# CSCI 136
# Data Structures &
# Advanced Programming

Lecture 11

Fall 2018

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# Administrative Details

- Lab 4 Wednesday: Sorting!
  - The lab has been posted on the Labs page
  - You may again work with a partner
    - Needn't be same partner as Lab 3
    - Fill out the Google Form!
  - Produce a design before lab
    - Both members of pair should produce their own

# Last Time

- Strong Induction
- The Comparable Interface
- Basic Sorting
  - Bubble, Insertion, Selection Sorts
  - Including time and space analysis

# This Time

- More Comparable Examples
- Better Sorting Methods
  - MergeSort
  - QuickSort
- More Flexible Comparing: Comparator Interface

# Comparable Interface

- Java provides the *Comparable* interface, which specifies a method *compareTo()*
  - Any class that implements Comparable, provides compareTo()

```
public interface Comparable<T> {
    //post: return < 0 if this smaller than other
            return 0 if this equal to other
            return > 0 if this greater than other
      int compareTo(T other);
}
```

# compareTo in Card Example

We could have written

```
public class CardRankSuit implements
      Comparable<CardRankSuit> {

  public int compareTo(CardRankSuit other) {
      if (this.getSuit() != other.getSuit())
         return getSuit().compareTo(other.Suit());
      else
         return getRank().compareTo(other.getRank());
  }
// rest of code for the class....
}
```

# compareTo in Card Example

Notes

- enum types automatically implement Comparable
- The magnitude of the values returned by compareTo are not important. We only care if value is positive, negative, or 0!
- compareTo defines a *"natural ordering"* of Objects
  - There's nothing "*natural*" about it….
- We use the *BubbleSort* algorithm to sort the cards in CardDeck.java

# Comparable & compareTo

- The Comparable interface (Comparable<T>) is part of the java.lang (not structure5) package.
- Other Java-provided structures can take advantage of objects that implement Comparable
  - See the Arrays class in java.util
  - Example JavaArraysBinSearch
- Users of Comparable are urged to ensure that *compareTo()* and *equals()* are *consistent*. That is,
  - x.compareTo(y) == 0 exactly when x.equals(y) == true
- Note that Comparable limits user to a *single ordering*
- The syntax can get kind of dense
  - See BinSearchComparable.java : a generic binary search method
  - And even more cumbersome….

# ComparableAssociation

- Suppose we want an *ordered* Dictionary, so that we can use binary search instead of linear
- Structure5 provides a ComparableAssociation class that implements Comparable.
- The class declaration for ComparableAssociation is

…wait for it...

```
public class ComparableAssociation<K extends Comparable<K>, V>
        Extends Association<K,V> implements
        Comparable<ComparableAssociation<K,V>>
```

(Yikes!)

- Example: Since Integer implements Comparable, we can write
  - ComparableAssociation<Integer, String> myAssoc =
    new ComparableAssociation( new Integer(567), "Bob");
- We could then use Arrays.sort on an array of these

# Faster Sorting: Merge Sort

- A *divide and conquer* algorithm
- Typically used on arrays
- Merge sort works as follows:
  - If the array is of length 0 or 1, then it is already sorted.
  - Divide the unsorted array into two arrays of about half the size of original.
  - Sort smaller arrays recursively by re-applying merge sort.
  - Merge the two smaller arrays back into one sorted array.
- Time Complexity?
  - Spoiler Alert! We'll see that it's O(n log n)
- Space Complexity?
  - O(n)

# Merge Sort

- [8 14 29 1 17 39 16 9]
- [8 14 29 1] [17 39 16 9] split
- [8 14] [29 1] [17 39] [16 9] split
- [8] [14] [29] [1] [17] [39] [16] [9] split
- [8 14] [1 29] [17 39] [9 16] merge
- [1 8 14 29] [9 16 17 39] merge
- [1 8 9 14 16 17 29 39] merge

# Merge Sort

- How would we implement it?
- First pass…

```
// recursively mergesorts A[from .. To] "in place"
void recMergeSortHelper(A[], int from, int to)
    if ( from ≤ to )
        mid = (from + to)/2
        recMergeSortHelper(A, from, mid)
        recMergeSortHelper(A, mid+1, to)
        merge(A, from, to)
```

But *merge* hides a number of important details….

# Merge Sort = O(n log n)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| [8 | 14 | 29 | 1 | 17 | 39 | 16 | 9] | |
| [8 | 14 | 29 | 1] | [17 | 39 | 16 | 9] | split |
| [8 | 14] | [29 | 1] | [17 | 39] | [16 | 9] | split |
| [8] | [14] | [29] | [1] | [17] | [39] | [16] | [9] | split |
| [8 | 14] | [1 | 29] | [17 | 39] | [9 | 16] | merge |
| [1 | 8 | 14 | 29] | [9 | 16 | 17 | 39] | merge |
| [1 | 8 | 9 | 14 | 16 | 17 | 29 | 39] | merge |

split × 3: log n

merge × 3: log n

merge takes at most n comparisons per line

# Time Complexity Proof

- Prove for $n = 2^k$ (true for other n but harder)
- That is, MergeSort for performs at most
  - $n * \log(n) = 2^k * k$ comparisions of elements
- Base cases $k \leq 1$: 0 comparisons: $0 < 1 * 2^1$ ✓
- Induction Step: Suppose true for all integers smaller than k. Let T(k) be # of comparisons for $2^k$ elements. Then
- $\underline{T(k) \leq} \; 2^k + 2 * T(k-1) \leq 2^k + 2(k-1)2^{k-1} \leq \underline{k * 2^k}$ ✓

# Problems with Merge Sort

- Need extra temporary array
  - If data set is large, this could be a problem
- Waste time copying values back and forth between original array and temporary array
- Can we avoid this?

# Quick Sort

- Quick sort is designed to behave much like Merge sort, without requiring extra storage space

| Merge Sort | Quick Sort |
|---|---|
| Divide list in half | Partition* list into 2 parts |
| Sort halves | Sort parts |
| Merge halves | Join* sorted parts |

# Quick Sort

```
public void quickSortRecursive(Comparable data[],
                    int low, int high) {
    // pre: low <= high
    // post: data[low..high] in ascending order
        int pivot;
        if (low >= high) return;

        /* 1 - place pivot */
        pivot = partition(data, low, high);
        /* 2 - sort small */
        quickSortRecursive(data, low, pivot-1);
        /* 3 - sort large */
        quickSortRecursive(data, pivot+1, high);
}
```

# Partition

1. Put first element (pivot) into sorted position
2. All to the left of "pivot" are smaller and all to the right are larger
3. Return index of "pivot"

# Partition

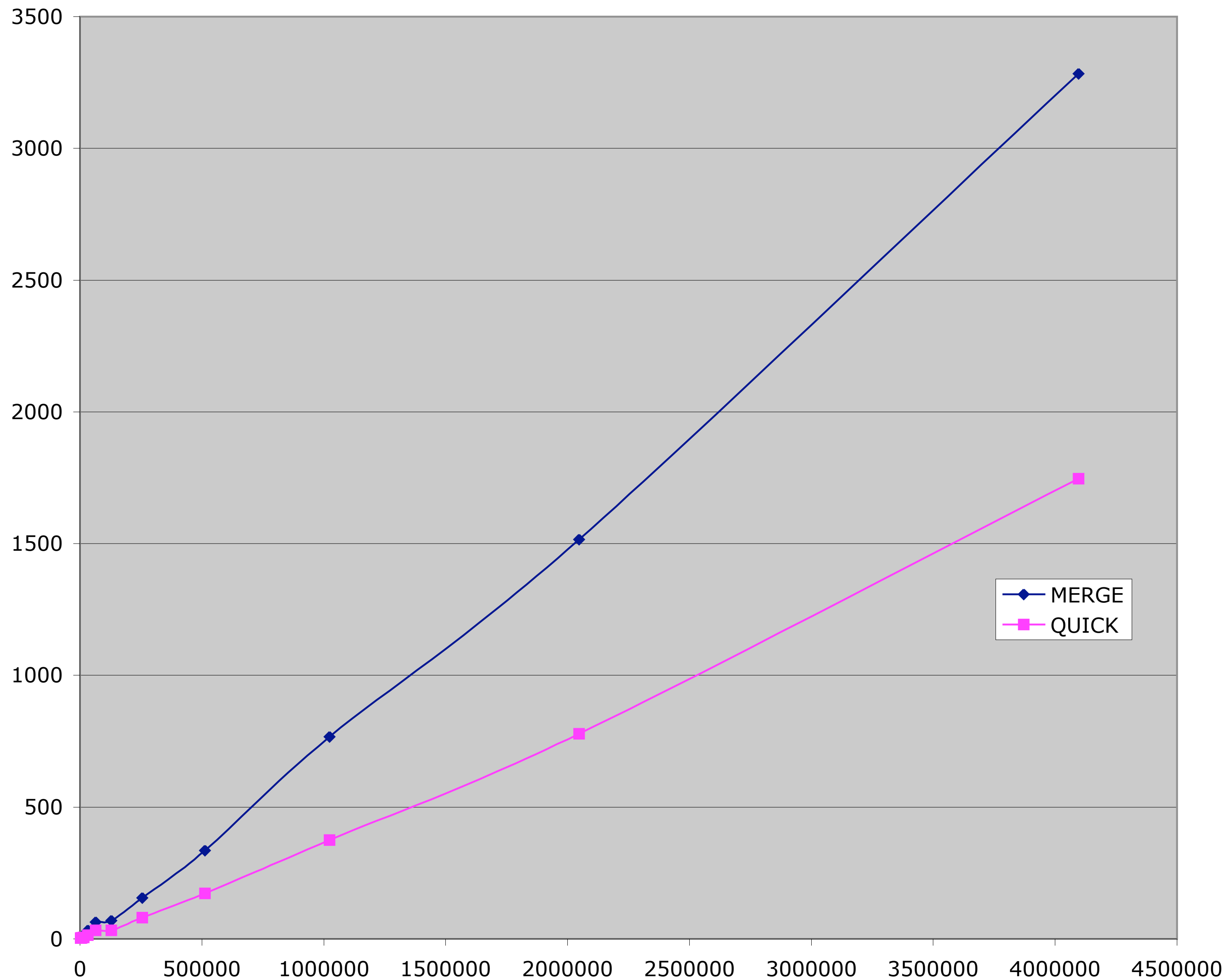
```
int partition(int data[], int left, int right) {
  while (true) {
    while (left < right && data[left] < data[right])
      right--;
    if (left < right) {
      swap(data,left++,right);
    } else {
      return left;
    }

    while (left < right && data[left] < data[right])
      left++;
    if (left < right) {
      swap(data,left,right--);
    } else {
      return right;
    }
  }
}
```

# Complexity

- Time:
  - Partition is O(n)
  - If partition breaks list exactly in half, same as merge sort, so O(n log n)
  - If data is already sorted, partition splits list into groups of 1 and n-1, so $O(n^2)$
- Space:
  - O(n) (so is MergSort)
    - In fact, it's n + c compared to 2n + c for MergeSort

# Merge vs. Quick (Average Time)



# Food for Thought…

- How to avoid picking a bad pivot value?
  - Pick median of 3 elements for pivot (heuristic!)
- Combine selection sort with quick sort
  - For small n, selection sort is faster
  - Switch to selection sort when elements is <= 7
  - Switch to selection/insertion sort when the list is almost sorted (partitions are very unbalanced)
    - Heuristic!

# Sorting Wrapup

| | Time | Space |
|---|---|---|
| Bubble | Worst: $O(n^2)$<br>Best: O(n) - if "optimiazed" | O(n) : n + c |
| Insertion | Worst: $O(n^2)$<br>Best: O(n) | O(n) : n + c |
| Selection | Worst = Best: $O(n^2)$ | O(n) : n + c |
| Merge | Worst = Best:: O(n log n) | O(n) : 2n + c |
| Quick | Average = Best: O(n log n)<br>Worst: $O(n^2)$ | O(n) : n + c |

# More Skill-Testing (Try these at home)

Given the following list of integers:

9 5 6 1 10 15 2 4

1) Sort the list using Bubble sort. Show your work!
2) Sort the list using Insertion sort. . Show your work!
3) Sort the list using Merge sort. . Show your work!
4) Verify the best and worst case time and space complexity for each of these sorting algorithms as well as for selection sort.

# Comparators

- Limitations with Comparable interface
  - Only permits one order between objects
  - What if it isn't the desired ordering?
  - What if it isn't implemented?
- Solution: Comparators

# Comparators (Ch 6.8)

- A comparator is an object that contains a method that is capable of comparing two objects
- Sorting methods can be written to apply a comparator to two objects when a comparison is to be performed
- Different comparators can be applied to the same data to sort in different orders or on different keys

```
public interface Comparator <E> {
    // pre: a and b are valid objects
    // post: returns a value <, =, or > than 0 determined by
    // whether a is less than, equal to, or greater than b
    public int compare(E a, E b);
}
```

# Example

```
class Patient {
    protected int age;
    protected String name;
    public Patient (String s, int a) {name = s; age = a;}
    public String getName() { return name; }
    public int getAge() {return age;}
}

class NameComparator implements Comparator <Patient>{
    public int compare(Patient a, Patient b) {
       return a.getName().compareTo(b.getName());
    }
} // Note: No constructor; a "do-nothing" constructor is added by Java
```

---

```
public void sort(T a[], Comparator<T> c) {
   …
   if (c.compare(a[i], a[max]) > 0) {…}
}
```

---

```
sort(patients, new NameComparator());
```

# Comparable vs Comparator

- Comparable Interface for class X
  - Permits just one order between objects of class X
  - Class X must implement a compareTo method
  - Changing order requires rewriting compareTo
    - And recompiling class X
- Comparator Interface
  - Allows creation of "Compator classes" for class X
  - Class X isn't changed or recompiled
  - Multiple Comparators for X can be developed
    - Sort Strings by length (alphabetically for equal-length)

# Selection Sort with Comparator

```
public static <E> int findPosOfMax(E[] a, int last,
              Comparator<E> c) {
      int maxPos = 0      // A wild guess
      for(int i = 1; i <= last; i++)
              if (c.compare(a[maxPos], a[i]) < 0) maxPos = i;
      return maxPos;
}
public static <E> void selectionSort(E[] a, Comparator<E> c) {
      for(int i = a.length - 1; i>0; i--) {
          int big= findPosOfMin(a,i,c);
          swap(a, i, big);
      }
}
```

- The same array can be sorted in multiple ways by passing different Comparator<E> values to the sort method;