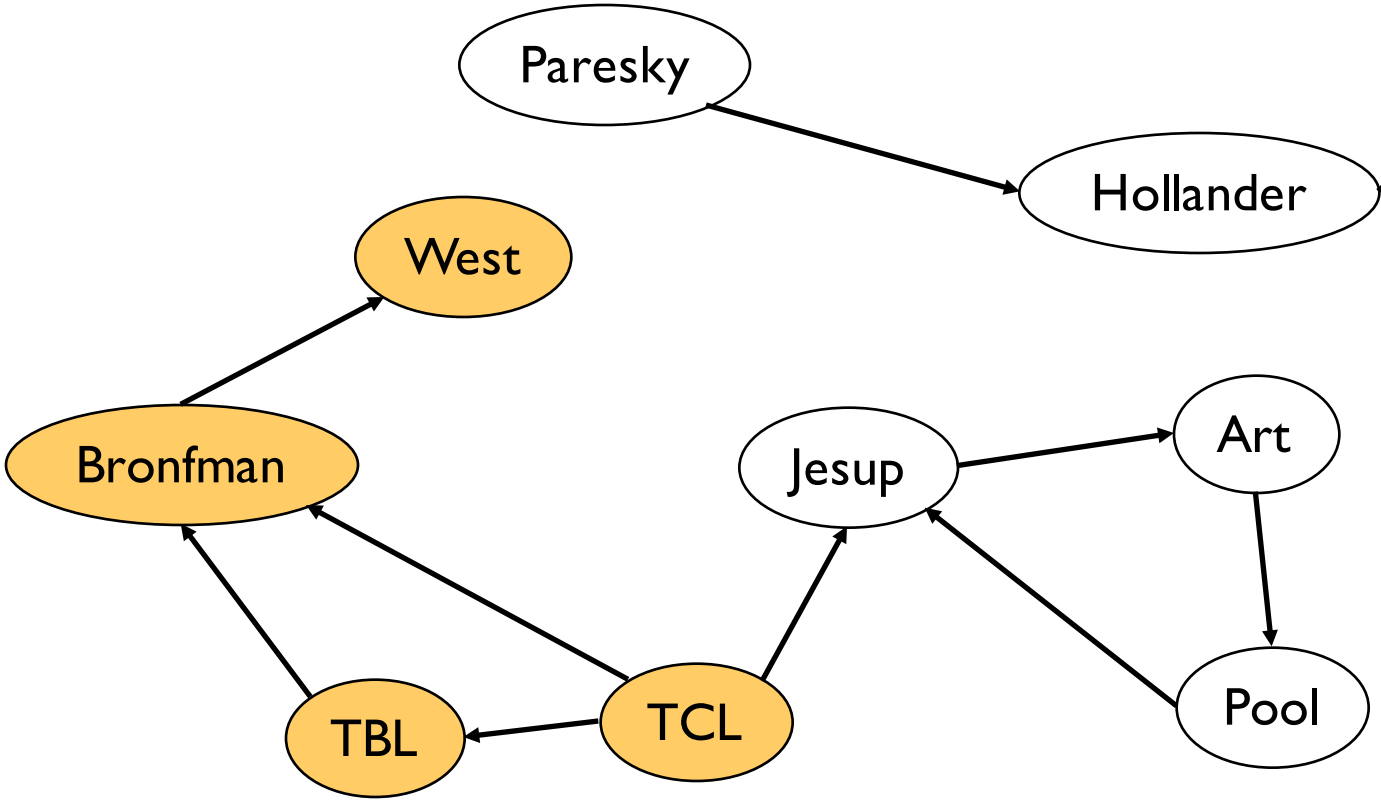




Find first neighbor of Bronfman…

todo stack

Depth-First Search

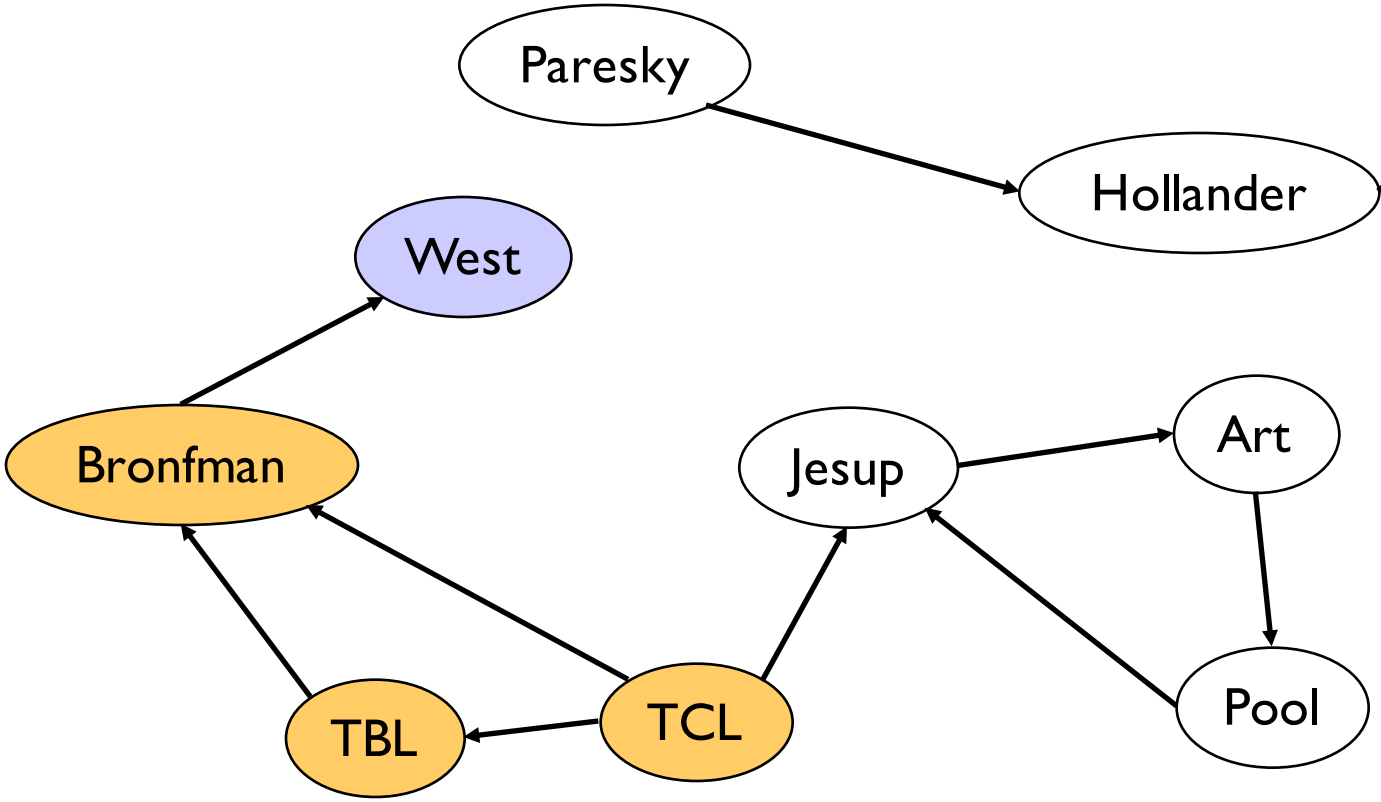


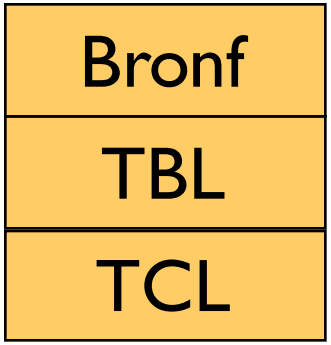
West
Bronf
TBL
TCL

todo stack

Find first neighbor of West…none exist! Pop stack…

Depth-First Search

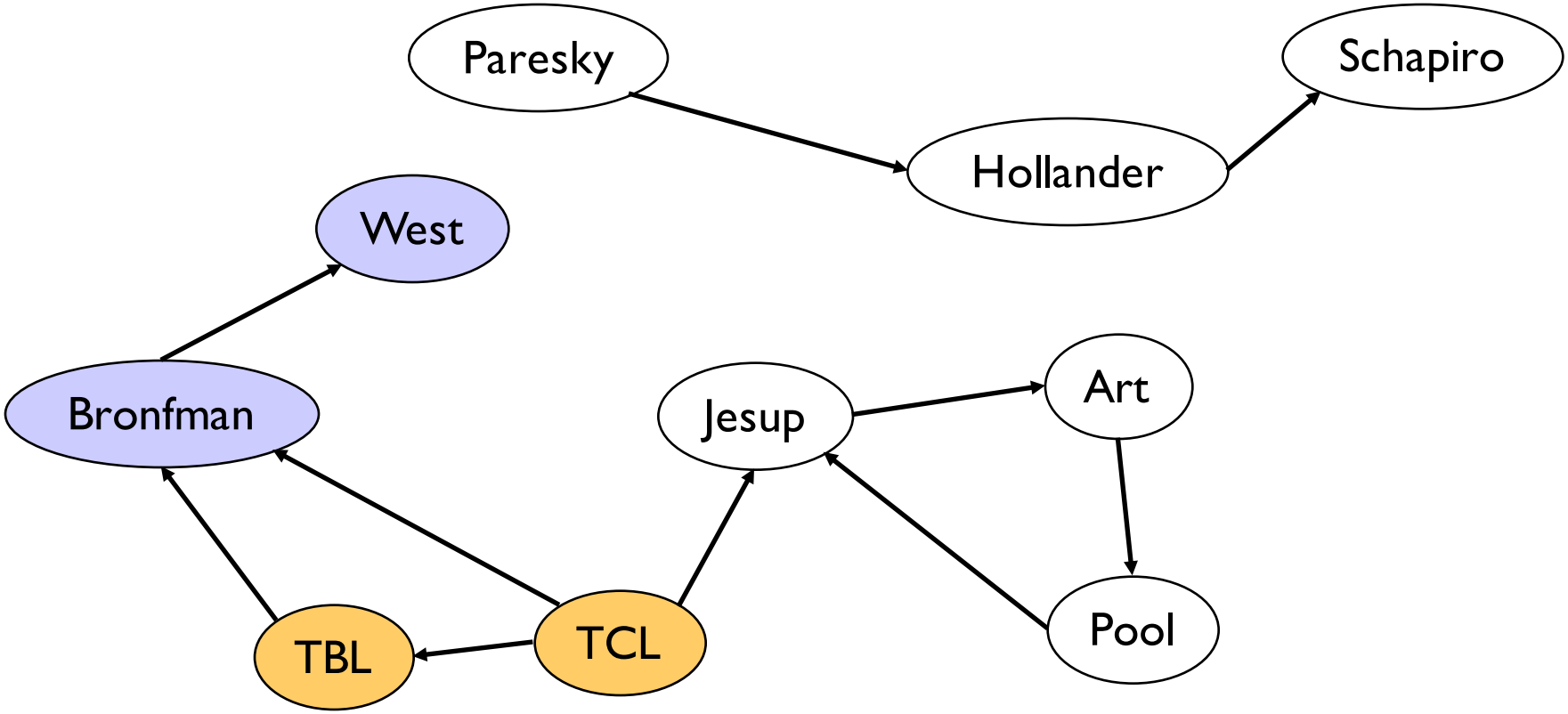


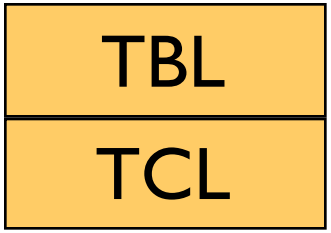


Find next neighbor of Bronf…none exist! Pop stack…

todo stack

Depth-First Search

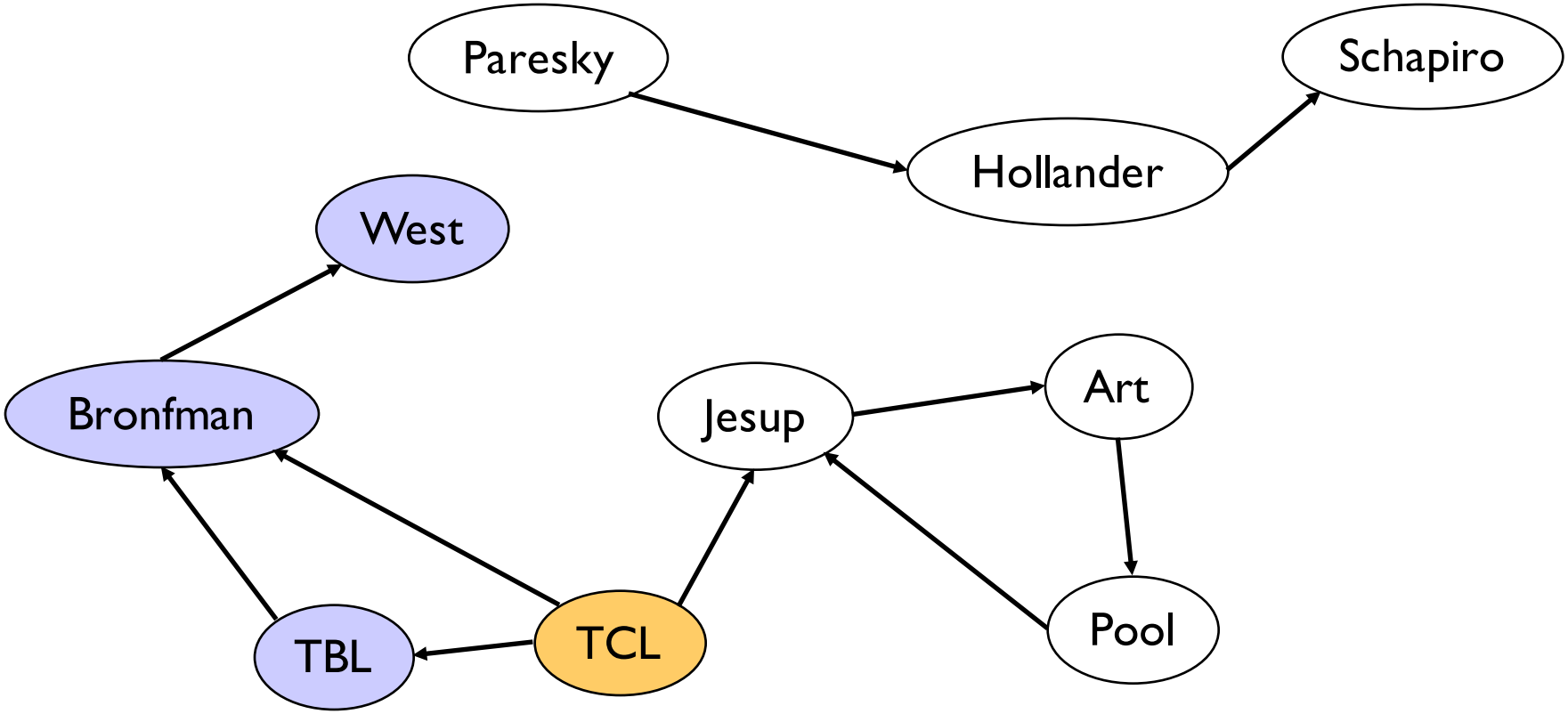




Find next neighbor of TBL…none exist! Pop stack…

todo stack

Depth-First Search

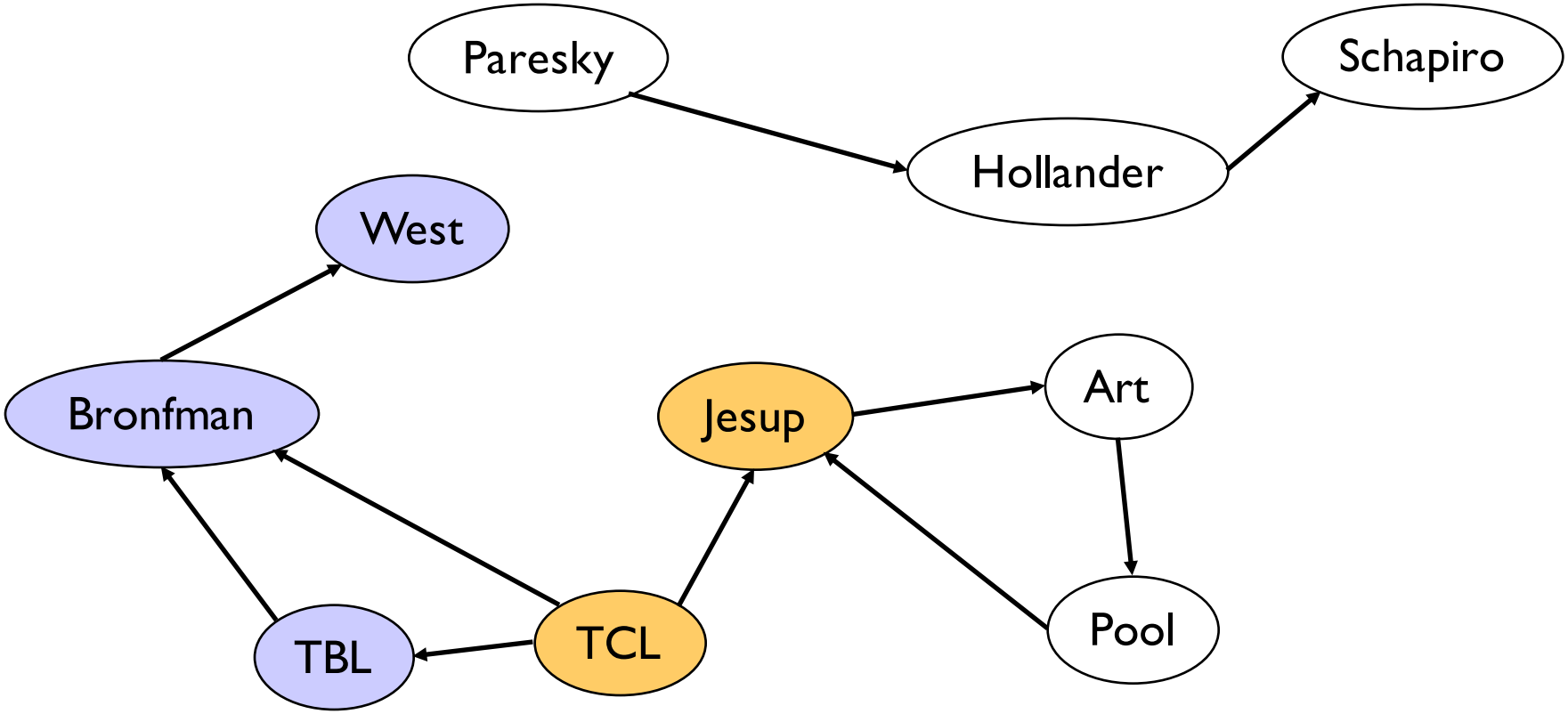


TCL

Find next unvisited neighbor of TCL…

todo stack

Depth-First Search

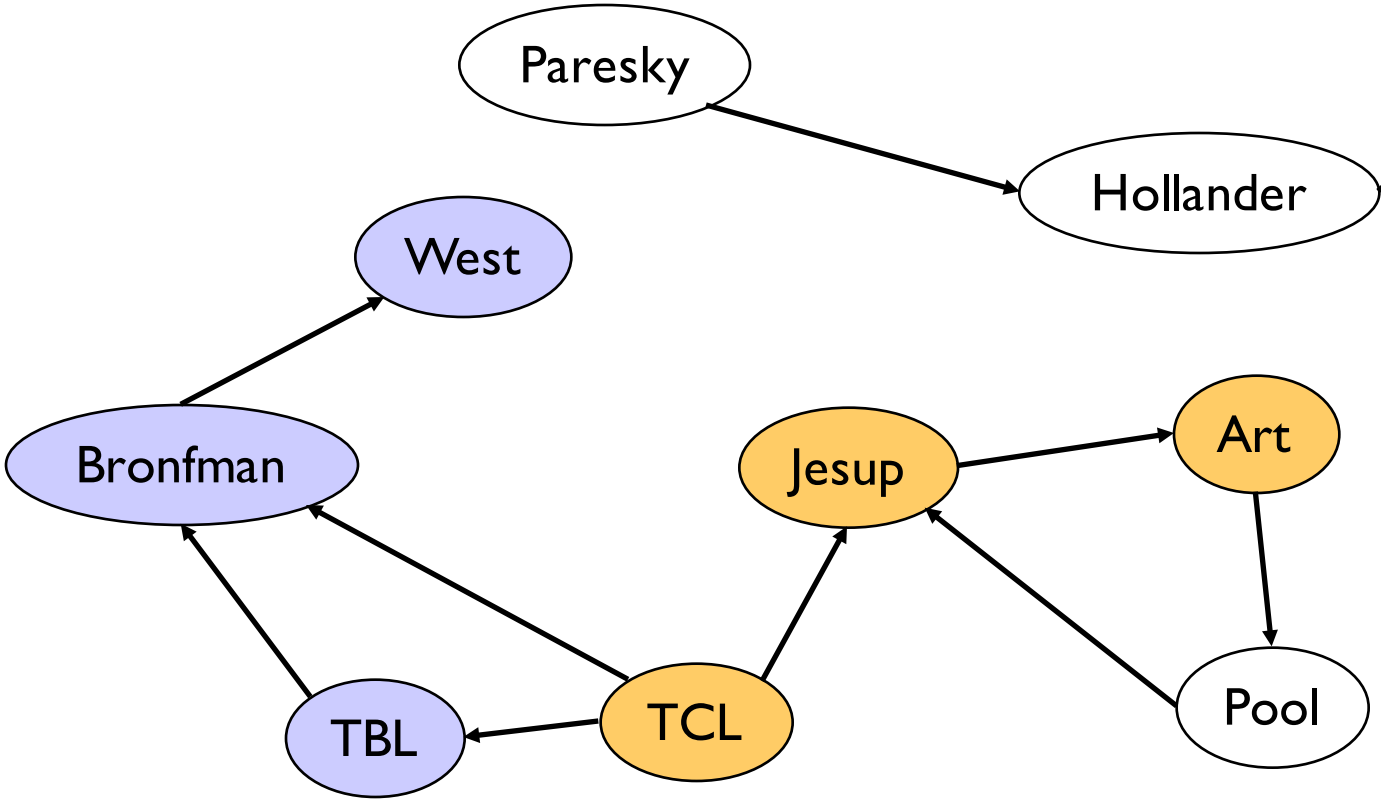


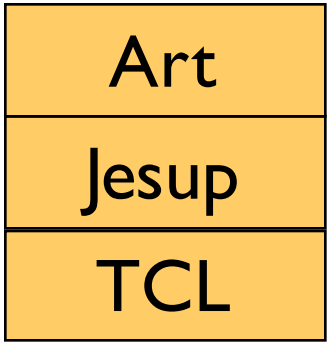
Jesup
TCL

todo stack

Find first neighbor of Jesup…

Depth-First Search

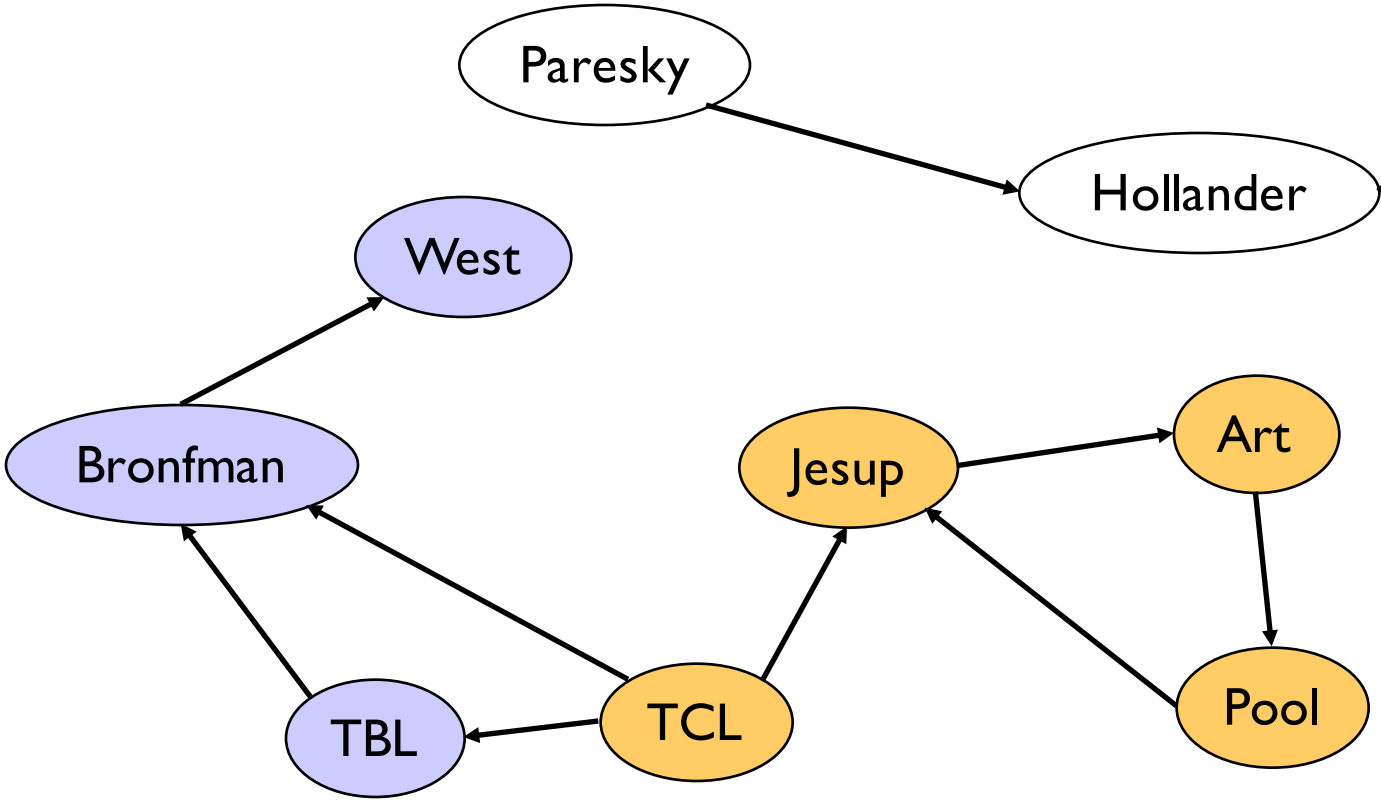


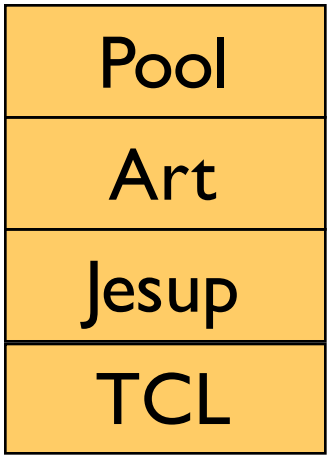


todo stack

Find first neighbor of Art…

Depth-First Search

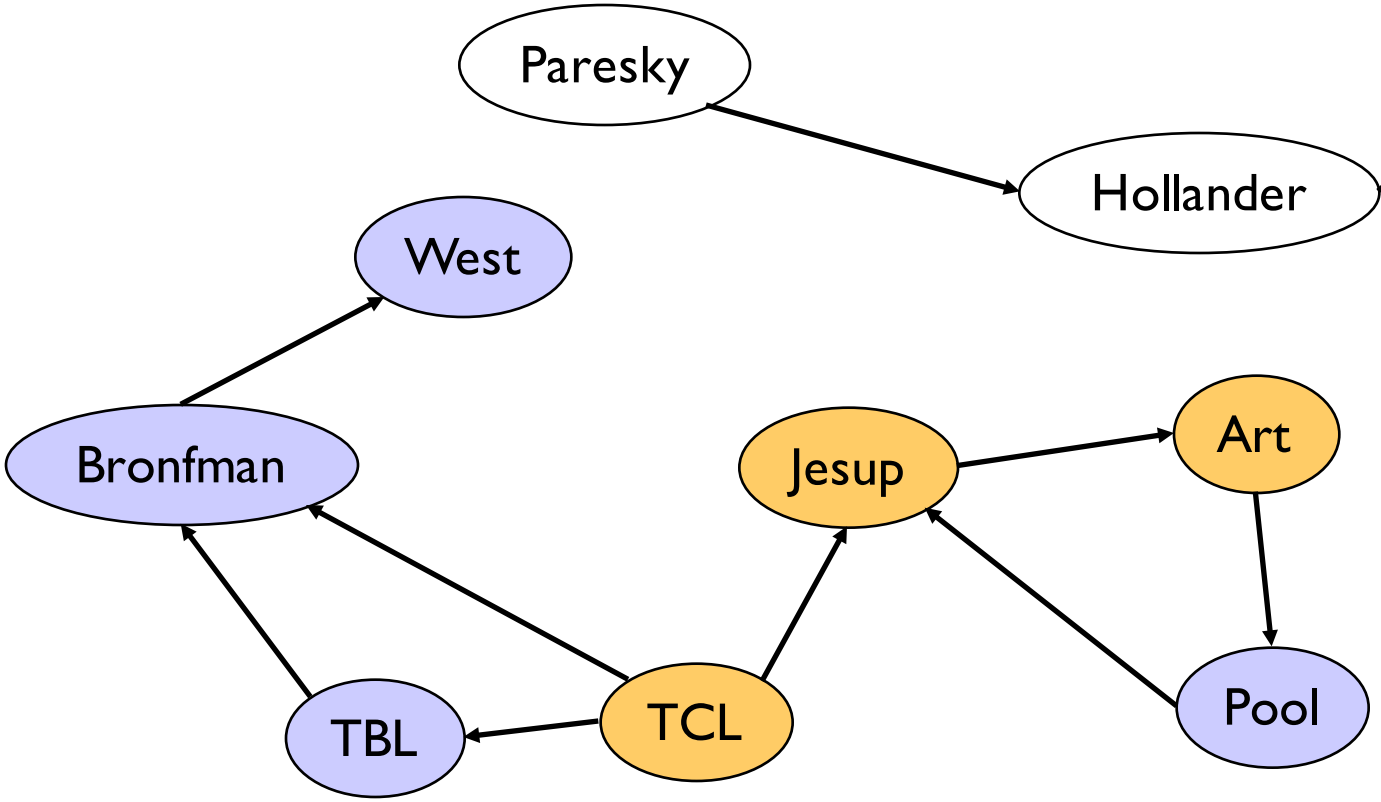




todo stack

Find first unvisited neighbor of Pool…none exist!

Depth-First Search

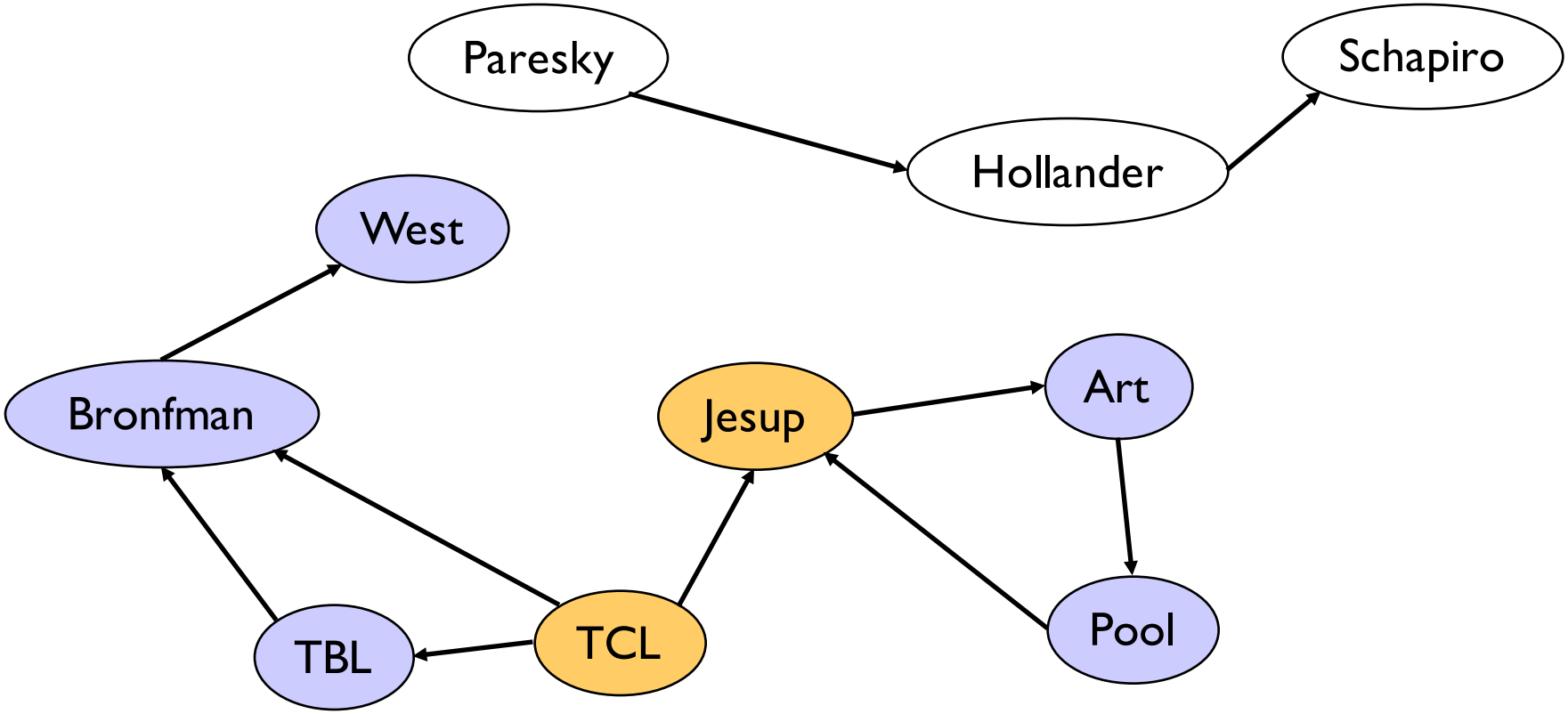


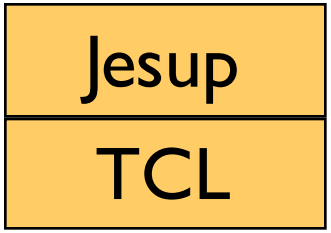
Art
Jesup
TCL

todo stack

Find next neighbor of Art…none exist!

Depth-First Search

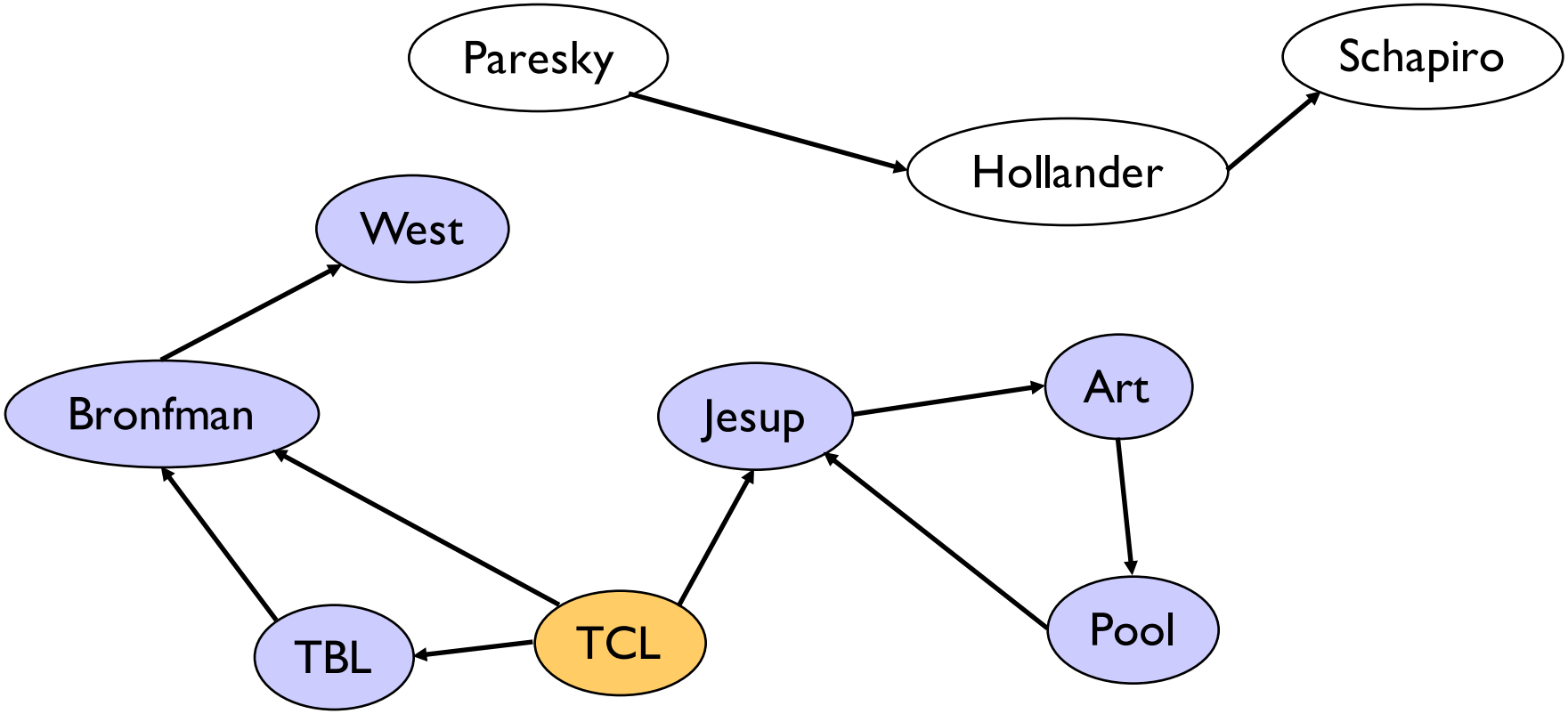




Find next neighbor of Jesup…none exist!

todo stack

Depth-First Search

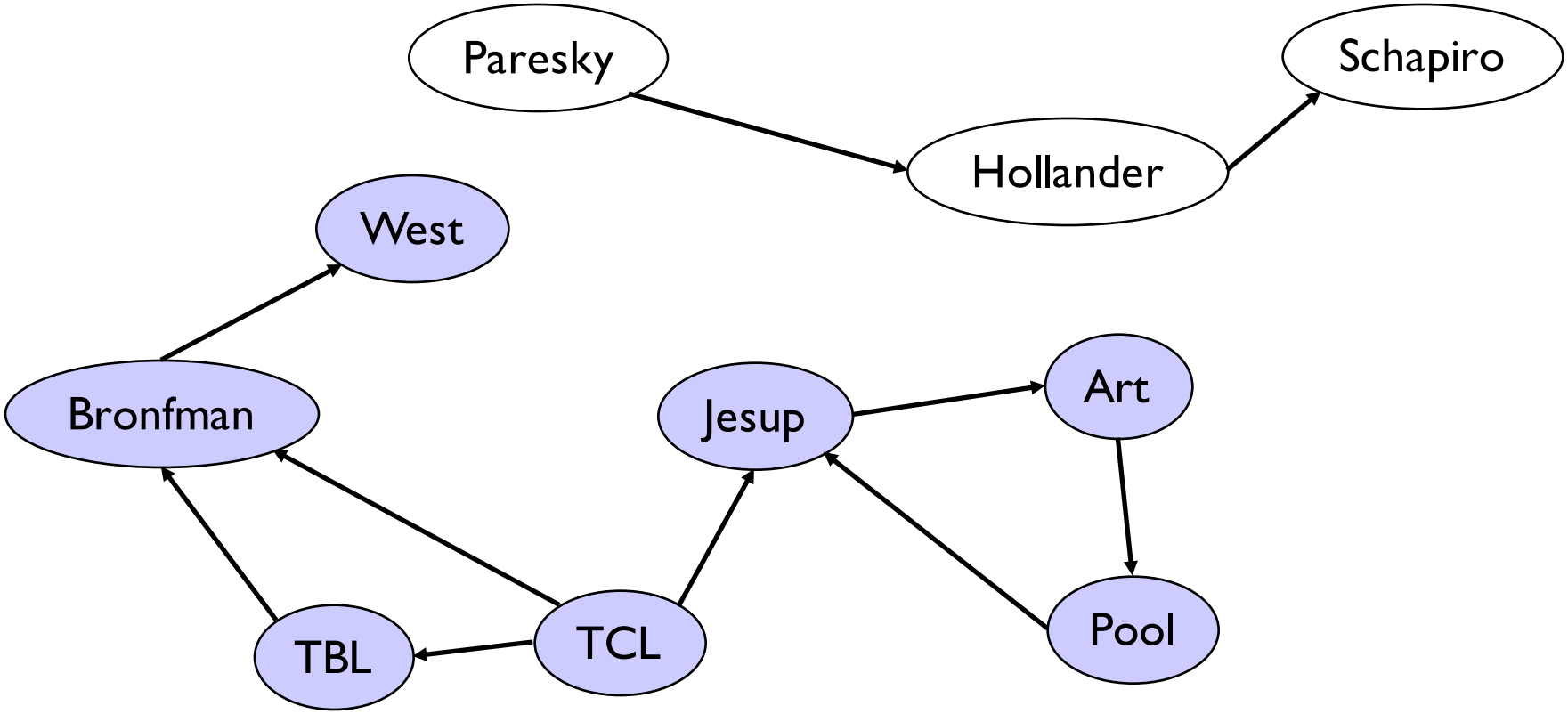


TCL

todo stack

Find next neighbor of TCL…none exist!

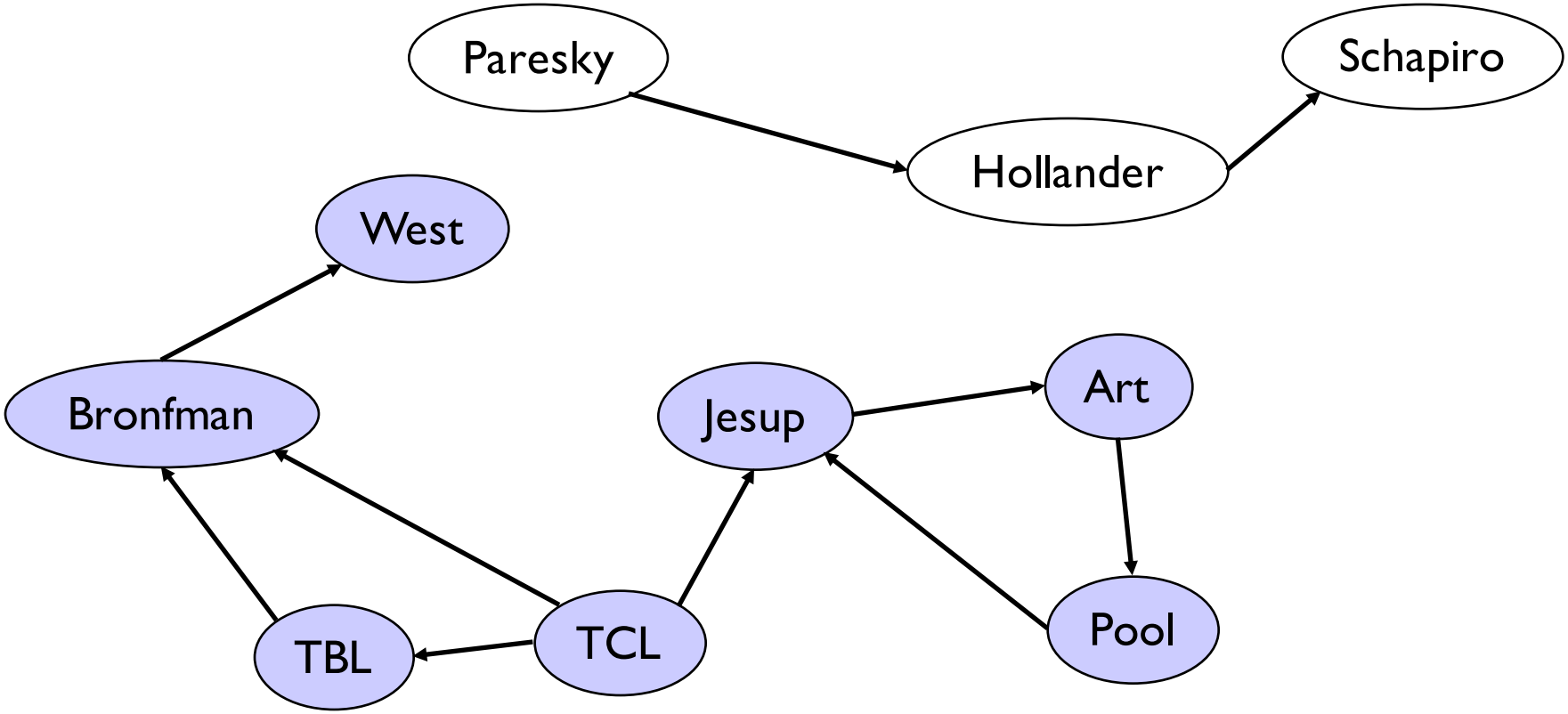
Depth-First Search



We're done!

todo stack

Depth-First Search

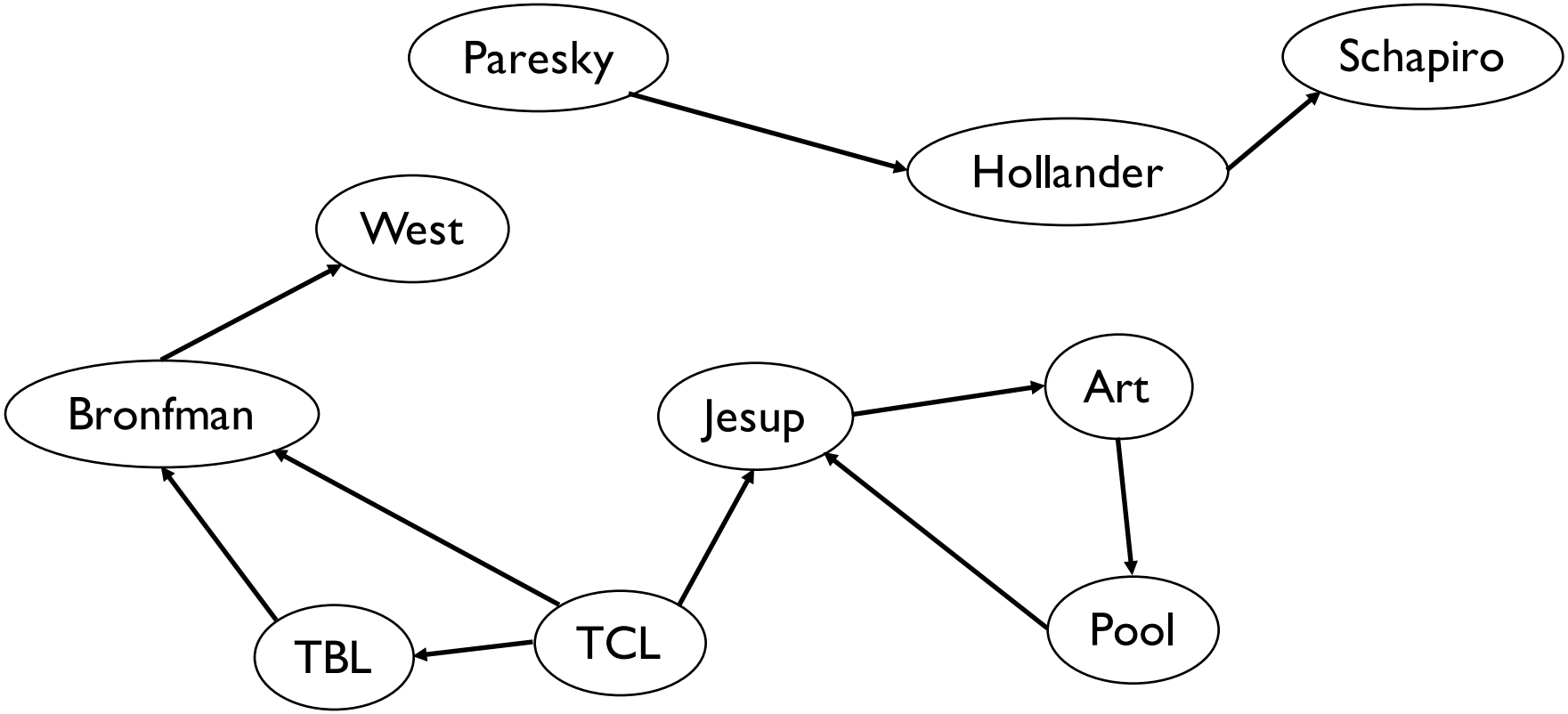


todo stack

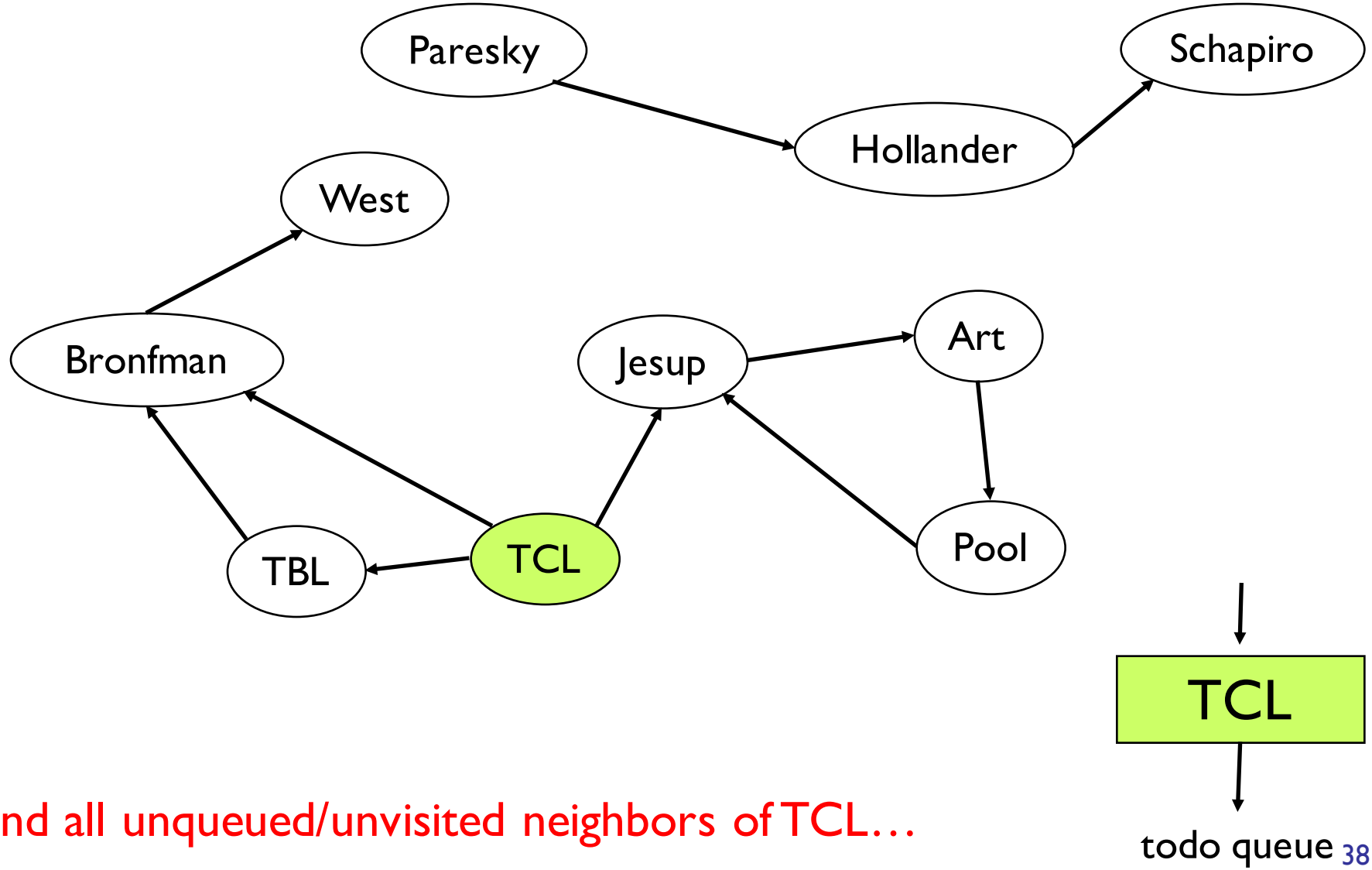
Breadth-First Search

```
void reachableFrom(Graph<V,E> g, V src) {
  Queue<V> todo = new QueueList<V>();
  g.visit(src);
  todo.enqueue(src);
  while (!todo.isEmpty()) {
    V node = todo.dequeue();
    Iterator<V> neighbors = g.neighbors(node);
    while (neighbors.hasNext()) {
       V next = neighbors.next();
       if (!g.visited(next)) {
          g.visit(next);
          todo.enqueue(next);
       }
    }
  }
}
```

Breadth-First Search

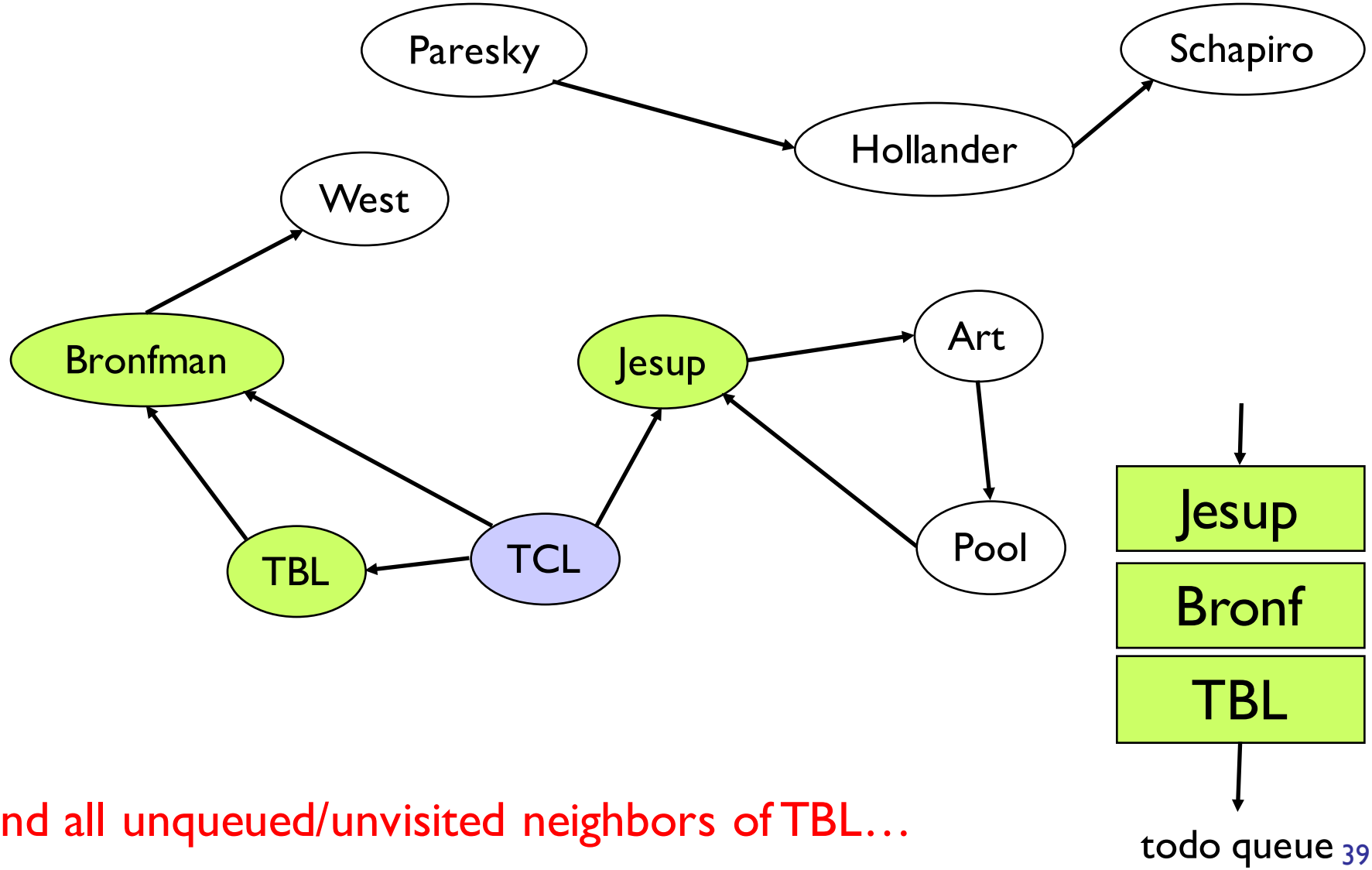


Breadth-First Search



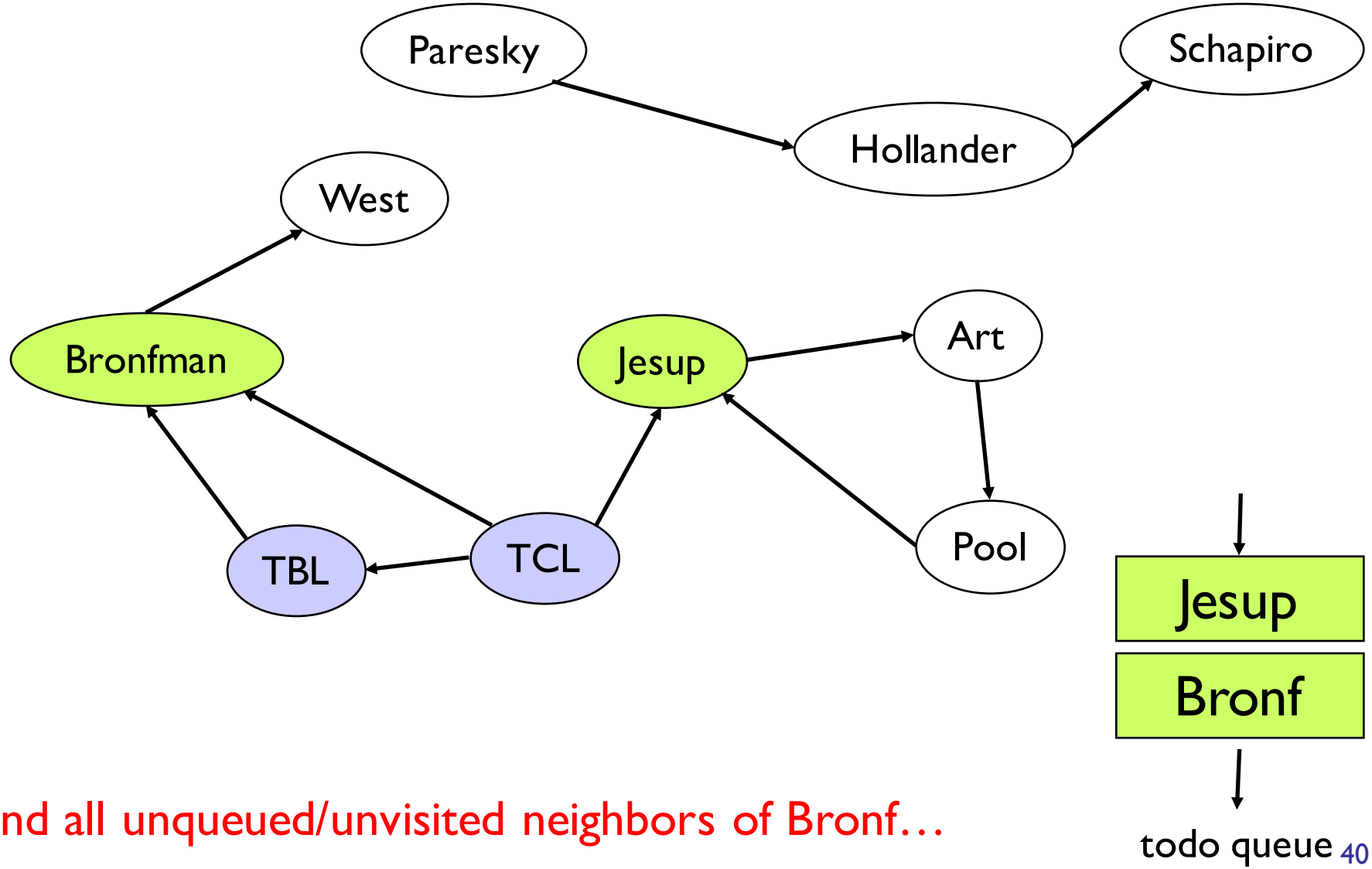
Find all unqueued/unvisited neighbors of TCL…

Breadth-First Search



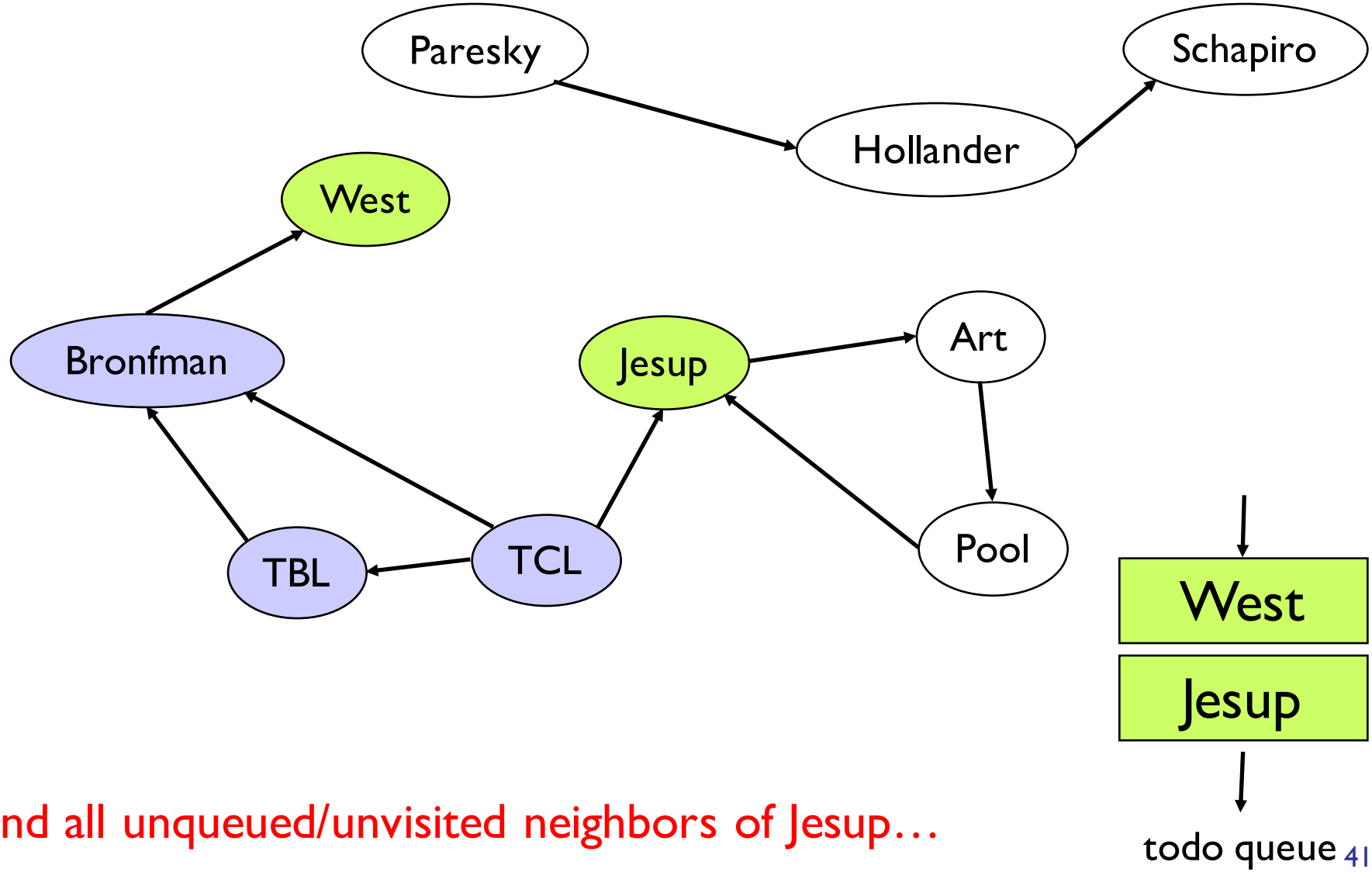
Find all unqueued/unvisited neighbors of TBL…

Breadth-First Search



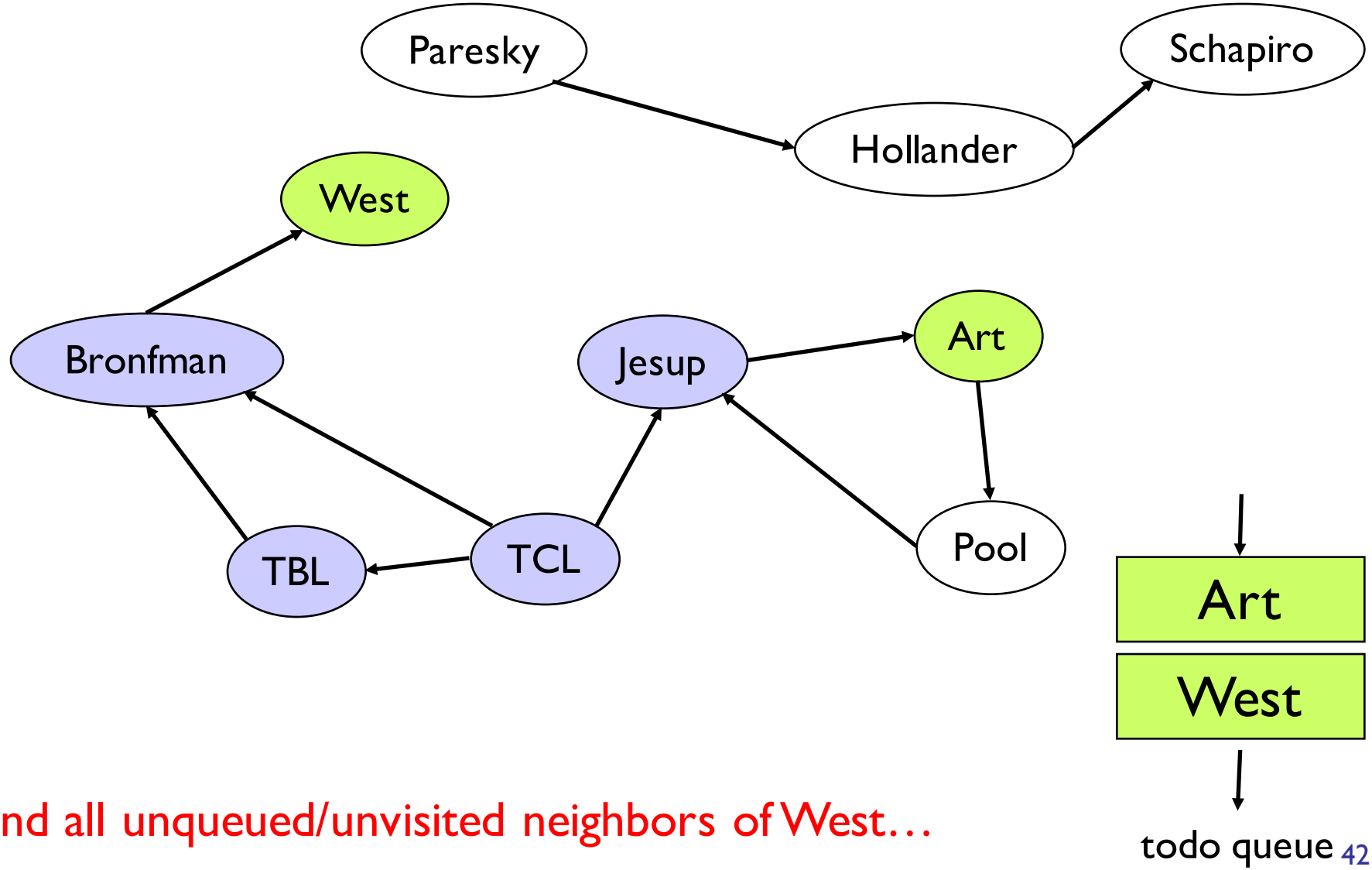
Find all unqueued/unvisited neighbors of Bronf…

Breadth-First Search



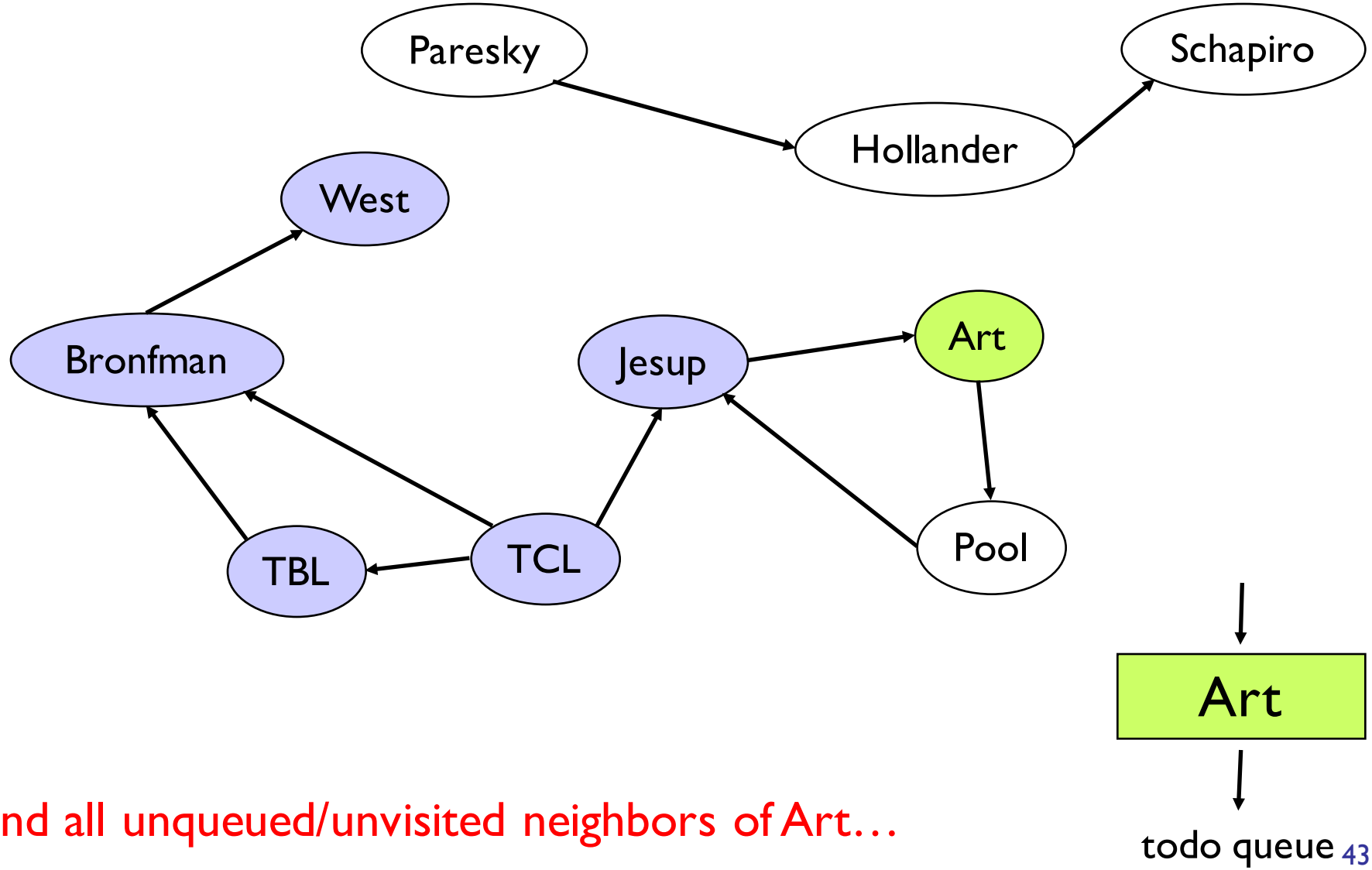
Find all unqueued/unvisited neighbors of Jesup…

Breadth-First Search



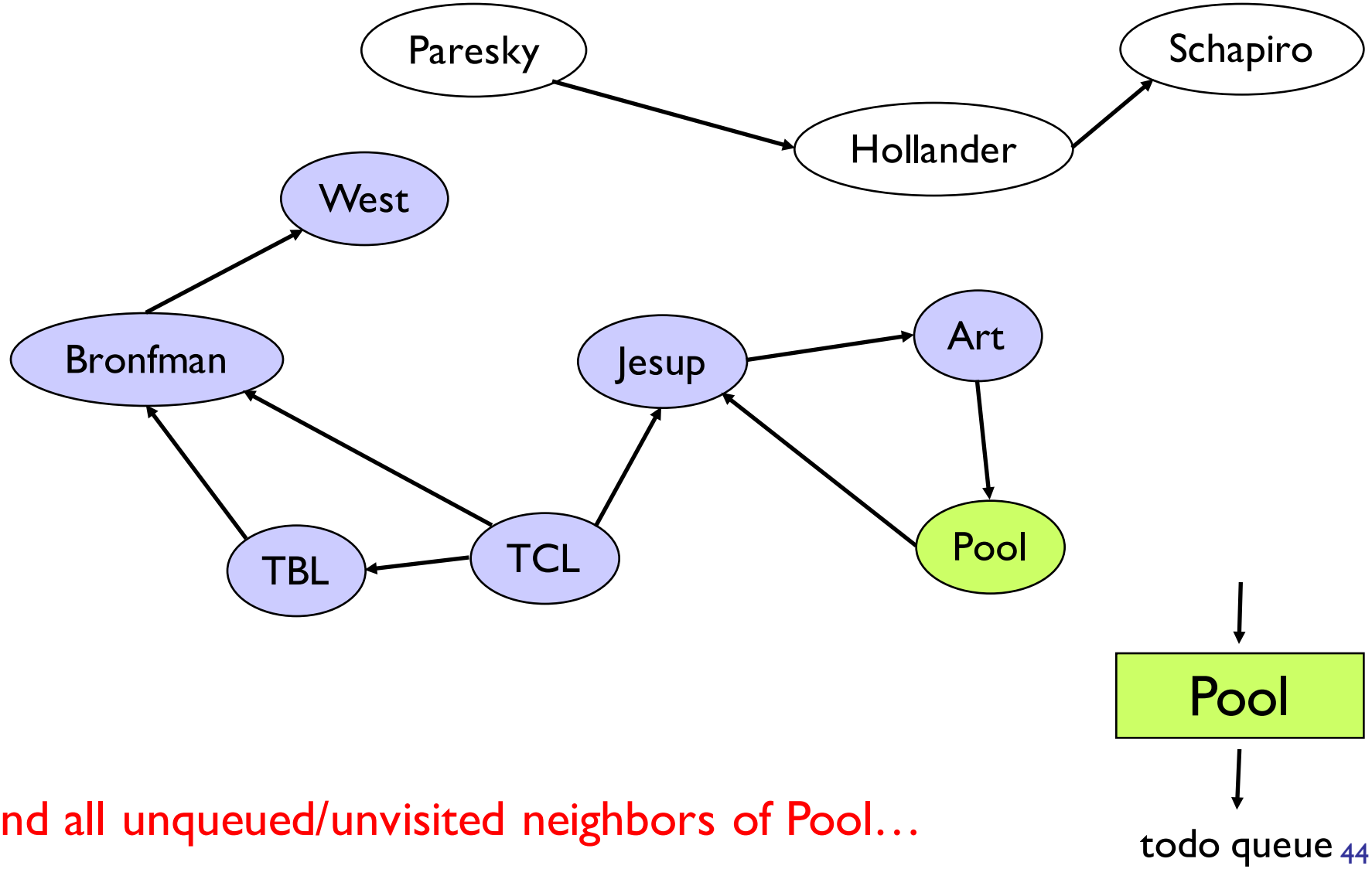
Find all unqueued/unvisited neighbors of West…

Breadth-First Search



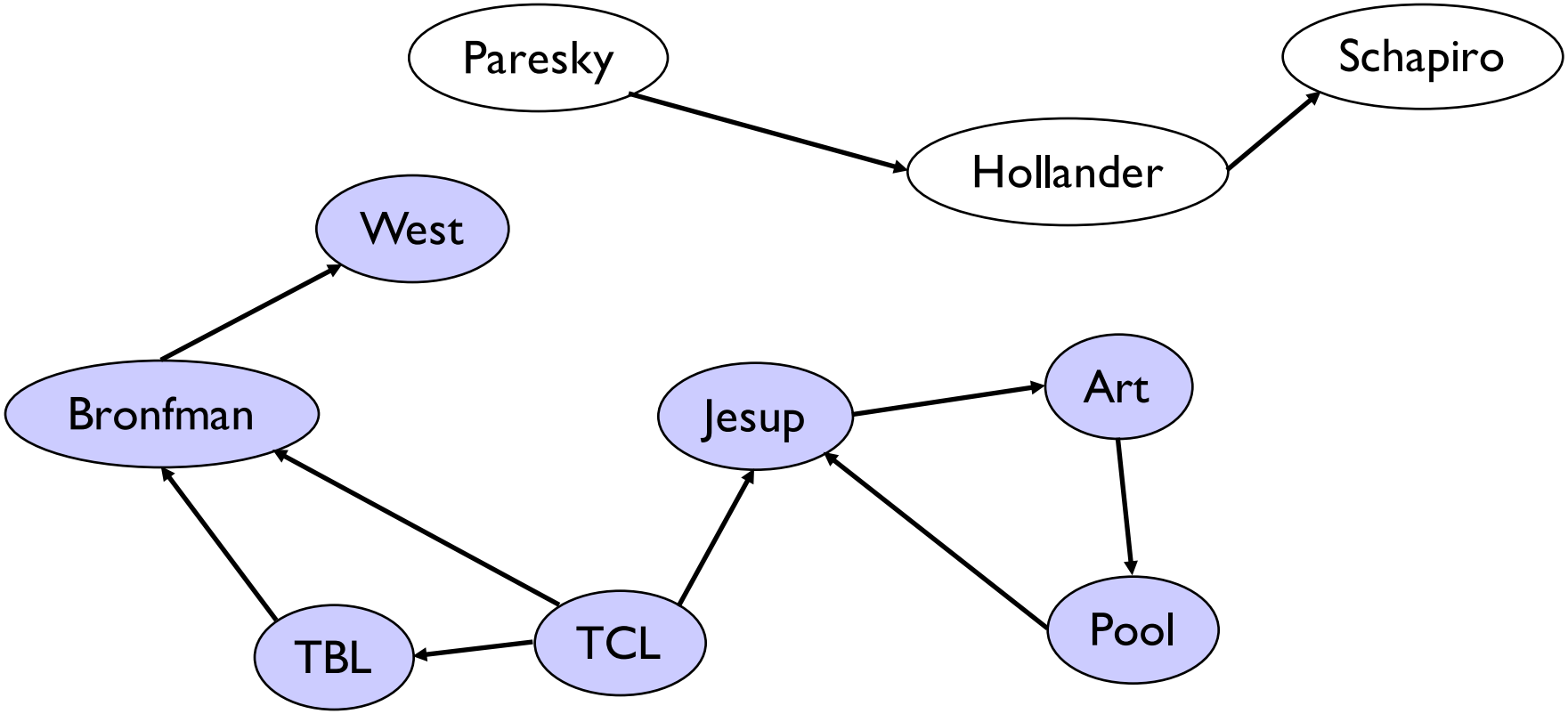
Find all unqueued/unvisited neighbors of Art…

Breadth-First Search



Find all unqueued/unvisited neighbors of Pool…

Breadth-First Search



todo queue