

**CSCI 136**  
**Data Structures &**  
**Advanced Programming**

**Fall 2017**

**Lecture 33**

**The 2070567s**

# Administrative Details

## Reminders

- No lab this week
- Final exam
  - Thursday, December 14 at 9:30 in TBL 112
  - Study guide, sample exam will be posted online
  - TAs available this weekend (see course calendar)
  - “Bills review” Tuesday from 1:30-2:30 in Physics 205

# Last Time

- Prim's algorithm wrapup
- Hash tables
  - `Object.hashCode()` maps objects to bins
  - Linear probing to resolve collisions

# Today's Outline

- External Chaining to resolve collisions
- Fun hashing applications (not on exam)
  - Cuckoo hashing
  - Bloom Filters
  - Verification/integrity
  - Deduplication

# One Last Note on Graphs

- In an undirected graph, each edge connects two vertices
  - Which contributes 1 to the degree of each of those vertices
  - Since each edge will be counted by two vertices, the sum of all of the degrees of all vertices is twice the number of edges

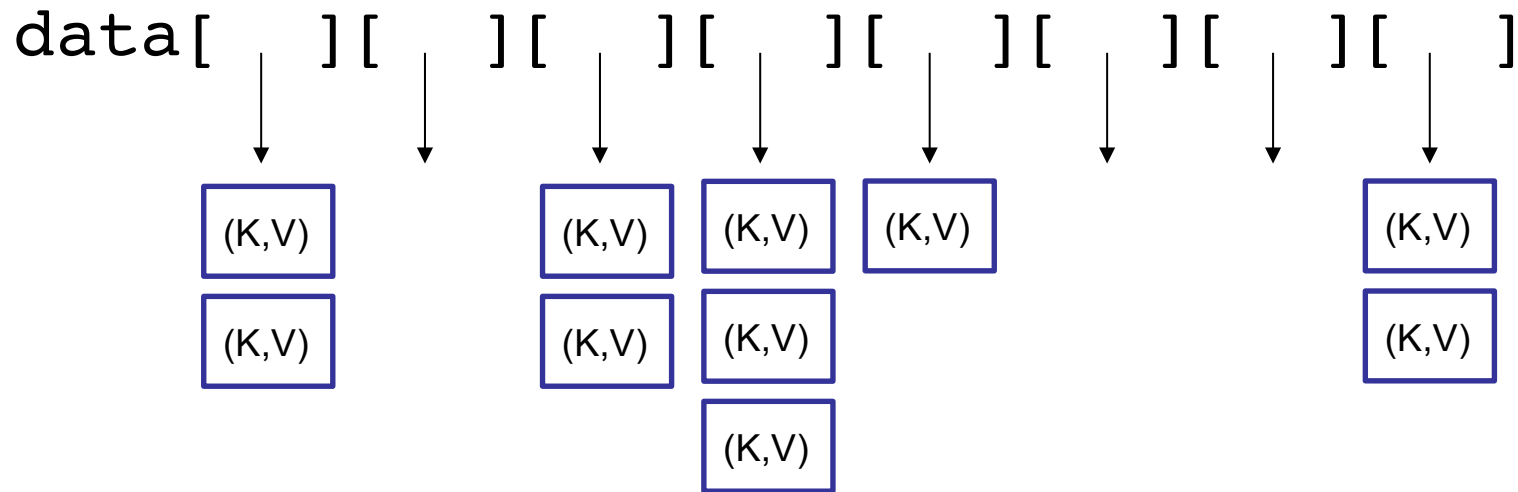
$$\sum \deg(v) = 2 |E|$$

# Hashtable Core Concept

- A hash function maps a **key** to an **index**
- The **index** specifies the **bin** where the **key-value** pair should be stored
- If two keys hash to the same bin, we have a **collision**
- **Linear probing** scans and places the collided element in the first empty bin, creating a **run**
  - When we remove, must add a placeholder

# External Chaining

- Instead of runs, we store a list in each bin



- Everything that hashes to  $\text{bin}_i$  goes into  $\text{list}_i$ 
  - `get()`, `put()`, and `remove()` only need to check one slot's list
  - No placeholders!

# Probing vs. Chaining

What is the performance of:

- `put (K, V)`
  - LP:  $O(l + \text{run length})$
  - EC:  $O(l + \text{chain length})$
- `get (K)`
  - LP:  $O(l + \text{run length})$
  - EC:  $O(l + \text{chain length})$
- `remove (K)`
  - LP:  $O(l + \text{run length})$
  - EC:  $O(l + \text{chain length})$
- Runs/chains are important. How do we control cluster/chain length?



# Load Factor

- Need to keep track of how full the table is
  - Why?
  - What happens when array fills completely?
- Load factor is a measure of how full the hash table is
  - $LF = (\# \text{ elements}) / (\text{table size})$
- When LF reaches some threshold, double size of array (typically threshold = 0.6)
  - Challenges?

# Doubling Array

- Cannot just copy values
  - Why?
  - Hash values may change
  - Example: suppose (`key.hashCode() == 11`)
    - $11 \% 7 = 4$ ;
    - $11 \% 13 = 11$ ;
- Result: must recompute all hash codes, reinsert into new array

# Good Hashing Functions

- Important point:
  - All of this hinges on using “good” hash functions that spread keys “evenly”
- Good hash functions:
  - Are fast to compute
  - Distribute keys uniformly
- We almost always have to test “goodness” empirically

# Example Hash Functions

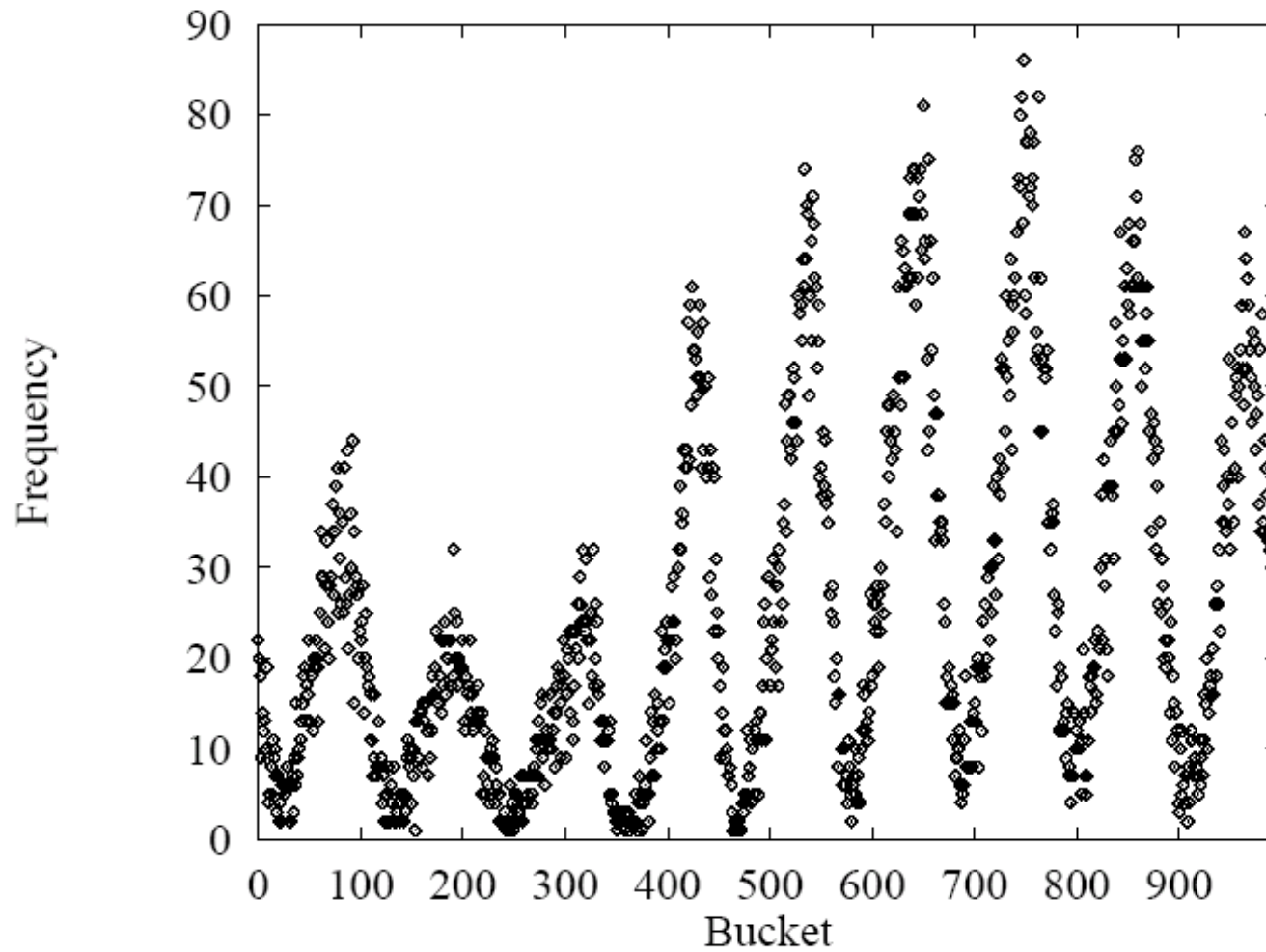
- What are some feasible hash functions for Strings?
  - First char ASCII value mapping
    - 0-255 only
    - Not uniform (some letters more popular than others)
  - Sum of ASCII characters
    - Not uniform - lots of small words
    - smile, limes, miles, slime are all the same

# Example Hash Functions

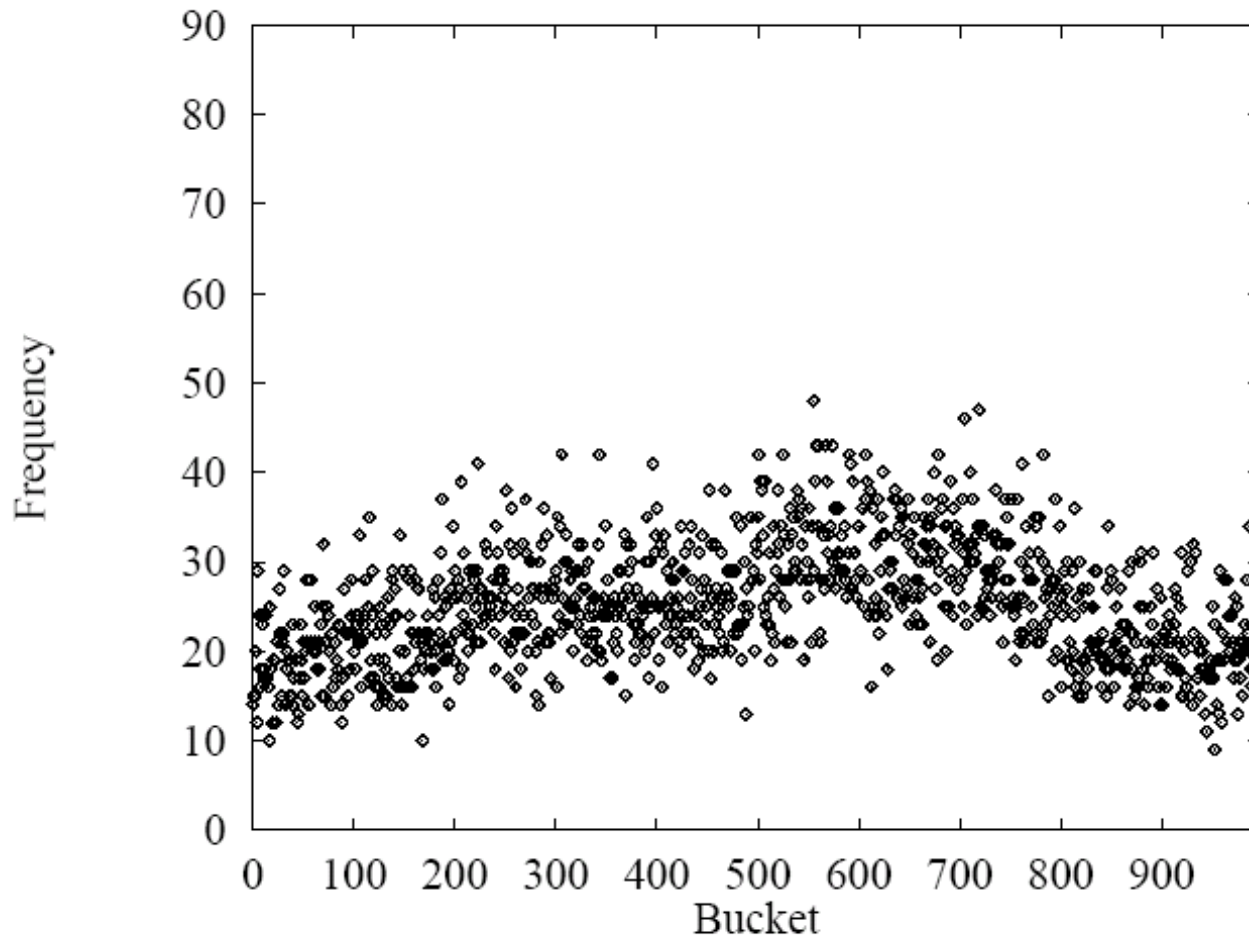
- String hash functions commonly use weighted sums
  - Character values weighted by position in string
    - Long words get bigger codes
    - Distributes keys better than non-weighted sum
  - Let's look at different weights...

$$\sum_{i=0}^{n=s.length()} s.charAt(i)$$

Hash of all words in UNIX  
spelling dictionary (997  
buckets)

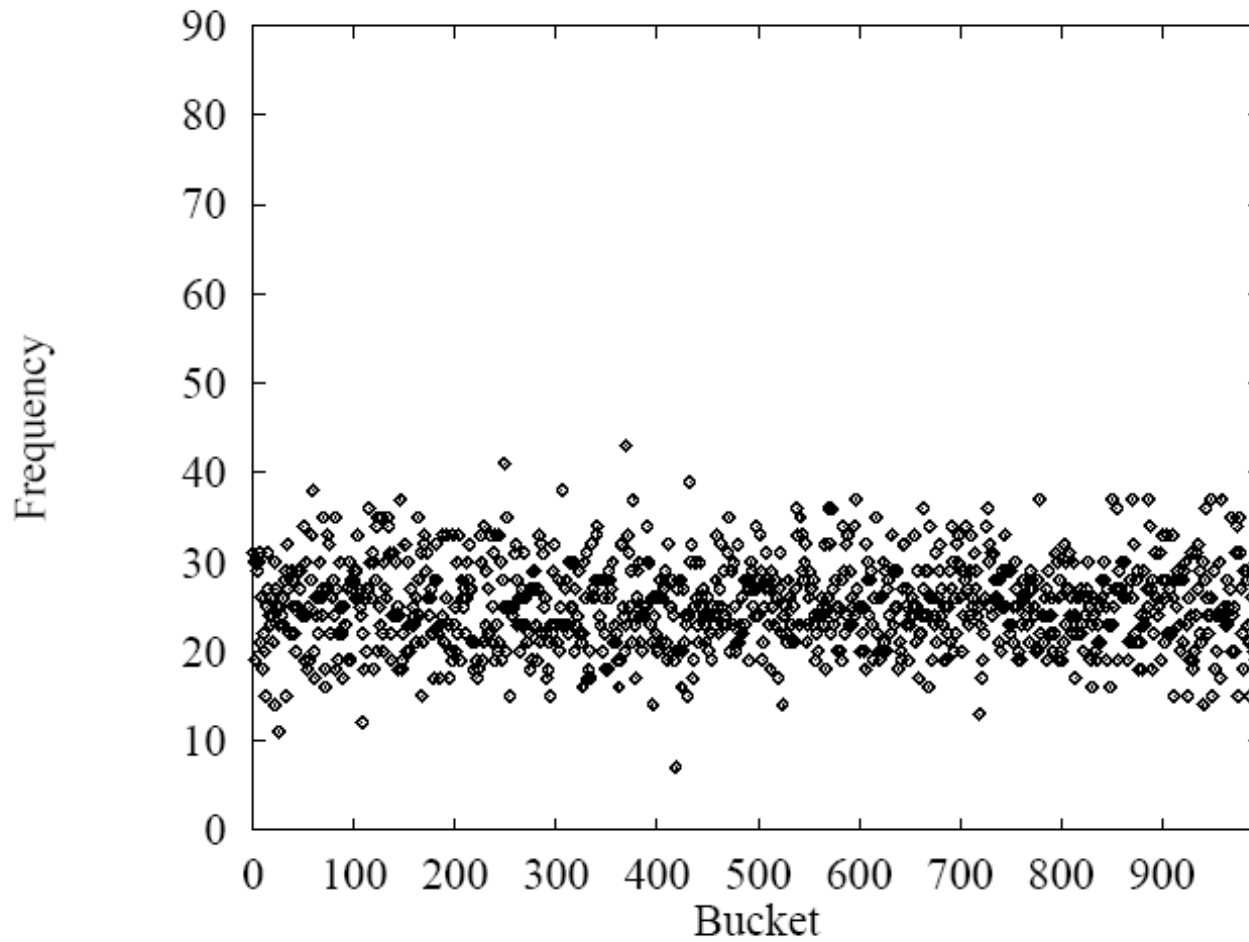


$$\sum_{i=0}^n s.\text{charAt}(i) * 2^i$$



$$\sum_{i=0}^n \text{s.charAt}(i) * 256^i$$

This looks pretty good, but  $256^i$  is big...

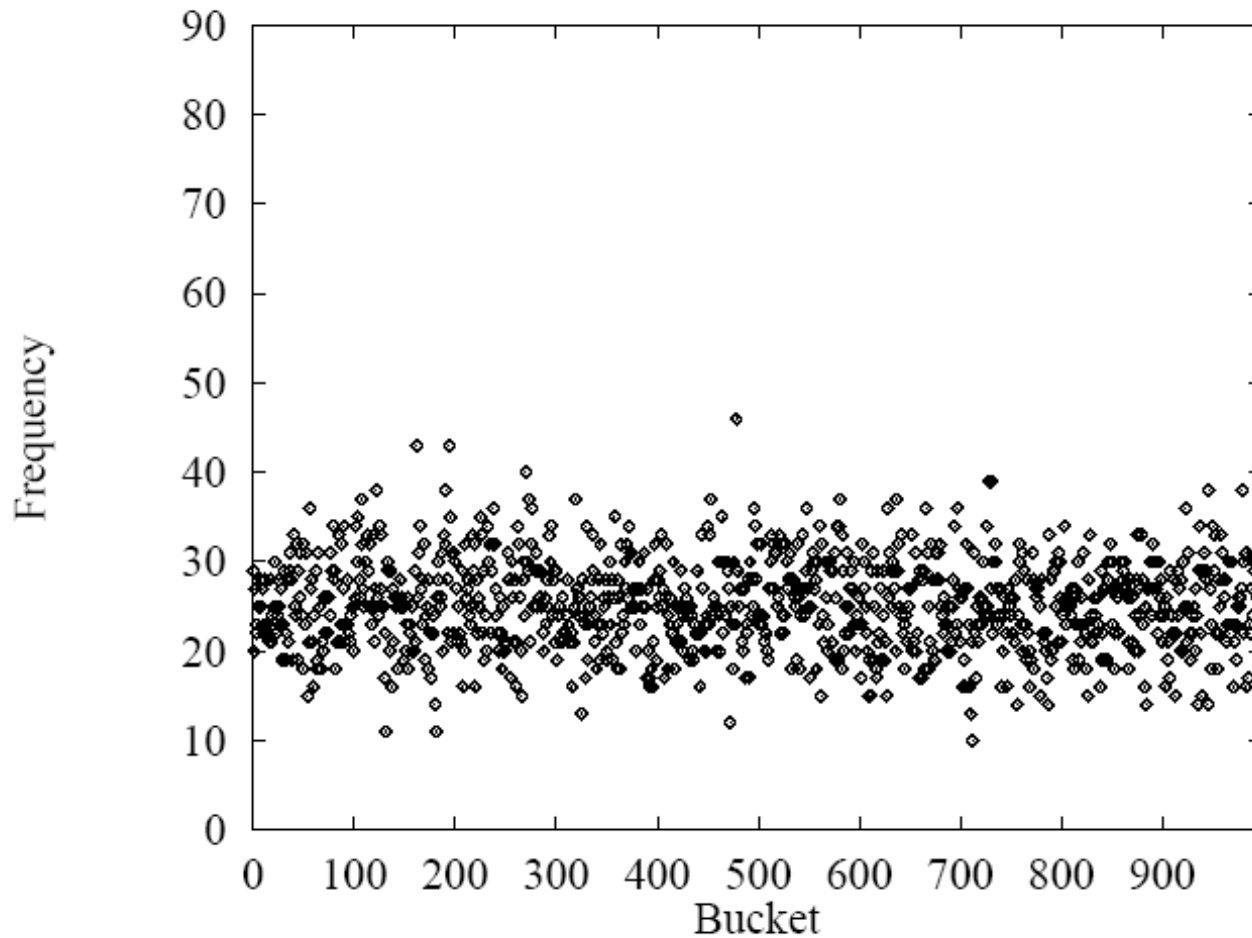




$$\sum_{i=0}^n \text{s.charAt}(i) * 31^i$$

Java uses:

$$\sum_{i=0}^n \text{s.charAt}(i) * 31^{(n-i-1)}$$



# Hashtables: $O(1)$ operations?

- How long does it take to compute a String's hashCode?
  - $O(s.length())$
- Given an object's hash code, how long does it take to find that object?
  - $O(\text{run length})$  or  $O(\text{chain length})$  PLUS cost of `.equals()` method
- Conclusion: for a good hash function (fast, uniformly distributed) and a low load factor (short runs/chains), we *say* hashtables are  $O(1)$

# Summary

	put	get	space
unsorted vector	$O(n)$	$O(n)$	$O(n)$
unsorted list	$O(n)$	$O(n)$	$O(n)$
sorted vector	$O(n)$	$O(\log n)$	$O(n)$
balanced BST	$O(\log n)$	$O(\log n)$	$O(n)$
array indexed by key	$O(1)$	$O(1)$	$O(\text{key range})$