CSCI 136
Data Structures & Advanced Programming

Lecture 30

Fall 2018

Instructors: Bills are Back

Last Time

- Graph Data Structures: Implementation
    - Adjacency Array Implementation Details
        - GraphMatrix Abstract Base Class

Today’ s Outline

- GraphMatrixDirected Implementation
- Greedy Algorithms for Optimization
- Lab 10 : Exam Scheduling
  - Defining the problem
  - Sketching a design
- Adjacency List Implementation Details
- More Fundamental Graph Properties
- An Important Algorithm: Minimum-cost spanning subgraph

GraphMatrixDirected

- Completes the implementation of GraphMatrix to ensure graph is directed
- GraphMatrixUndirected is very similar…
- How do we implement GraphMatrixDirected?
  - We'll discuss some methods
  - Read Ch 16 for complete details…

GraphMatrixDirected

- Constructor

```
public GraphMatrixDirected(int size) {
   // pre: size > 0
   // post: constructs an empty graph that may be
   //       expanded to at most size vertices. Graph
   //       is directed if dir true and undirected
   //       otherwise

   // call GraphMatrix constructor
   super(size,true);
}
```

GraphMatrixDirected

- addEdge

```
// pre: vLabel1 and vLabel2 are labels of existing vertices
public void addEdge(V vLabel1, V vLabel2, E label) {
   GraphMatrixVertex<V> vtx1,vtx2;
   vtx1 = dict.get(vLabel1);
   vtx2 = dict.get(vLabel2);
   Edge<V,E> e = new Edge<V,E>(vtx1.label(), vtx2.label(),
                               label, true);
   data[vtx1.index()][vtx2.index()] = e;
}
```

GraphMatrixDirected

- removeEdge

```
// pre: vLabel1 and vLabel2 are labels of existing vertices
public E removeEdge(V vLabel1, Vlabel2) {
    // get indices
   int row = dict.get(vLabel1).index();
   int col = dict.get(vLabel2).index();
   // cache old value
   Edge<V,E> e = (Edge<V,E>)data[row][col];
   // update matrix
   data[row][col] = null;
   if (e == null) return null;
   else return e.label(); // return old value
}
```

GraphMatrix Efficiency

- Assume Map operations are O(1) (for now)
  - |E| = number of edges
  - |V| = number of vertices
- Runtime of add, addEdge, getEdge, removeEdge, remove?
- Space usage?
- Conclusions
  - Matrix is good for dense graphs
  - Have to commit to maximum # of vertices in advance

Efficiency : Assuming Fast Map

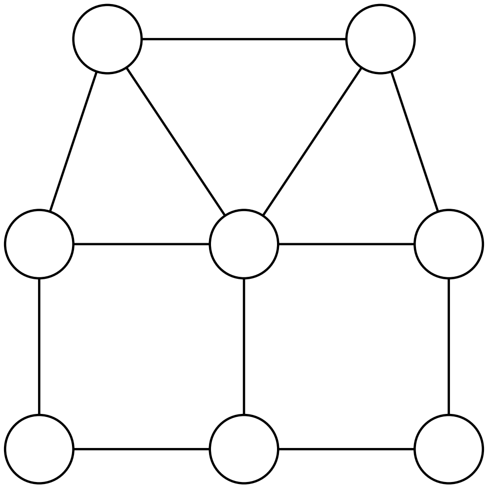
	GraphMatrix
add	O(1)
addEdge	O(1)
getEdge	O(1)
removeEdge	O(1)
remove	O(|V|)
space	$O(\|V\|^2)$

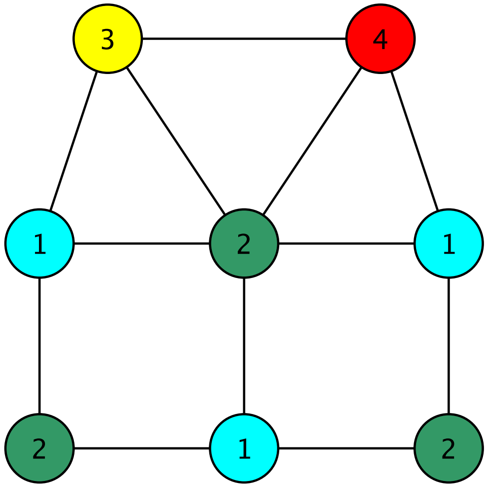
Lab 10 Overview:
Graph Algorithms using structure5

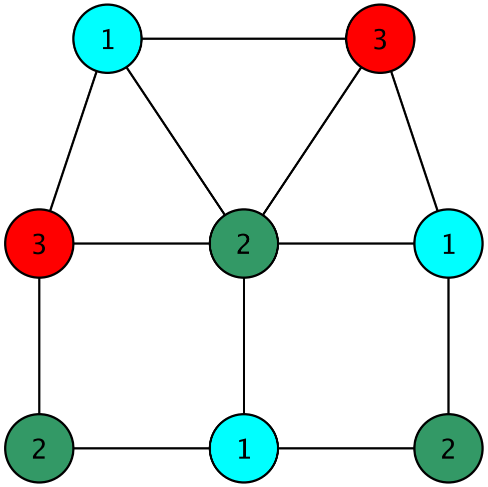
Greedy Algorithms

- A *greedy algorithm* attempts to find a globally optimum solution to a problem by making locally optimum (greedy) choices
- Example: Graph Coloring
  - A *(proper) coloring* of a graph G = (V,E) is an assignment of a value (color) to each vertex so that adjacent vertices get different values (colors)
  - Typically one strives to minimize the number of colors used

Greedy Coloring







Greedy Coloring : Math

Here's a greedy coloring algorithm

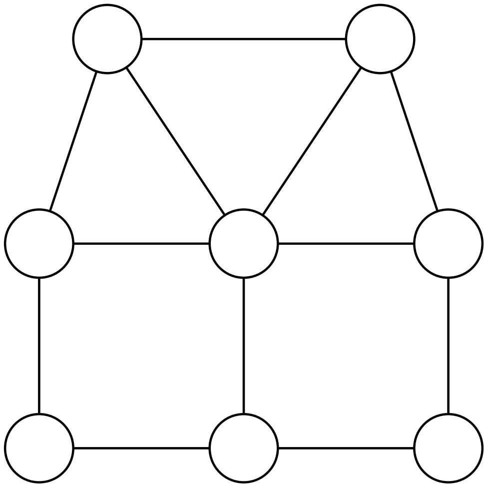
```
Build a collection $C = \{C_1, \ldots, C_k\}$ of sets of vertices
$i = 0$; $C_i = \{\}$ // empty set
while G is has more vertices
        for each vertex u in G
                if u is not adjacent to any vertex of $C_i$
                        remove u from G and add u to $C_i$
        add $C_i$ to C
        i++;
Return C as the coloring
```

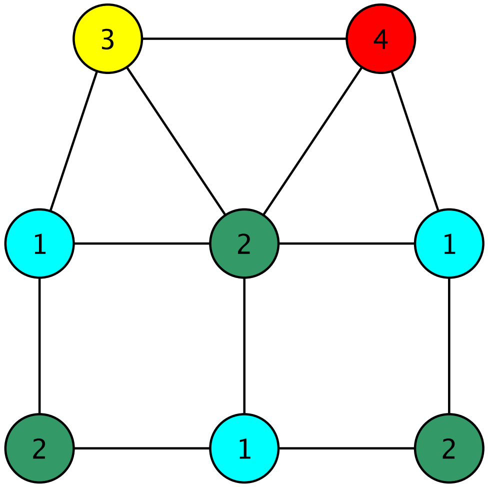
Greedy Coloring : CS

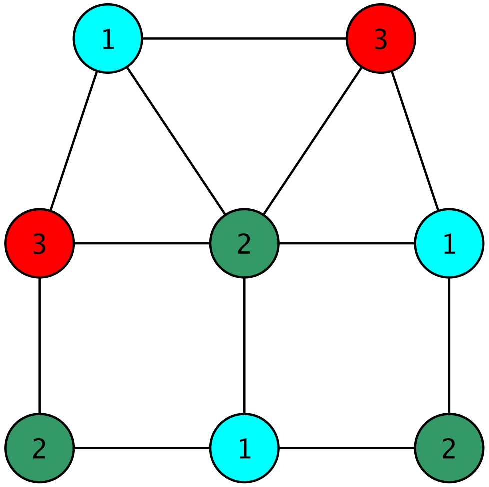
Here's a greedy coloring algorithm

```
Create a structure C to hold a collection of lists
while G is not empty
    pick a vertex v in G; create an empty list L; add v to L
    for each vertex $u \neq v$ in G
        if u is not adjacent to any vertex of L
            add u to L
    remove all vertices of L from G
    add L to C
Return C as the coloring
```

Greedy Coloring







Greedy Coloring

Some observations

- Each list (color class) L is a set of vertices no two of which are adjacent (an *independent set*)
- Each color class is maximal: cannot be made any larger
  - The hope is that this results in fewer colors being needed
  - But the solution is not always optimum!
  - This is a *very hard problem*
- The coloring problem is the same as finding a *partition* of the vertex set into independent sets
  - Partition means union of disjoint sets

Lab 10 : Exam Scheduling

Find a schedule (set of time slots) for exams so that

- No student has two exams in the same slot
- Every course is in a slot
- The number of slots is as small as possible

This is just the graph coloring problem in disguise!

- Each course is a vertex
- Two vertices are adjacent if the courses share students
- A slot must be an independent set of vertices (that is, a color class)

Lab 10 Notes: Using Graphs

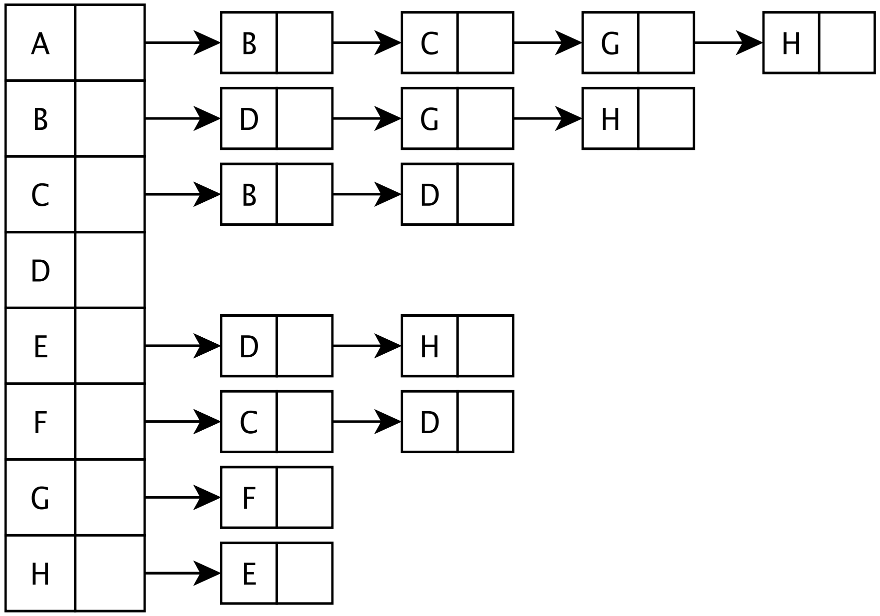
- Create a new graph in structure5
    - GraphListDirected, GraphListUndirected,
    - GraphMatrixDirected, GraphMatrixUndirected

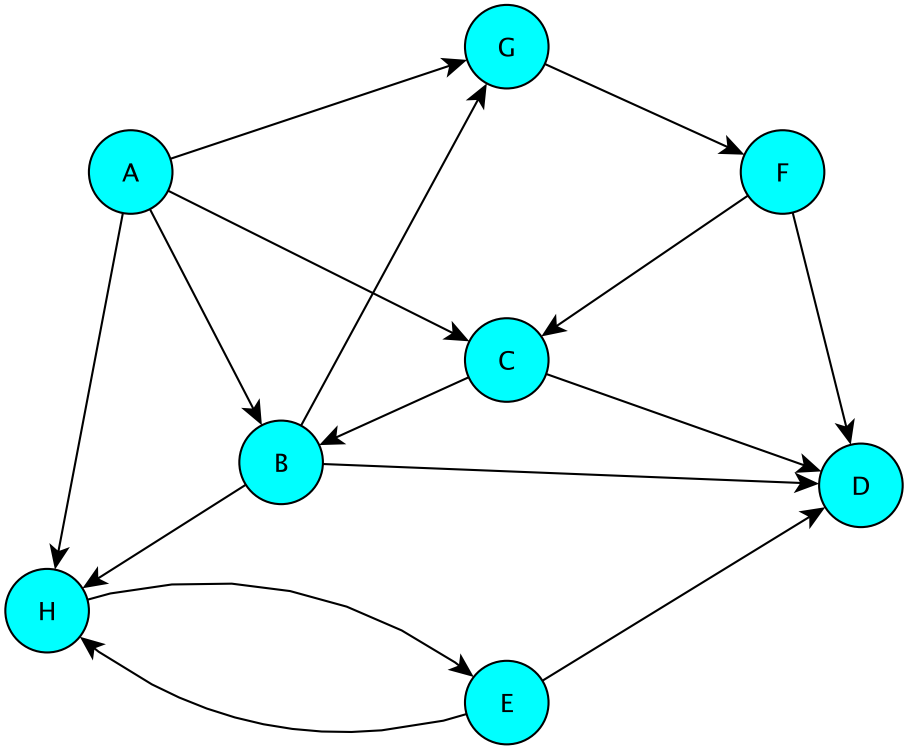
- Graph<V,E> conflictGraph = new GraphListUndirected<V,E>();

Lab 10 : Useful Graph Methods

- `void add(V label)`
  - add vertex to graph
- `void addEdge(V vtx1, V vtx2, E label)`
  - add edge between vtx1 and vtx2
- `Iterator<V> neighbors(V vtx1)`
  - Get iterator for all neighbors to vtx1
- `boolean isEmpty()`
  - Returns true iff graph is empty
- `Iterator<V> iterator()`
  - Get vertex iterator
- `V remove(V label)`
  - Remove a vertex from the graph
- `E removeEdge(V vLabel1, V vLabel2)`
  - Remove an edge from graph

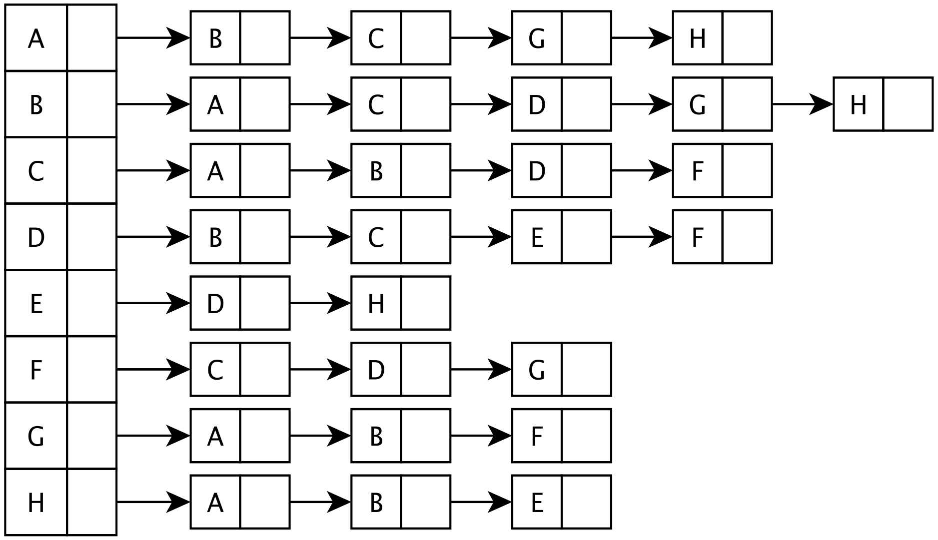
Adjacency List : Directed Graph



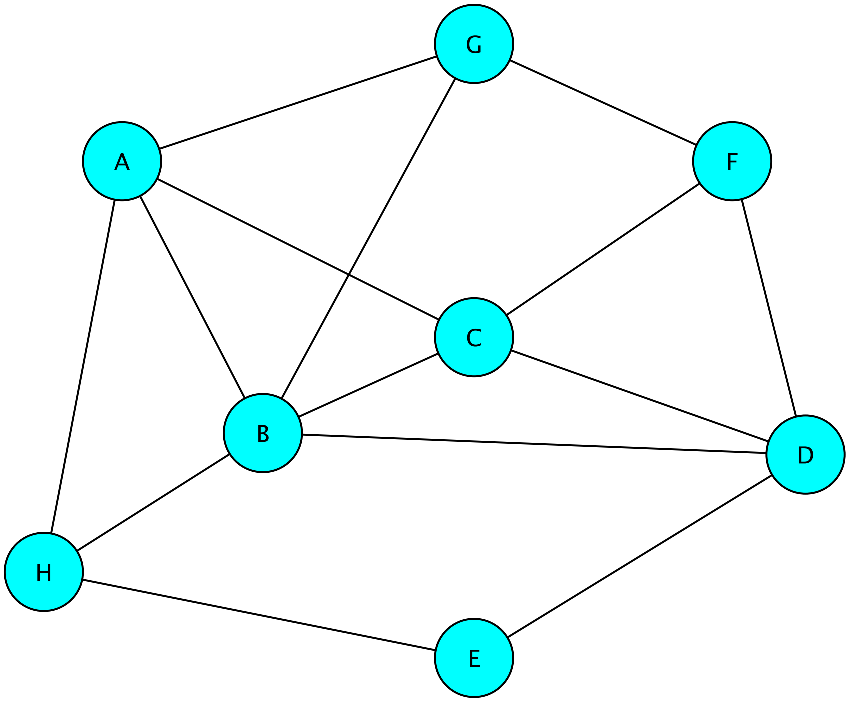


The vertices are stored in an array V[]
V[] contains a linked list of edges having a given source

Adjacency List : Undirected Graph



A → B → C → G → H
B → A → C → D → G → H
C → A → B → D → F
D → B → C → E → F
E → D → H
F → C → D → G
G → A → B → F
H → A → B → E



The vertices are stored in an array V[]
V[] contains a linked list of edges incident to a given vertex

GraphList

- Maintain an *adjacency list of* edges at each vertex (no adjacency matrix)
  - Keep only outgoing edges for directed graphs
- Support both directed and undirected graphs (GraphListDirected, GraphListUndirected)

Vertex and GraphListVertex

- We use the same Edge class for all graph types
- We extend Vertex to include an Edge list
- GraphListVertex class adds to Vertex class
  - A Structure to store edges adjacent to the vertex

    protected Structure<Edge<V,E>> adjacencies; // adjacent edges

    – adjacencies is created as a SinglyLinkedList of edges
  - Several methods

```
public void addEdge(Edge<V,E> e)
public boolean containsEdge(Edge<V,E> e)
public Edge<V,E> removeEdge(Edge<V,E> e)
public Edge<V,E> getEdge(Edge<V,E> e)
public int degree()
// and methods to produce Iterators...
```

GraphListVertex

```
public GraphListVertex(V key){
        super(key); // init Vertex fields
        adjacencies = new SinglyLinkedList<Edge<V,E>>();
}

public void addEdge(Edge<V,E> e){
        if (!containsEdge(e)) adjacencies.add(e);
}

public boolean containsEdge(Edge<V,E> e){
        return adjacencies.contains(e);
}

public Edge<V,E> removeEdge(Edge<V,E> e) {
        return adjacencies.remove(e);
}
```

GraphListVertex Iterators

```
// Iterator for incident edges
public Iterator<Edge<V,E>> adjacentEdges() {
        return adjacencies.iterator();
}

// Iterator for adjacent vertices
public Iterator<V> adjacentVertices() {
        return new GraphListAIterator<V,E>
            (adjacentEdges(), label());
}
```

GraphListAIterator creates an Iterator over *vertices* based on the Iterator over *edges* produced by `adjacentEdges()`

GraphListAIterator

GraphListAIterator uses two instance variables

```
    protected AbstractIterator<Edge<V,E>> edges;
    protected V vertex;

public GraphListAIterator(Iterator<Edge<V,E>> i, V v) {
        edges = (AbstractIterator<Edge<V,E>>)i;
        vertex = v;
}

public V next() {
        Edge<V,E> e = edges.next();
        if (vertex.equals(e.here()))
            return e.there();
        else { // could be an undirected edge!
            return e.here();
}
```

GraphListEIterator

GraphListEIterator uses one instance variable

```
protected AbstractIterator<Edge<V,E>> edges;
```

GraphListEIterator

- Takes the Map storing the vertices
- Uses it to build a linked list of all edges
- Gets an iterator for this linked list and stores it, using it in its own methods