Model 1: Networks and flows

A

B
Model 1: (continued)
Consider graph $A$. Once again we have a directed graph with weighted edges. However, instead of thinking of the weights as some sort of length, we will now think of them as a capacity: the “maximum amount of stuff” that the edge can carry. For example, the capacity might be used to model things like:

- maximum gallons of water per minute that can flow through a pipe;
- maximum number of trucks per hour that can drive along a road; or
- maximum number of times a certain resource can be used before it is all used up.

1. Consider graph $B$. How is it related to graph $A$?

2. What do the blue edges in graph $B$ all have in common?

3. What do you think the labels on the edges of graph $B$ represent?

4. Now consider graph $C$. Why do you think some of the edges are green?

5. Graph $D$ is invalid! In fact, there are two things wrong with it. What are they?
Definition 1. A flow network is a directed graph \( G = (V, E) \) with

- a distinguished source vertex \( s \in V \), with only outgoing edges;
- a distinguished sink or target vertex \( t \in V \), with only incoming edges;
- a capacity function \( c : E \to \mathbb{R}^+ \) assigning a non-negative real number capacity \( c(e) \) to each edge \( e \in E \).

6 Is graph \( A \) a flow network? Why or why not?

Now let’s define a flow. Both graphs \( B \) and \( C \) depict valid flows on \( A \); graph \( D \) does not.

Definition 2. A flow on a flow network \( G \) is a function \( f : E \to \mathbb{R}^+ \) assigning a non-negative flow \( f(e) \) to each edge, such that

1. \( \text{__________} \leq f(e) \leq \text{__________} \) for every \( e \in E \)

2. At each vertex \( v \in V \) other than \( s \) and \( t \), \( \text{_________________________} \)

Definition 3. The value of a flow, \( v(f) \), is the sum of the flow on all edges leaving \( s \).

7 What is the value of the flow on graph \( B \)?

8 What is the value of the flow on graph \( C \)?

9 Make a conjecture about the relationship between the value of a flow and the amount of flow entering \( t \).

10 For each amount, say whether you can construct a flow on graph \( A \) with the given value.

(a) 15
What is the value of the biggest flow you can construct on graph $A$?