CSCI 136
Data Structures &
Advanced Programming

Lecture 19

Fall 2018

Instructor: Bills

Last Time

- Trees
  - Expression Trees
    - Recursive evaluation
  - Implementation

Today

- Recursion/Induction on Trees
- Applications: Decision Trees
- Trees with more than 2 children
  - Representations
- Traversing Binary Trees
  - As methods taking a BinaryTree parameter
  - With Iterators

BT Questions/Proofs

- Prove
  - The number of nodes at depth n is at most 2^n.
  - The number of nodes in tree of height n is at most $2^{(n+1)}-1$.
  - A tree with n nodes has exactly n-1 edges
  - The size() method works correctly
  - The height() method works correctly
  - The isFull() method works correctly

BT Questions/Proofs

Prove: Number of nodes at depth $d \geq 0$ **is at most** 2^d.

Idea: Induction on depth d of nodes of tree

Base case: d= 0: 1 node. $1 = 2^0$ ✓

Induction Hyp.: For some $d \geq 0$, **there are at** most 2^d nodes at depth d.

Induction Step: Consider depth d+1. There are at most 2 nodes at depth d+1 for every node at depth d.

Therefore it has at most $2*2^d = 2^{d+1}$ nodes ✓

BT Questions/Proofs

Prove that any tree on n≥**1 nodes has n-1** edges

Idea: Induction on number of nodes

Base case: n = 1. There are no edges✓

Induction Hyp: Assume that, for some n ≥ **1,** every tree on n nodes has exactly n-1 edges.

Induction Step: Let T have n+1 nodes. Show it has exactly n edges.

- Remove a leaf v (and its single edge) from T
- Now T has n nodes, so it has n-1 edges
- Now add v (and its single edge) back, giving n+1 nodes and n edges.

BT Questions/Proofs

Prove that BinaryTree method size() is correct.

- Let n be the number of nodes in the tree T
- Alert: Strong Induction Ahead...

Base case: n = 0. T is empty---size() returns 0✓

Induction Hyp: Assume size() is correct for *all trees* having *at most* n nodes.

Induction Step: Assume T has n+1 nodes

- Then left/right subtrees each have *at most* n nodes
- So size() returns correct value for each subtree
- And the size of T is 1 + size of left subtree + size of right subtree✓

Representing Knowledge

- Trees can be used to represent knowledge
  - Example: InfiniteQuestions game
    - Let's play!
- We often call these trees decision trees
  - Leaf: object
  - Internal node: question to distinguish objects
- Two methods: play() and learn()
  - Play: Move down decision tree until we reach a leaf
    - Check to see if the leaf is correct
  - Learn: If not correct, add question, make new and old objects children
- Let's look at the code

Building Decision Trees

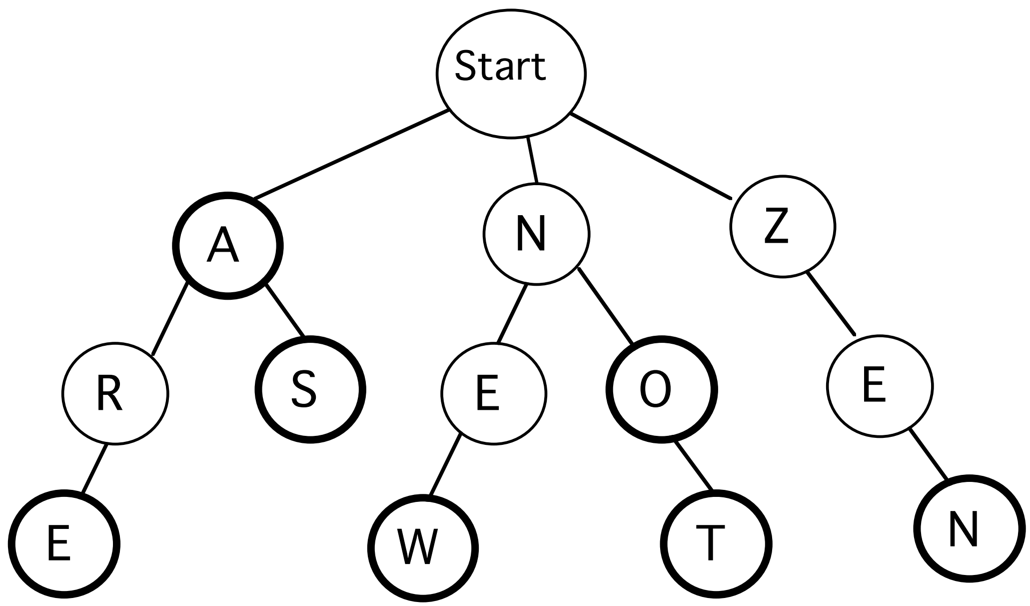
- Gather/obtain data
- Analyze data
  - Make greedy choices: Find good questions that divide data into halves (or as close as possible)
- Construct tree with shortest height
- In general this is a *hard* problem!
- Example

Representing Arbitrary Trees

- What if nodes can have many children?
  - Example: Game trees
- Replace left/right node references with a list of children (Vector, SLL, etc)
  - Allows getting "i^{th}" child
- Should provide method for getting degree of a node
- Degree 0   Empty list   No children   Leaf

Lab 9 Preview : Lexicon

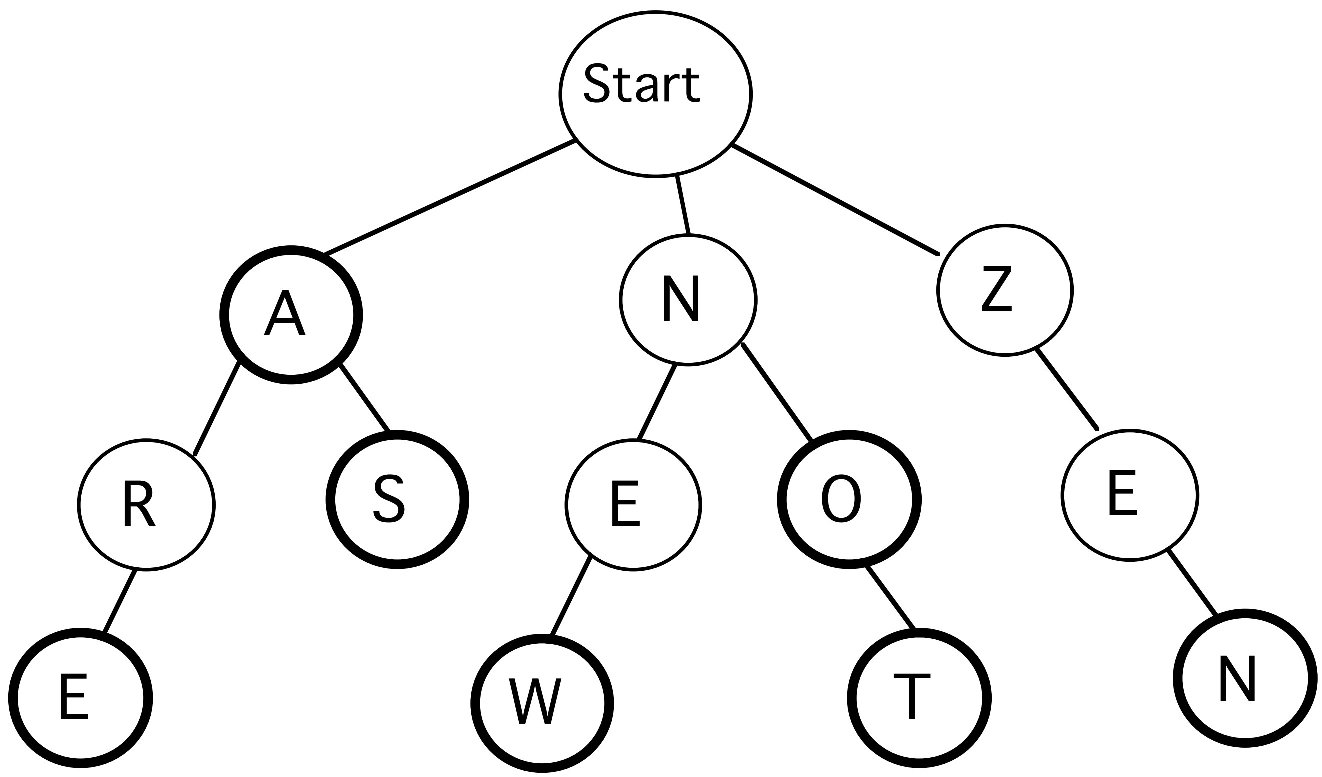
- Goal: Build a data structure that can efficiently store and search a large set of words
- A special kind of tree called a *trie*



Lab 9 Preview : Tries

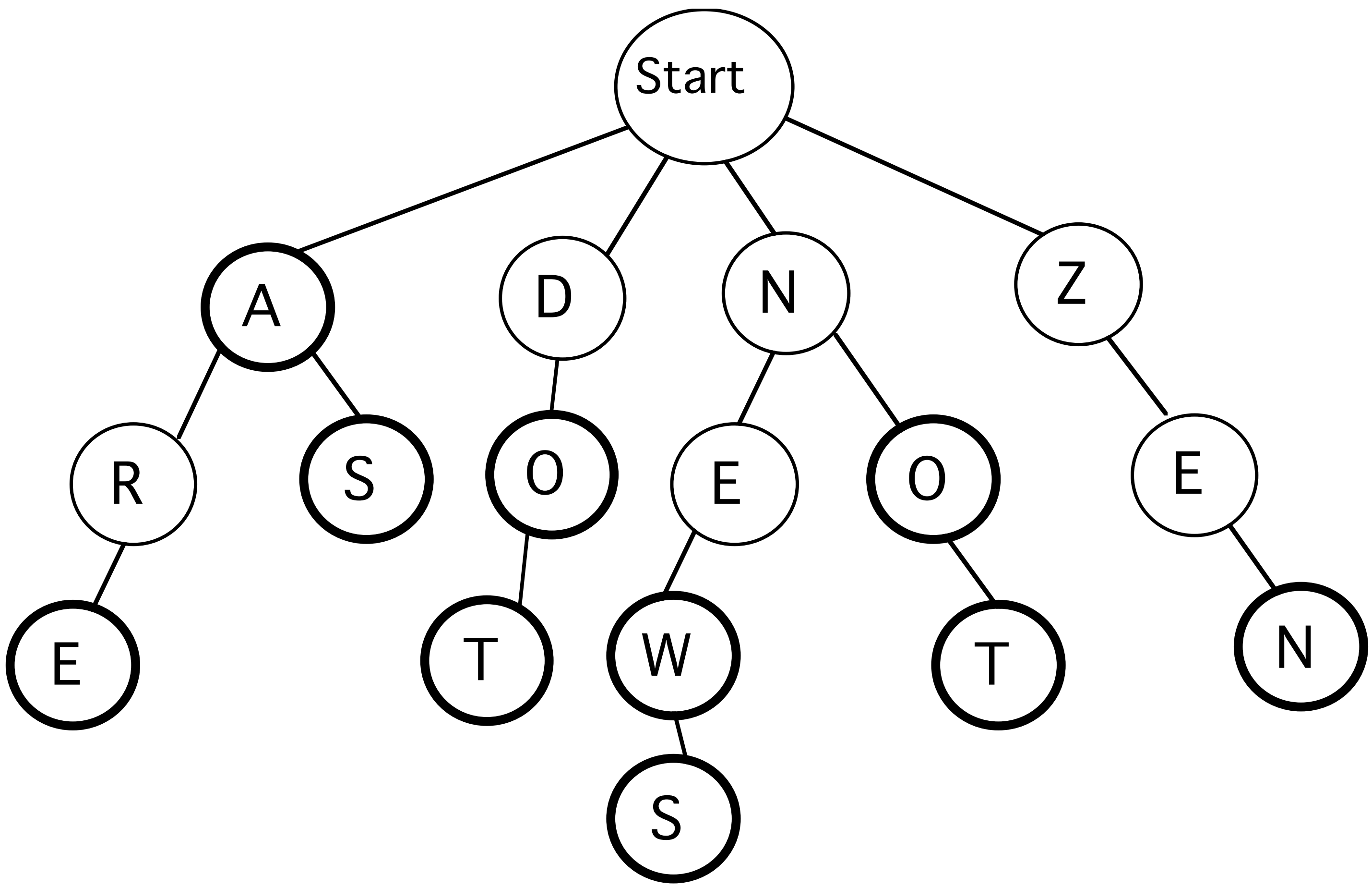
- A trie is a tree that stores words where
  - Each node holds a letter
  - Some nodes are "word" nodes (dark circles)
  - Any path from the root to a word node describes one of the stored words
  - All paths from the root form prefixes of stored words (a word is considered a prefix of itself)

Tries



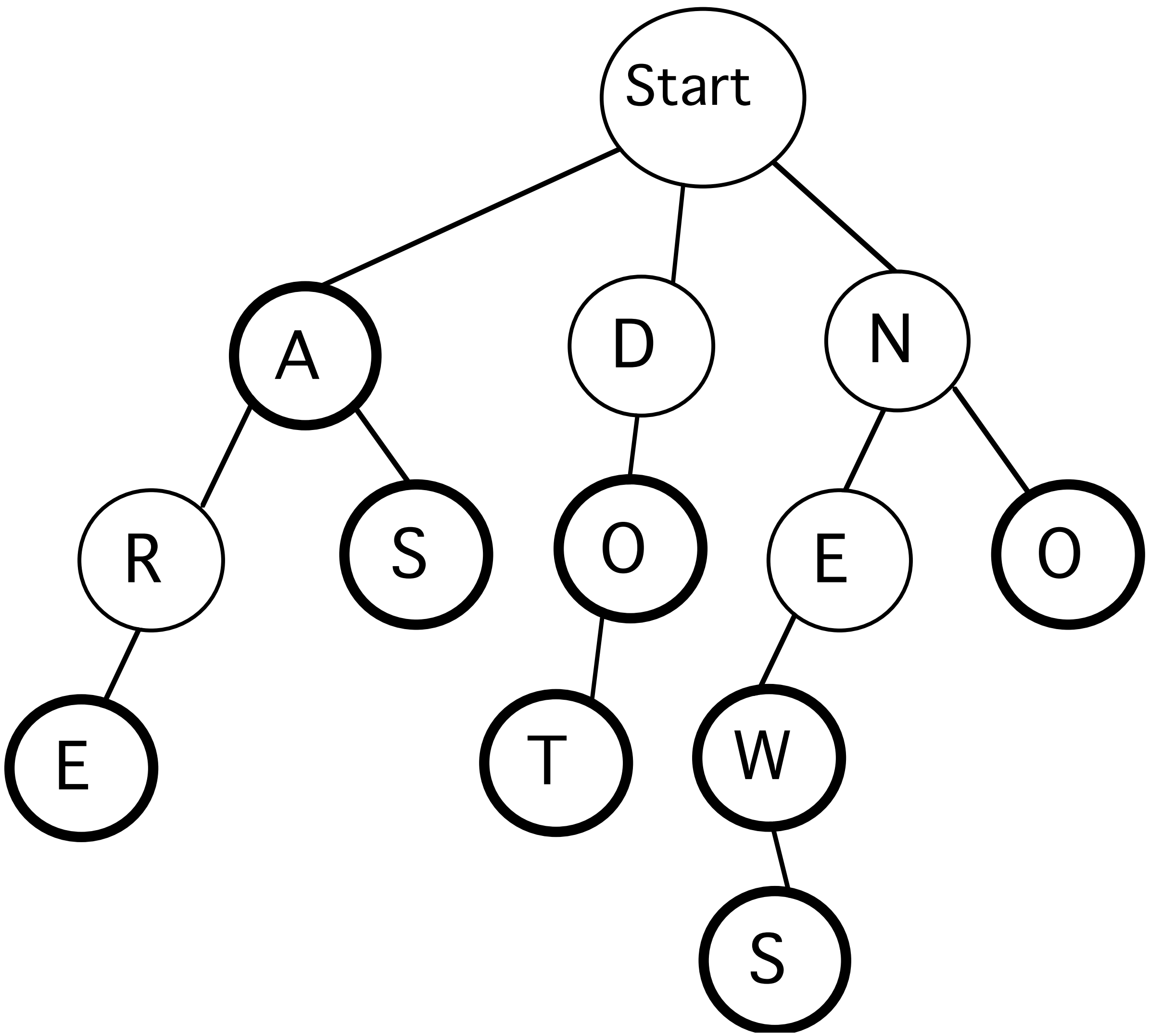
Now add “dot” and “news”

Tries



Now remove “not” and “zen”

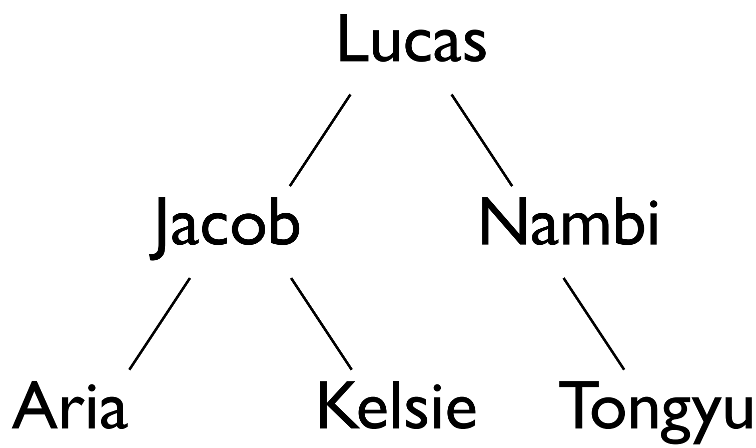
Tries



Tree Traversals

- In linear structures, there are only a few basic ways to traverse the data structure
  - Start at one end and visit each element
  - Start at the other end and visit each element
- How do we traverse binary trees?
  - (At least) four reasonable mechanisms

Tree Traversals



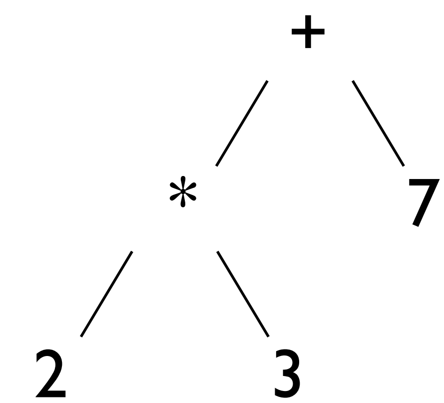
In-order: Aria, Jacob, Kelsie, Lucas, Nambi, Tongyu

Pre-order: Lucas, Jacob, Aria, Kelsie, Nambi, Tongyu

Post-order: Aria, Kelsie, Jacob, Tongyu, Nambi, Lucas,

Level-order: Lucas, Jacob, Nambi, Aria, Kelsie, Tongyu

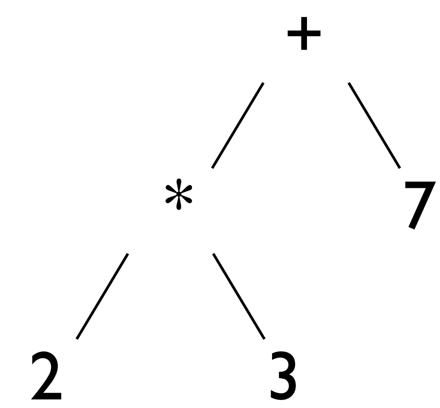
Tree Traversals



- Pre-order
  - Each node is visited before any children. Visit node, then each node in left subtree, then each node in right subtree. (node, left, right)
    - +*237
- In-order
  - Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree. (left, node, right)
    - 2*3+7

("pseudocode")

Tree Traversals

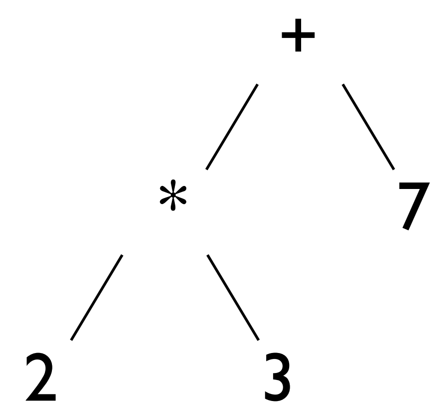


- Post-order
  - Each node is visited after its children are visited. Visit all nodes in left subtree, then all nodes in right subtree, then node itself. (left, right, node)
    - 23*7+
- Level-order (not obviously recursive!)
  - All nodes of level i are visited before nodes of level i+1. (visit nodes left to right on each level)
    - +*723

("pseudocode")

Tree Traversals

```
public void pre-order(BinaryTree t) {
      if(t.isEmpty()) return;
      touch(t); // some method
      preOrder(t.left());
      preOrder(t.right());
}
```



For in-order and post-order: just move touch(t)!

But what about level-order???