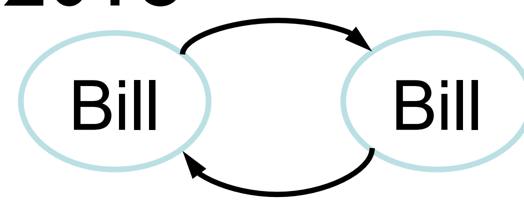
CSCI 136
Data Structures & Advanced Programming

Lecture 28

Fall 2018

Instructors: 

Announcements

- I have office hours today from 1:00-2:00pm

Last Time

- More on Graphs
  - Applications and Problems
    - Testing connectedness
    - Counting connected components
    - Breadth-first search
    - Depth-first search
      - And recursive depth-first search

Today's Outline

- Recursive Depth First Search
  - Why it works
- Directed Graphs
  - Definition and Properties
  - Reachability and (Strong) Connectedness
- Graph Data Structures: Implementation
  - Graph Interface
  - Adjacency Array Implementation Basic Concepts
  - Adjacency List Implementation Basic Concepts
  - Adjacency Array Implementation Details

Recursive Depth-First Search

// Before first call to DFS, set all vertices to unvisited
//Then call DFS(G,v)

```
DFS(G, v)
    Mark v as visited; count = 1;
    for each unvisited neighbor u of v:
        count += DFS(G,u);
    return count;
```

Is it even clear that this method does what we want?!

Let's prove some facts about it....

Recursive Depth-First Search

Claim: DFS visits all vertices w reachable from v

- Proof: Induction on length d of shortest path from v to w
  - Base case: d = 0: Then v = w ✓
  - Ind. Hyp.: Assume DFS visits all vertices w of distance at most d from v (for some d ≥ 0).
  - Ind. Step: Suppose now that w is distance d+1 from v. Consider a path of length d+1 from v to w and let u be the next-to-last vertex on the path

Recursive Depth-First Search

Claim: DFS visits all vertices w reachable from v

- Proof: Induction on length d of shortest path from v to w
  - The path is $v = v_0, v_1, v_2, \ldots, v_d = u, v_{d+1} = w$
    - The edges are implied so not explicitly written!
  - By Ind. Hyp., u is visited. At this point, if w has not yet been visited, it will be one of the unvisited vertices on which DFS() is recursively called, so it will then be visited.

Recursive Depth-First Search

Claim: DFS visits *only* vertices reachable from v

- Idea: Prove the following by induction on number of times DFS is called:
  - DFS is only called on vertices w reachable from v

Claim: DFS counts correctly the number of vertices reachable from v

- Idea: Induction on number of unvisited vertices reachable from v
  - DFS will never be called on same vertex twice

Recursive Depth-First Search

Claim: DFS(G,v) returns the number of unvisited nodes reachable from v

Proof: Uses previous two observations

- DFS visits every node reachable from v
- DFS doesn't visit any node *not* reachable from v

What *Exactly* Does DFS Do?

- Given a graph $G = (V, E)$, a vertex v, let $X \subseteq V$, where $v \notin X$.
- Assume X are exactly the vertices of V that have been marked as visited
- Claim: DFS(G,v) will visit exactly those vertices that are in the connected component of $G - X$ that contains v
  - $G - X$ is the graph obtained by deleting the vertices of X–and edges using X–from G
  - Prove by induction on $|V - X|$

Implementing Breadth-First Search

```
BFS(G, v)        // Do a breadth-first search of G starting at v
// pre: all vertices are marked as unvisited
// post: return number of visited vertices
count ← 0;
Create empty queue Q; enqueue v; mark v as visited; count++
While Q isn't empty
        current ← Q.dequeue();
        for each unvisited neighbor u  of current :
                add u to Q; mark u as visited; count++
return count;
```

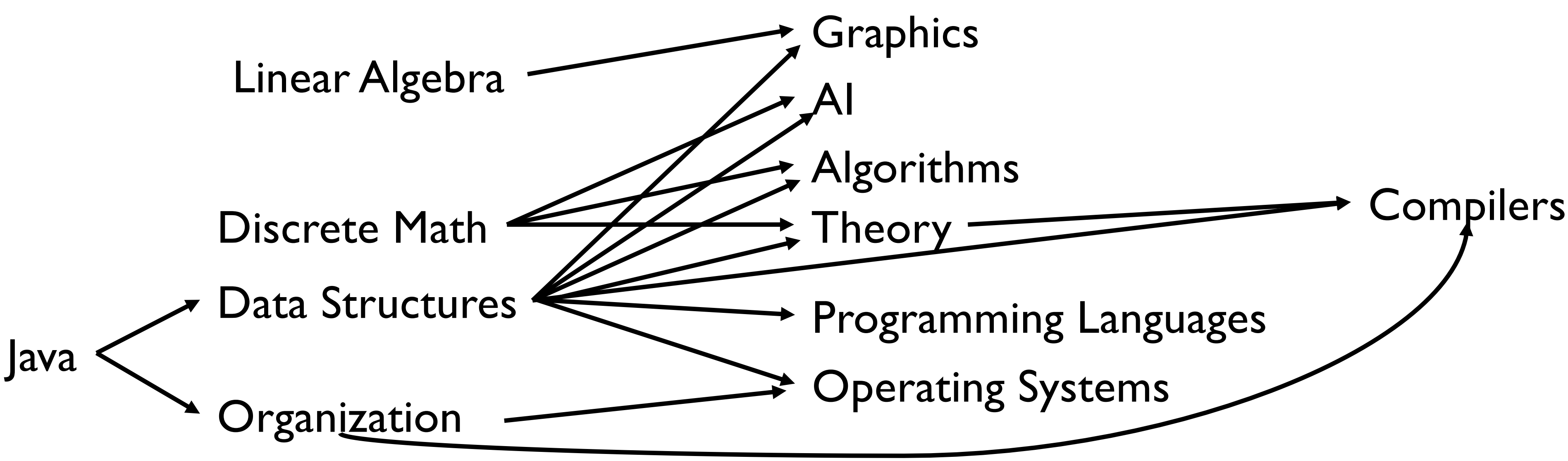
Breadth-First Search

```
int BFS(Graph<V,E> g, V src) {
  Queue<V> todo = new QueueList<V>(); int count = 0;
  g.visit(src); count++;
  todo.enqueue(src);
  while (!todo.isEmpty()) {
    V node = todo.dequeue();
    Iterator<V> neighbors = g.neighbors(node);
    while (neighbors.hasNext()) {
       V next = neighbors.next();
       if (!g.isVisited(next)) {
          g.visit(next); count++;
          todo.enqueue(next);
       }
    }
  }
  return count;
}
```

Breadth-First Search of Edges

```
int BFS(Graph<V,E> g, V src) {
  Queue<V> todo = new QueueList<V>(); int count = 0;
  g.visit(src); count++;
  todo.enqueue(src);
  while (!todo.isEmpty()) {
    V node = todo.dequeue();
    Iterator<V> neighbors = g.neighbors(node);
    while (neighbors.hasNext()) {
       V next = neighbors.next();
       if (!g.isVisitedEdge(node,next)) g.visitEdge(next,node);
       if (!g.isVisited(next)) {
          g.visit(next); count++;
          todo.enqueue(next);
       }
    }
  }
  return count;
}
```

Directed Graphs

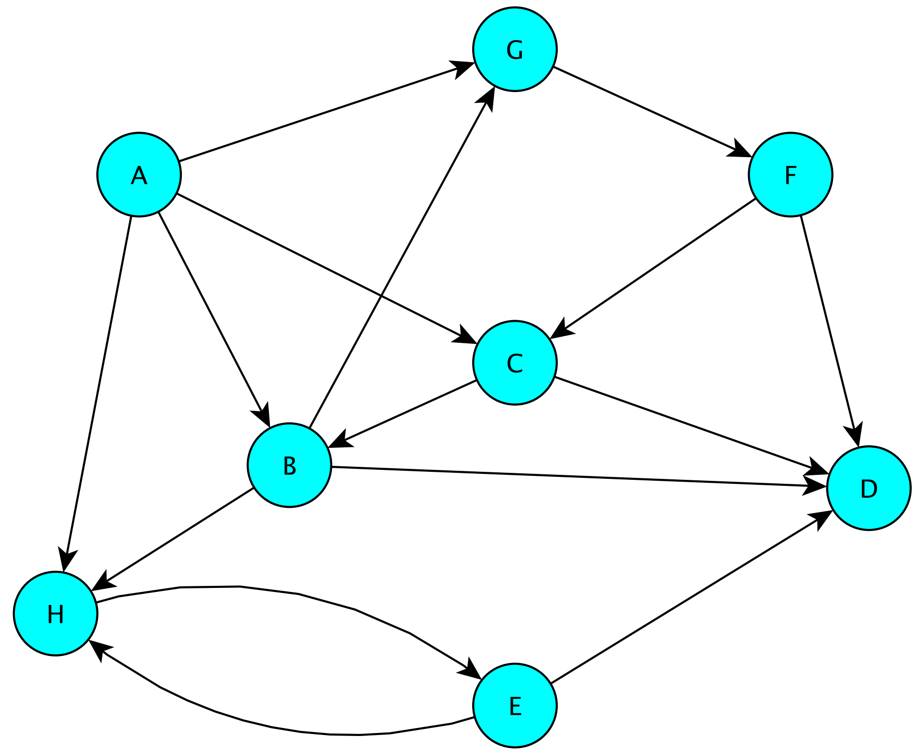


Def'n: In a *directed graph G = (V,E)*, each edge e in E is an *ordered* pair: e = (u,v) vertices: its *incident vertices*. The *source* of e is u; the *destination/target* is v.

Note: (u,v) ≠ (v,u)

Directed Graphs

- The (out) neighbors of B are D, G, H: B has out-degree 3
- The in neighbors of B are A, C: B has in-degree 2
- A has in-degree 0: it is a *source* in G; D has out-degree 0: it is a *sink* in G



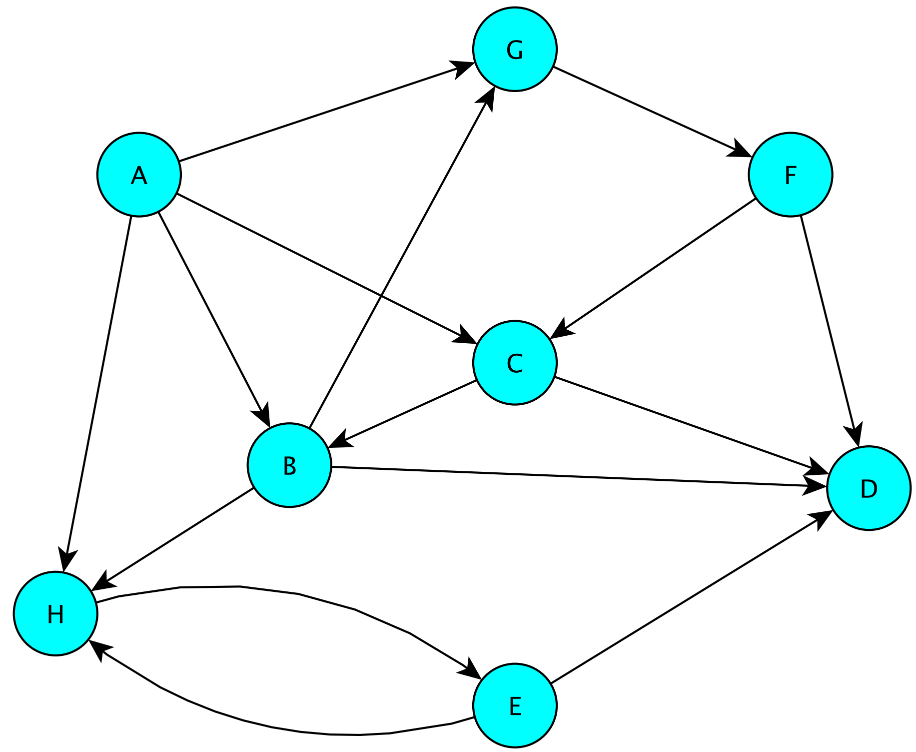
A walk is still an alternating sequence of vertices and edges

$$u = v_0, e_1, v_1, e_2, v_2, \ldots, v_{k-1}, e_k, v_k = v$$

but now $e_i = (v_{i-1}, v_i)$: all edges *point along direction* of walk

Directed Graphs

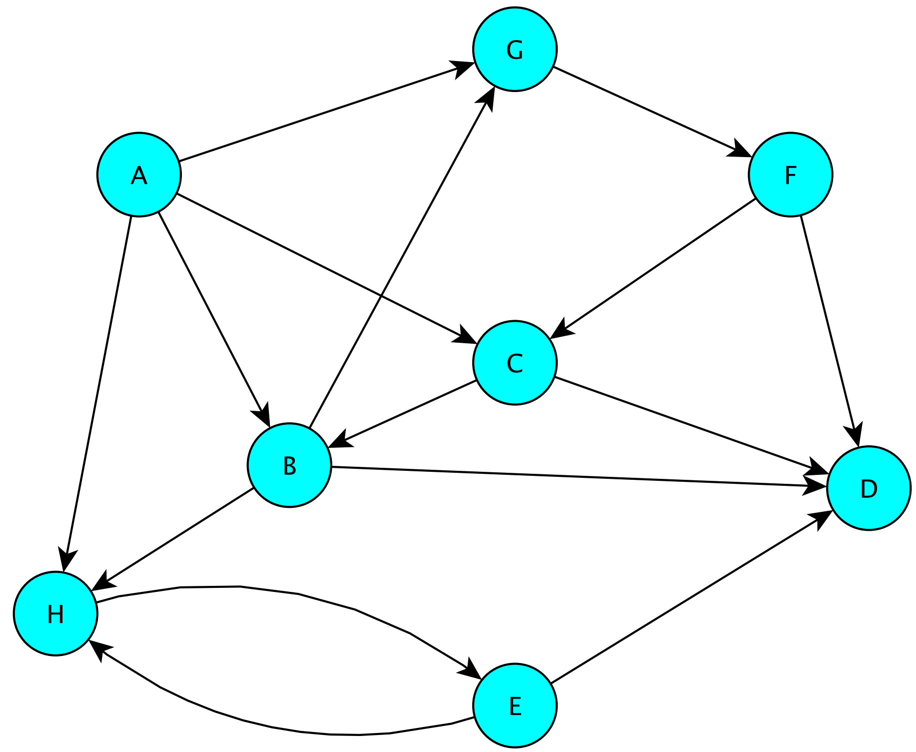
- A, B, H, E, D is a walk from A to D
- It's also a (simple) path
- D, E, H, B, A is *not* a walk from D to A
- B, G, F, C, B is a (directed) cycle (it's a 4-cycle)
- So is H, E, H (a 2-cycle)



- D is reachable from A (via path A, B, D), but A is not reachable from D
- In fact, every vertex is reachable from A

Directed Graphs

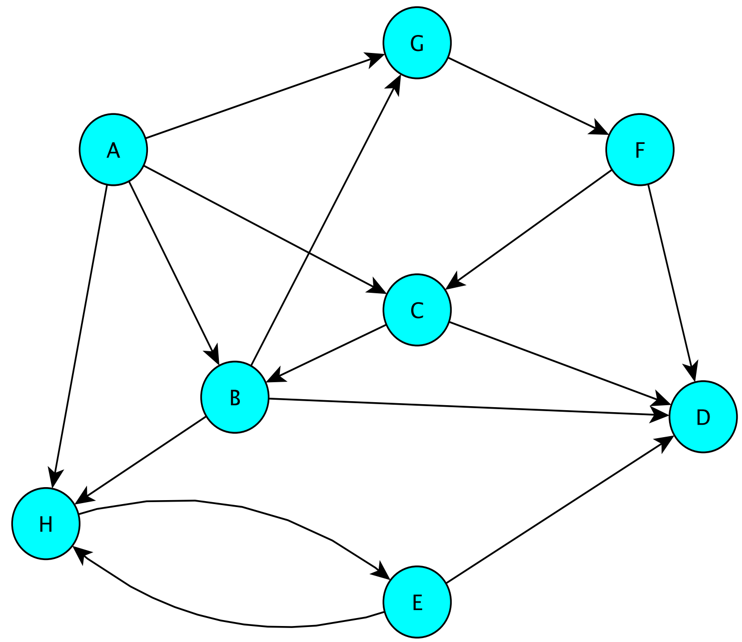
- A BFS of *G* from A visits every vertex
- A BFS of *G* from F visits all vertices but A
- A BFS of *G* from E visits only E, H, D

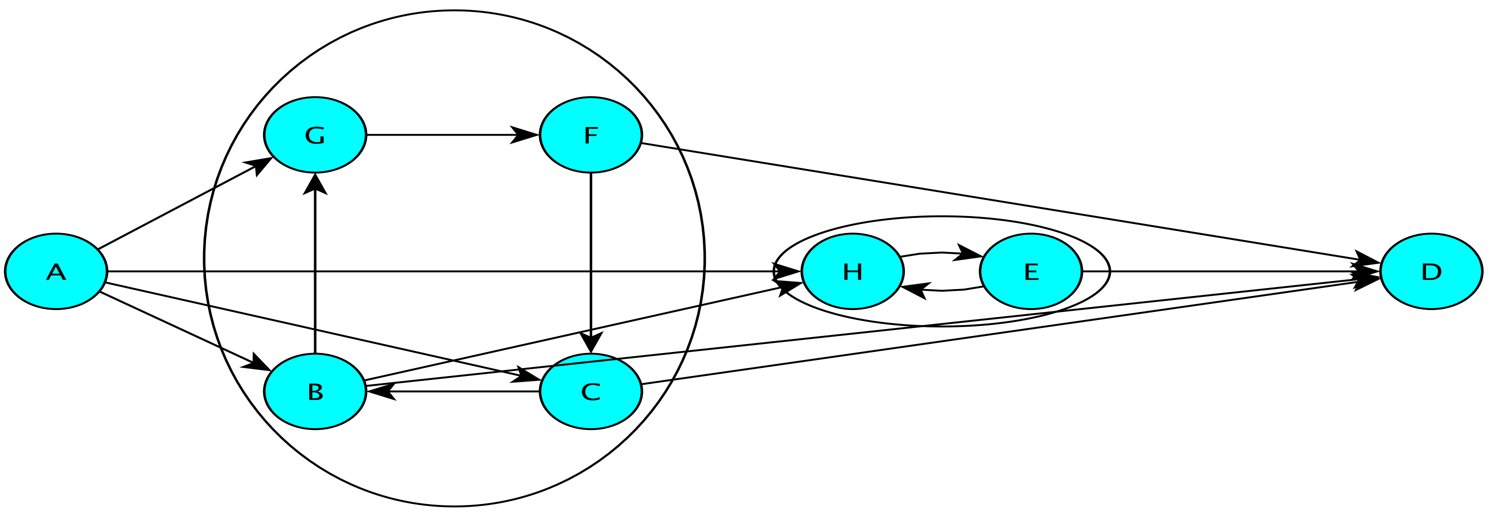


- Connectivity in directed graphs is more subtle than in undirected graphs!

Directed Graphs

- Vertices u and v are *mutually reachable* vertices if there are paths from u to v and v to u
- *Maximal* sets of mutually reachable vertices form *the strongly connected components* of G





Implementing Graphs

- Involves a number of implementation decisions, depending on intended uses
  - What kinds of graphs will be availabe?
    - Undirected, directed, mixed?
  - What underlying data structures will be used?
  - What functionality will be provided
  - What aspects will be public/protected/private
- We'll focus on popular implementations for undirected and directed graphs (separately)

Graphs in structure5

- We want to store information at vertices and at edges, but we favor vertices
  - Let V and E represent the types of information held by vertices and edges respectively
  - Interface Graph<V,E> extends Structure<V>
    - Vertices are the building blocks; edges depend on them
- Type V holds a *label* for a (hidden) vertex type
- Type E holds a *label* for an (available) edge type
  - Label: Application-specific data for a vertex/edge

Graphs in structure5

- The methods described in the Structure interface deal with *vertices*
  - but also impact edges: e.g., clear()
- We'll want to add a number of similar methods to provide information about edges, and the graph itself