CSCI 136
Data Structures &
Advanced Programming

Lecture 22

Fall 2018

Instructor: Bills

Last Time

- Lab 7: Two Towers
- Array Representations of (Binary) Trees
- Application: Huffman Encoding

Today

Improving Huffman's Algorithm

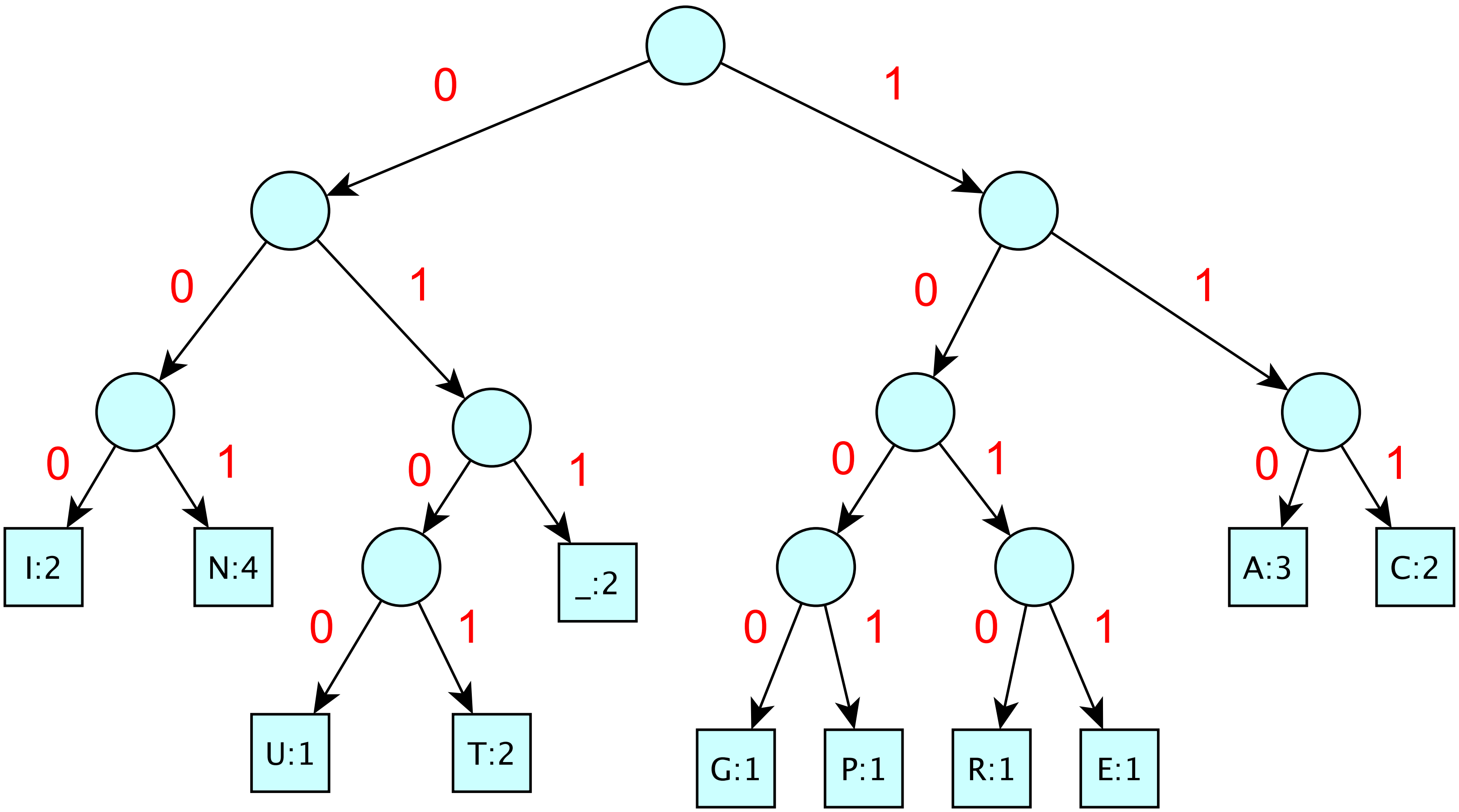
- Priority Queues & Heaps
  - A "somewhat-ordered" data structure
    - Conceptual structure
    - Efficient implementations

Huffman Codes

- Input: Text (a very long String!)
- Algorithm
  - Transform text into symbol frequency count
  - Build optimal encoding tree

A	C	E	G	I	N	P	R	T	U	_
3	2	1	1	2	4	1	1	2	1	2

An Encoding Tree



Left = 0; Right = 1

Huffman Encoding

- Input: symbols of alphabet with frequencies
- Huffman encode as follows
  - Create a single-node tree for each symbol: key is frequency; value is letter
  - while there is more than one tree
    - Find two trees T1 and T2 with lowest keys
    - Merge them into new tree T with dummy value and key= T1.key+ T2.key
- Theorem: The tree computed by Huffman is an optimal encoding for given frequencies

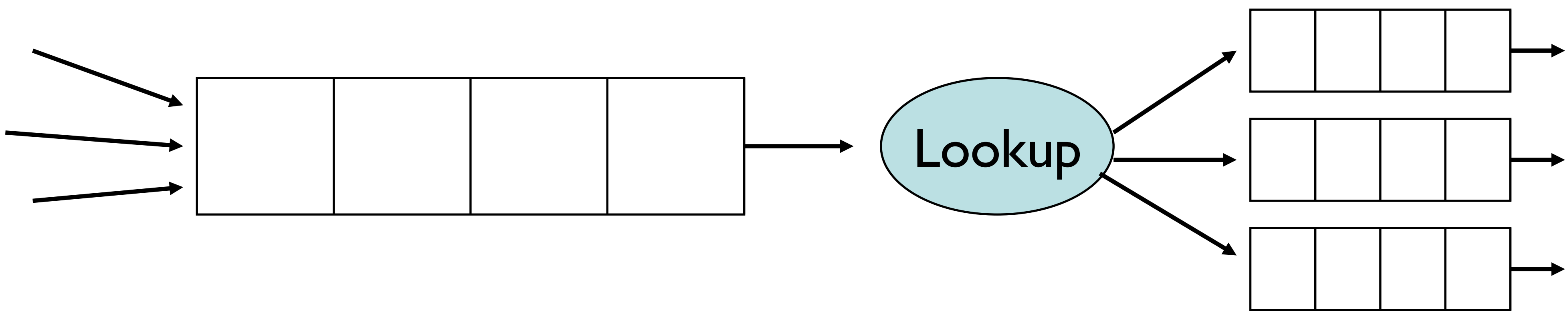
Recall : Huffman Encoding Algorithm

- Keep a Vector of Binary Trees
- Sort them by decreasing frequency
  - Removing two smallest frequency trees is fast
- Insert merged tree into correct (sorted) location in Vector
- Running Time:
  - O(n log n) for initial sorting
  - $O(n^2)$ for rest: O(n) for each re-insertion
- Can we do better...?

What Huffman Encoder Needs

- A structure S to hold items with *priorities*
- S should support operations
  - add(E item);   // add an item
  - E removeMin(); // remove min priority item
- S should be designed to make these two operations fast
- If, say, they both ran in O(log n) time, the Huffman while loop would take O(n log n) time instead of $O(n^2)$!
- We've seen this situation before….

Priority Queues



<u>Packet Sources May Be Ordered by Sender</u>

`sysnet.cs.williams.edu`	`priority = 1 (best)`
`bull.cs.williams.edu`	`2`
`yahoo.com`	`10`
`spammer.com`	`100 (worst)`

Priority Queues

- Priority queues are also used for:
  - Scheduling processes in an operating system
    - Priority is function of time lost + process priority
  - Order services on server
    - Backup is low priority, so don't do when high priority tasks need to happen
  - Scheduling future events in a simulation
  - Medical waiting room
  - Huffman codes - order by tree root "frequency"
  - A variety of graph/network algorithms
  - To roughly rank choices that are generated out of order

Priority Queues

- Name is misleading: They are **not FIFO**
- Always dequeue object with **highest priority** (smallest rank) regardless of when it was enqueued
- Data can be received/inserted in any order, but it is always returned/removed according to priority
- Like ordered structures (i.e., OrderedVectors and OrderedLists), PQs require comparisons of values

An Apology

- On behalf of computer scientists everywhere, I'd like to apologize for the confusion that inevitably results from the fact that

  Higher Priority ↔ Lower Rank

- The PQ removes the *lowest ranked* value in an ordering: that is, the *highest priority* value!

We're sorry!

PQ Interface

```
public interface PriorityQueue<E extends Comparable<E>> {
  public E getFirst(); // peeks at minimum element
  public E remove();   // removes minimum element
  public void add(E value); // adds an element
  public boolean isEmpty();
  public int size();
  public void clear();
}
```

Notes on PQ Interface

- Unlike previous structures, we do not extend any other interfaces
  - Many reasons: For example, it's not clear that there's an obvious iteration order
- PriorityQueue uses Comparables: methods *consume* Comparable parameters and *return* Comparable values
  - Could be made to use Comparators instead…

Implementing PQs

- Queue?
  - Wouldn't work so well because we can't insert and remove in the "right" way (i.e., keeping things ordered)
- OrderedVector?
  - Keep ordered vector of objects
  - O(n) to add/remove from vector
  - Details in book…
  - Can we do better than O(n)?
- Heap!
  - Partially ordered binary tree

Heap

- A heap is a special type of tree
  - Root holds smallest (highest priority) value
  - Subtrees are also heaps (recursive definition!)
- So values increase in priority (decrease in rank) from leaves to root (from descendant to ancestor)
- *Invariant for nodes:* For each child of each node
  - node.value() <= child.value()   // if child exists
- Several valid heaps for same data set (no unique representation)

Inserting into a PQ

- Add new value as a leaf
- “Percolate” it up the tree
  - while (value < parent’s value) swap with parent
- This operation preserves the heap property since new value was the only one violating heap property
- Efficiency depends upon speed of
  - Finding a place to add new node
  - Finding parent
  - Depth of newly added node

Removing From a PQ

- Find a leaf, delete it, put its *data* in the root
- "Push" *data* down through the tree
  - while (*data.value* > value of (at least) one child)
    - Swap *data* with data of **smallest** child
- This operation preserves the heap property
- Efficiency depends upon speed of
  - Finding a leaf
  - Finding locations of children
  - Height of tree

Implementing Heaps

- VectorHeap
  - Use conceptual array representation of BT (ArrayTree)
  - But use extensible Vector instead of array (makes adding elements easier)
  - Note:
    - Root of tree is location 0 of Vector
    - Children of node in location i are in locations 2i+1 (left) and 2i+2 (right)
    - Parent of node i is in location (i-1)/2

Implementing Heaps

- Features
  - No gaps in array (array is *complete*)-- why?
    - We always add in next available array slot (left-most available spot in binary tree;
    - We always remove using “final” leaf
  - *Heap Invariant becomes*
    - data[i] <= data[2i+1]; data[i]<=data[2i+2] (or kids might be null)
  - When elements are added and removed, do small amount of work to “re-heapify”
    - How small? Note: finding a node’s child or parent takes constant time, as does finding “final” leaf or next slot for adding
    - Since this heap corresponds to a full binary tree, the depth of the tree is O(log n), so percolate/pushDown takes O(log n) time!