CSCI 136
Data Structures & Advanced Programming

Fall 2018

Lecture 33

The 2070567s

Administrative Details

Reminders

- No lab this week
- Final exam
  - Monday, December 17 at 9:30 in TPL 203
  - Covers everything, with strong emphasis on post-midterm
  - Study guide, sample exam will be posted on handouts page

Topics Covered

- Vectors (and arrays)
- Complexity (big O)
- Recursion + Induction
- Searching
- Sorting
- Linked Lists (SLL & DLL)
- Stacks
- Queues
- Iterators
- Bitwise operations
- Comparables/Comparators
- OrderedStructures
- Binary Trees
- Priority Queues
- Heaps
- Binary Search Trees
- Graphs
- Maps/Hashtables

Last Time

- Graph applications (more in Ch 16)
  - Prim's algorithm for MCST
  - Dijkstra's Algorithm for shortest paths
    - Single source

Today's Outline

- Finish Dijkstra's algorithm
- Maps
  - Revisit Naïve implementation from Lab 2
  - structure5.Hashtable (finally)
    - Hash functions
    - "Load factor"
    - Collisions and how to handle them
  - You should also read Ch 15 for more info

Single Source Shortest Paths

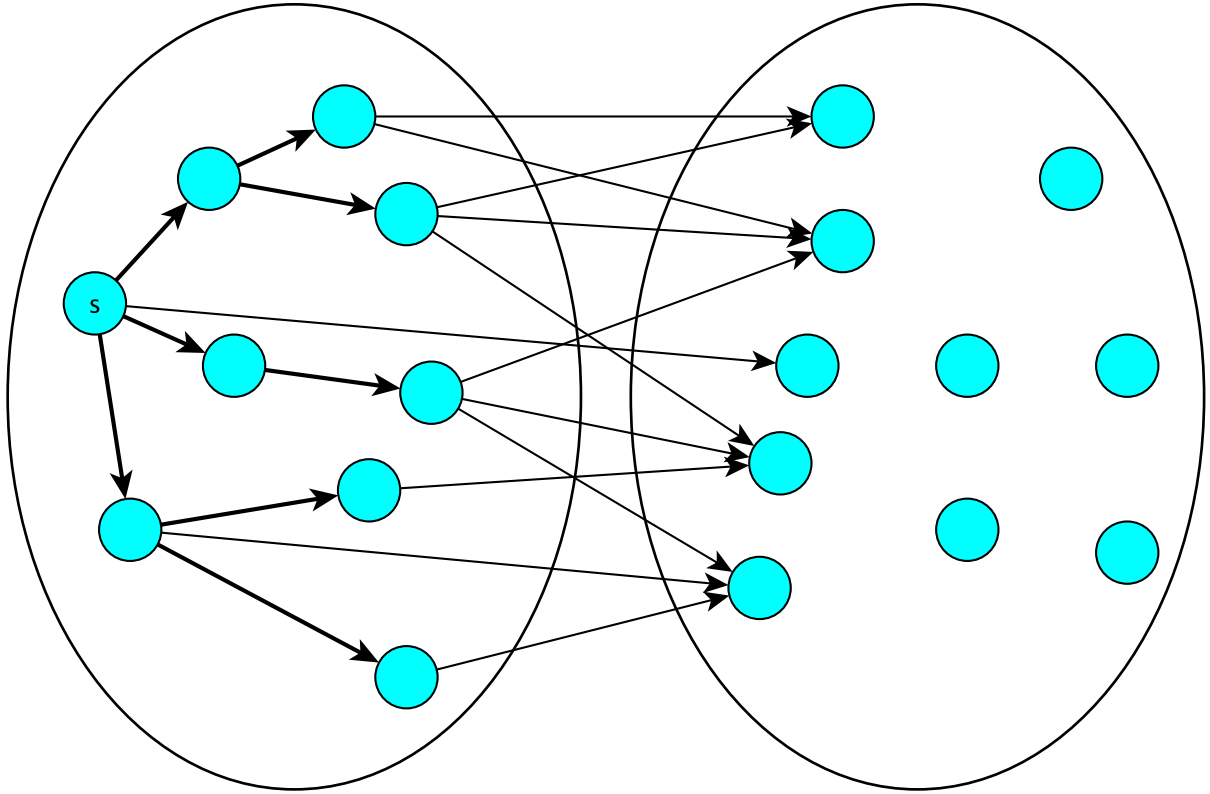
The Input: A graph G such that each edge has a positive cost and a starting vertex v.

The Output: For *each* vertex u ≠ v reachable from v, a shortest path P_u from v to u.

Graph can be directed or undirected

The method: Dijkstra's Algorithm: Grow a tree

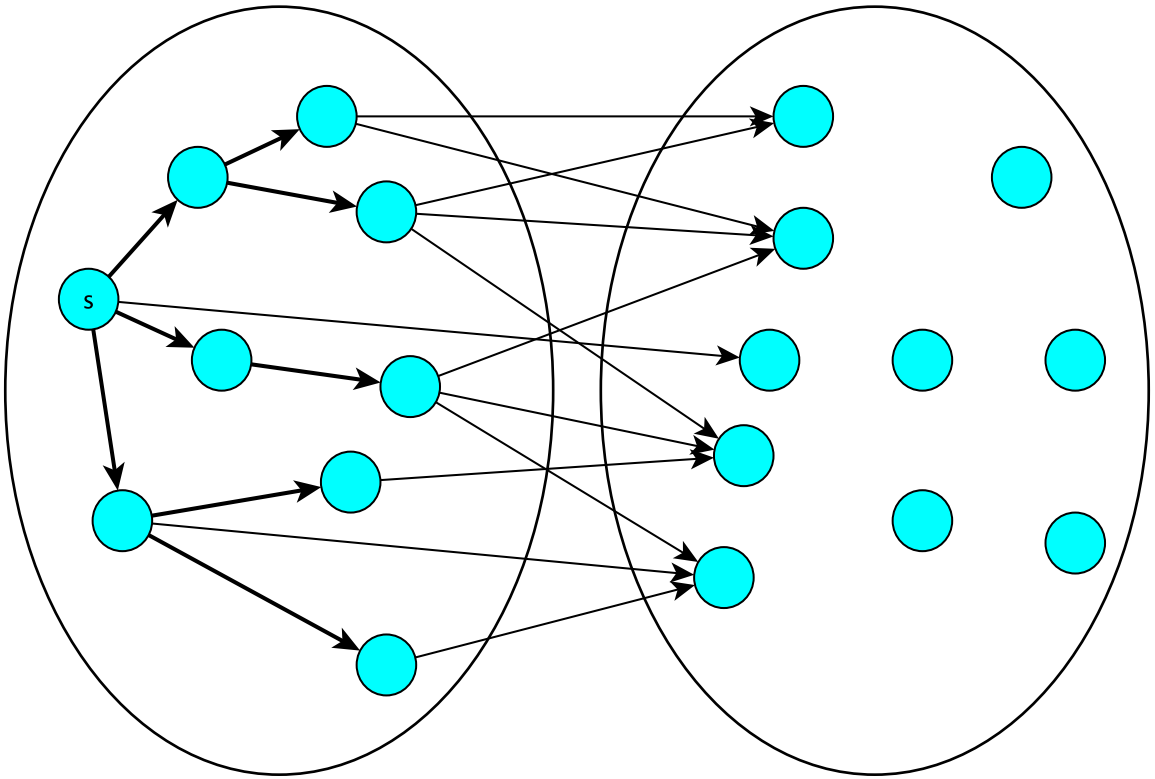
Finding the Best Vertex to Add to T_k



Not all edges are displayed

Question: Can we find the next closest vertex to s?

What's a Good Greedy Choice?



Idea: Pick edge e from u in T_k to v in G-T_k that minimizes the length of the tree path from s up to–and through–e

Now add v and e to T_k to get tree T_{k+1}

Now T_{k+1} is a tree consisting of shortest paths from s to the k vertices closest to s! [Proof?] Repeat!

Some Notation Reminders

- l(e) : length (weight) of edge e
- d(u,v) : *distance* from u to v
    - Length of shortest path from u to v

- The priority queue stores an *estimate* of the distance from s to w by storing, for some edge (v,w), d(s,v) + l(v,w)
    - The estimate is always an *upper bound* on d(s,w)

Dijkstra: What Do We Return?

- As we find a new vertex v to add to the tree T from some u in T, add info to a PQ and a Map.
- Precisely:
  - Use a PQ of association(X,Y) edgeInfo where
    - X is d(s,v) + l(v,w)
    - Y is the edge e=(v,w)
  - Add all edges from v to w, w not in T, to the PQ
  - Add the key/value pair (v, u) to the Map
- So the map entry with key v tells us the vertex u that precedes v on shortest path from s to v

Dijkstra's Algorithm

Dijkstra(G, s) // l(e) is the length of edge e

let T←({s}, ∅) and PQ be an empty priority queue

for each neighbor v of s, add edge (s,v) to PQ with priority l(e)

while T doesn't have all vertices of G and PQ is non-empty

repeat

e ← PQ.removeMin() // skip edges with both ends in T

until PQ is empty or e=(u,v) for u∈T, v ∉ T

if e=(u,v) for u∈T, v ∉ T

add e (and v) to T

for each neighbor w of v

add edge (v,w) to PQ with weight/key d(s,v) + l(v,w)

Dijkstra: Space Complexity

- Graph: O(|V| + |E|)
  - Each vertex and edge uses a constant amount of space
- Priority Queue: O(|E|)
  - Each edge takes up constant amount of space
- Map: O(|V|)
- Result: O(|V| + |E|)
  - Optimal in Big-O sense!

Dijkstra : Time Complexity

Assume Map ops are O(1) time

Across *all* iterations of outer while loop

- Edges are added to and removed from the priority queue
  - But any edge is added (and removed) at most once!
  - Total PQ operation cost is O(|E| log |E|) time
    - Which is O(|E| log |V|) time
  - All other operations take constant time
- Thus time complexity is O(|E| log |V|)

Final Topic: Maps and Hashing

Map Interface

Methods for `Map<K, V>`

- `int size()` - returns number of entries in map
- `boolean isEmpty()` - true iff there are no entries
- `boolean containsKey(K key)` - true iff key exists in map
- `boolean containsValue(V val)` - true iff val exists at least once in map
- `V get(K key)` - get value associated with key
- `V put(K key, V val)` - insert mapping from key to val, returns value replaced (old value) or null
- `V remove(K key)` - remove mapping from key to val
- `void clear()` - remove all entries from map

Map Interface

Other methods for Map<K,V>:

- `void putAll(Map<K,V> other)` - puts all key-value pairs from Map other in map
- `Set<K> keySet()` - return set of keys in map
- `Set<Association<K,V>> entrySet()` - return set of key-value pairs from map
- `Structure<V> valueSet()` - return set of values
- `boolean equals()` - used to compare two maps
- `int hashCode()` - returns hash code associated with values in map (stay tuned…)

Dictionary.java

```
public class Dictionary {

    public static void main(String args[]) {
         Map<String, String> dict = new Hashtable<String, String>();
         …
        dict.put(word, def);
         …
         System.out.println("Def: " + dict.get(word));
     }

}
```

What's missing from the Map API that a dictionary needs?

`successor(key), predecessor(key)`

Maps do NOT preserve order!

Simple Implementation: MapList

- Uses a SinglyLinkedList of Associations as underlying data structure
  - Think back to Lab 2, but a List instead of a Vector
- How would we implement `get(K key)`?
- How would we implement `put(K key, V val)`?

MapList.java

```
public class MapList<K, V> implements Map<K, V>{

    //instance variable to store all key-value pairs
    SinglyLinkedList<Association<K,V>> data;

    public V put (K key, V value) {
       Association<K,V> temp =
                              new Association<K, V> (key, value);
       // Association equals() just compares keys
       Association<K,V> result = data.remove(temp);

       data.addFirst(temp);
       if (result == null)
              return null;
       else
              return result.getValue();
    }
}
```

Simple Map Implementation

- What is MapList's running time for:
  - containsKey(K key)?
  - containsValue(V val)?

- Bottom line: not O(1)!

Search/Locate Revisited

- How long does it take to search for objects in Vectors and Lists?
  - O(n) on average
- How about in BSTs?
  - O(log n)
- Can this be improved?
  - Hash tables can locate objects in *really quickly*!
    - (we will cover two reasons that O(1) performance is a fuzzy claim)

Example from Bailey

"We head to a local appliance store to pick up a new freezer. When we arrive, the clerk asks us for the last two digits of our home telephone number! Only then does the clerk ask for our last name. Armed with that information, the clerk walks directly to a bin in a warehouse of hundreds of appliances and comes back with the freezer in tow."

- Thoughts?
  - What is Key? What is Value?
  - Are names evenly distributed?
  - Are the last 2 phone digits evenly distributed?

Hashing in a Nutshell

- Assign objects to "bins" based on key
- When searching for object, go directly to appropriate bin (and ignore the rest)
- If there are multiple objects in bin, then search for the correct one
- Important Insight: Hashing works best when objects are evenly distributed among bins
  - Phone numbers are randomly assigned, last names are not (there were a lot of Smiths in Smithsville!)

Implementing a HashTable

- How can we represent bins?
- Slots in array (or Vector, but arrays are faster)
  - Initial size of array is a prime number
- How do we find a key's bin number?
  - We use a *hash function* that converts keys into integers
  - In Java, all Objects have `public int hashCode()`
    - Hashing function is one way: key fingerprint
    - Hashing function is deterministic

hashCode() rules

The general contract of `hashCode` is:

- Whenever it is invoked on the same object more than once during an execution of a Java application, the `hashCode` method must consistently return the same integer, provided no information used in `equals` comparisons on the object is modified. This integer need not remain consistent from one execution of an application to another execution of the same application.
- If two objects are equal according to the `equals(Object)` method, then calling the `hashCode` method on each of the two objects must produce the same integer result.
- It is *not* required that if two objects are unequal according to the `equals(java.lang.Object)` method, then calling the `hashCode` method on each of the two objects must produce distinct integer results. However, the programmer should be aware that producing distinct integer results for unequal objects may improve the performance of hash tables.

https://docs.oracle.com/javase/7/docs/api/java/lang/Object.html#hashCode()

Implementing HashTable

- How do we add Associations to the array?
  - `array[o.hashCode() % array.length] = o;` ?
    - What's "aaaaaa".hashCode() ?
- Collisions make life hard
- Two approaches
  - Open Addressing
    - Linear or Quadratic Probing
    - Double Hashing
  - External chaining