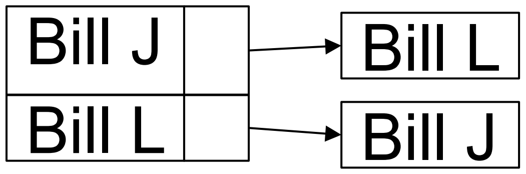
CSCI 136 Data Structures & Advanced Programming

Lecture 30

Fall 2017

Instructors:



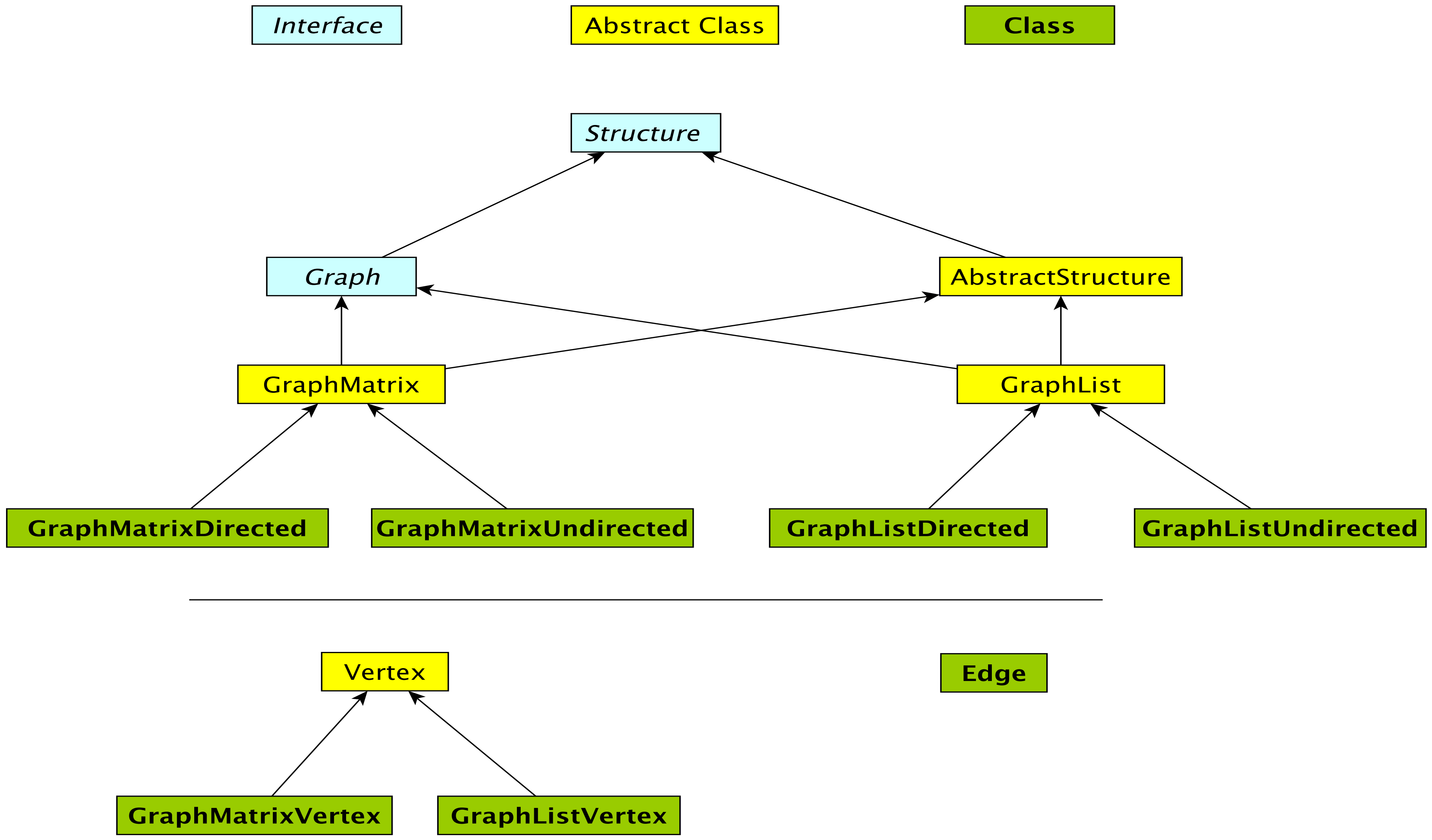
Last Time

- Graph Interface
  - Adjacency Array Implementation Basic Concepts
  - Adjacency List Implementation Basic Concepts
- Structure5 Graph classes + hierarchy

Today’s Outline

- Graph Data Structures: Implementation
  - Adjacency Array Implementation Details
- Greedy Algorithms for Optimization
- Lab 11 : Exam Scheduling
  - Defining the problem
  - Sketching a design

Graph Classes in structure5



Graph Classes in structure5

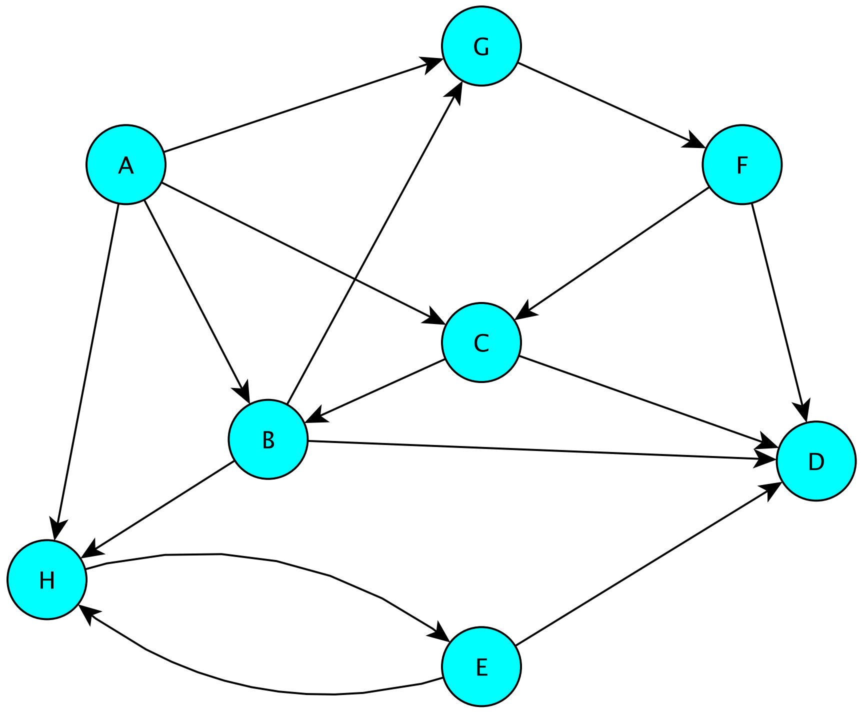
Why so many?!

- There are two types of graphs: undirected & directed
- There are two implementations: arrays and lists
- Strategy: implement as much code as can be written without assuming directedness
    - (Un)Directed Subclasses implement the rest

We'll tackle array-based graphs first....

Adjacency Array: Directed Graph

	A	B	C	D	E	F	G	H
A	0	1	1	0	0	0	1	1
B	0	0	0	1	0	0	1	1
C	0	1	0	1	0	0	0	0
D	0	0	0	0	0	0	0	0
E	0	0	0	1	0	0	0	1
F	0	0	1	1	0	0	0	0
G	0	0	0	0	0	1	0	0
H	0	0	0	0	1	0	0	0



Challenges to having our rows/columns be "vertices"
- Can't use Objects as array indices
- How does adding/deleting a vertex work?!

Adjacency Array: Undirected Graph

Halving the Space (not in structure5)

	0	1	2	3	4	5	6
0	0	1	1	0	0	0	1
1	1	0	1	1	0	0	1
2	1	1	0	1	0	1	0
3	0	1	1	0	1	1	0
4	0	0	0	1	0	0	0
5	0	0	1	1	0	0	1
6	1	1	0	0	0	1	0

	0	1	2	3	4	5	6
0	0	1	1	0	0	0	1
1		0	1	1	0	0	1
2			0	1	0	1	0
3				0	1	1	0
4					0	0	0
5						0	1
6							0

0	1	2	3	4	5	6	7	8	9	…																	
0	1	1	0	0	0	1	0	1	1	0	0	1	0	1	0	1	0	0	1	1	0	0	0	0	0	1	0

`(i,j)` maps to `i*7+j`

in general case: `(i,j)` maps to `i*|V|+7`

Vertex and GraphMatrixVertex

- We need to define a Vertex class
  - Unlike the Edge class, Vertex class **is not public**
  - Useful Vertex methods:

    ```
    V label(), boolean visit(),
    boolean isVisited(), void reset()
    ```
  - GraphMatrixVertex class adds one more useful attribute to Vertex class
    - Index of node (int) in adjacency matrix

      ```
      int index()
      ```
    - Why do we only need one int to represent index?

Choosing a Dictionary Structure

- We need a structure that will let us retrieve the index of a vertex given the vertex label (a dictionary)
- Many choices
  - `Vector` of `Associations`:
    - `Vector<Association<V, GraphMatrixVertex<V>>>`
  - `OrderedVector` of `Associations`
  - `BinarySearchTree` of `Associations`
- Problem: We don't want to allow multiple vertices with same label.... [Why?]
- We'll use the Map Interface [Chapter 15]
  - Maps require a unique key for each entry

Digression : Map Interface

- Maps *unique* keys to values (V is value not vertex!!!)
- Methods for Map<K, V>
  - int size() - returns number of entries in map
  - boolean isEmpty() - true iff there are no entries
  - boolean containsKey(K key) - true iff key exists in map
  - boolean containsValue(V val) - true iff val exists at least once in map
  - V get(K key) - get value associated with key
  - V put(K key, V val) - insert mapping from key to val, returns value replaced (old value) or null
  - V remove(K key) - remove mapping from key to val
  - void clear() - remove all entries from map
- We'll study this more in a week or so....

Implementing the Matrix Model

- Abstract class – partially implements Graph

```
public abstract class GraphMatrix<V,E> implements Graph<V,E>
```

- This class will implement features common to directed and undirected graphs
- Instance variables

```
 protected int size;  //max size of matrix
 protected Object data[][];  //matrix of edges
 protected Map<V, GMV<V>> dict; //labels -> vertices
 // This is structure5.Map, NOT java.util.Map!
 protected List<Integer> freeList; //avail indices
 protected boolean directed;
```

GraphMatrix Constructor
(Yes, abstract classes can have constructors!)

```
protected GraphMatrix(int size, boolean dir) {
    this.size = size; // set maximum size
    directed = dir; // fix direction of edges

    // the following constructs a size x size matrix
    // (the “Objects” will be “Edges”)
    // (can’t use generics with arrays!)
    data = new Object[size][size];

    // label→index translation table
    dict = new Hashtable<V,GraphMatrixVertex<V>>(size);

    // put all indices in the free list
    freeList = new SinglyLinkedList<Integer>();
    for (int row = size-1; row >= 0; row--)
        freeList.add(new Integer(row));
}
```

GraphMatrix add()

```
 public void add(V label) {
      // if there already, do nothing
      if (dict.containsKey(label)) return;

      Assert.pre(!freeList.isEmpty(), "Matrix not full");
      // allocate a free row and column
      int row = freeList.removeFirst().intValue();
      // add vertex to dictionary
      dict.put(label, new GraphMatrixVertex<V>(label, row));
}
```

GraphMatrix remove()

```
public V remove(V label) {
       // find and extract vertex
       GraphMatrixVertex<V> vert;
       vert = dict.remove(label);
       if (vert == null) return null;
       // remove vertex from matrix
       int index = vert.index();
       // clear row and column entries
       for (int row=0; row<size; row++) {
           data[row][index] = null;
           data[index][row] = null;
       }
       // add node index to free list
       freeList.add(new Integer(index));
       return vert.label();
}
```

Neighbors Iterator : GraphMatrix

neighbors Iterator

```
public Iterator<V> neighbors(V label) {
      GraphMatrixVertex<V> vert = dict.get(label);
      List<V> list = new SinglyLinkedList<V>();
      for (int row=size-1; row>=0; row--) {
            Edge<V,E> e = (Edge<V,E>)data[vert.index()][row];
            if (e != null)
                  if (e.here().equals(vert.label()))
                        list.add(e.there());
                        else list.add(e.here());
      }
      return list.iterator();
   }
```

GraphMatrixDirected

- Completes the implementation of GraphMatrix to ensure graph is directed
- GraphMatrixUndirected is very similar…
- How do we implement GraphMatrixDirected?
  - We'll discuss some methods
  - Read Ch 16 for complete details…

GraphMatrixDirected

- Constructor

```
public GraphMatrixDirected(int size) {
   // pre: size > 0
   // post: constructs an empty graph that may be
   //       expanded to at most size vertices. Graph
   //       is directed if dir true and undirected
   //       otherwise

   // call GraphMatrix constructor
   super(size,true);
}
```

GraphMatrixDirected

- addEdge

```
// pre: vLabel1 and vLabel2 are labels of existing vertices
public void addEdge(V vLabel1, V vLabel2, E label) {
   GraphMatrixVertex<V> vtx1,vtx2;
   vtx1 = dict.get(vLabel1);
   vtx2 = dict.get(vLabel2);
   Edge<V,E> e = new Edge<V,E>(vtx1.label(), vtx2.label(),
                               label, true);
   data[vtx1.index()][vtx2.index()] = e;
}
```

GraphMatrixDirected

- removeEdge

```
// pre: vLabel1 and vLabel2 are labels of existing vertices
public E removeEdge(V vLabel1, Vlabel2) {
    // get indices
   int row = dict.get(vLabel1).index();
   int col = dict.get(vLabel2).index();
   // cache old value
   Edge<V,E> e = (Edge<V,E>)data[row][col];
   // update matrix
   data[row][col] = null;
   if (e == null) return null;
   else return e.label(); // return old value
}
```

GraphMatrix Efficiency

- Assume Map operations are O(1) (for now)
  - |E| = number of edges
  - |V| = number of vertices
- Runtime of add, addEdge, getEdge, removeEdge, remove?
- Space usage?
- Conclusions
  - Matrix is good for dense graphs
  - Have to commit to maximum # of vertices in advance

Efficiency : Assuming Fast Map

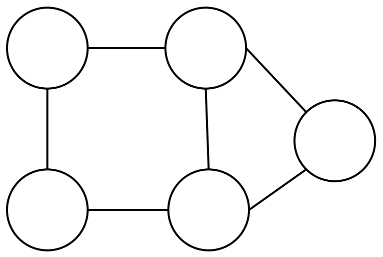
	GraphMatrix
add	O(1)
addEdge	O(1)
getEdge	O(1)
removeEdge	O(1)
remove	O(|V|)
space	$O(\|V\|^2)$

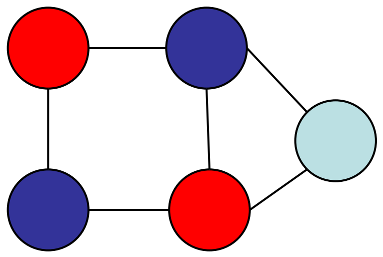
Lab 11 Overview:
Graph Algorithms using structure5

Greedy Algorithms

- A *greedy algorithm* attempts to find a globally optimum solution to a problem by making locally optimum (greedy) choices
- Example: Walking in Manhattan
- Example: Graph Coloring
  - A *(proper) coloring* of a graph `G=(V,E)` is an assignment of a value (color) to each vertex so that adjacent vertices get different values (colors)
  - Typically one strives to minimize the number of colors used

Graph Coloring Example





Greedy Coloring : Math

Here's a greedy coloring algorithm

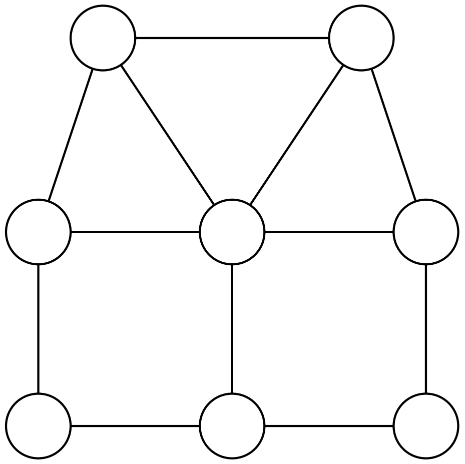
```
Build a collection $C = \{C_1, \ldots, C_k\}$ of sets of vertices
$i = 0$;  $C_i = \{\}$  // empty set
while G is has more vertices
    for each vertex u in G
        if u is not adjacent to any vertex of $C_i$
            remove u from G and add u to $C_i$
    add $C_i$ to C
    i++;
Return C as the coloring
```

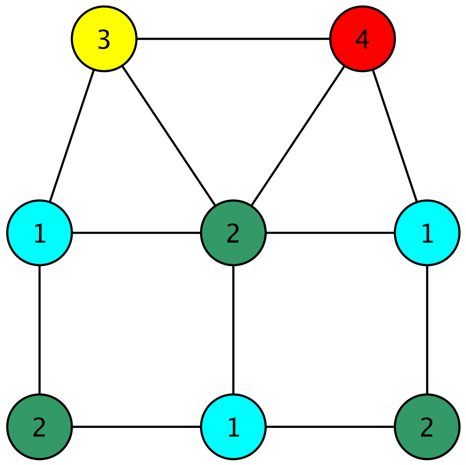
Greedy Coloring : CS

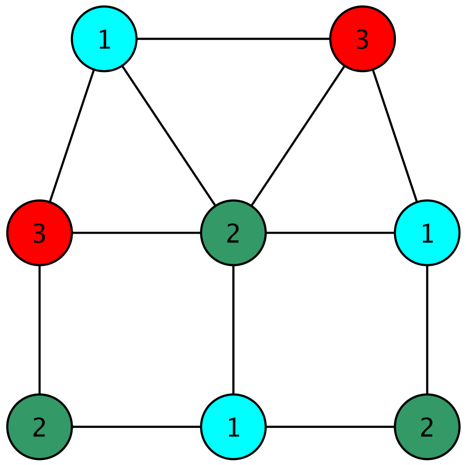
Here's a greedy coloring algorithm

```
Create a structure C to hold a collection of lists
while G is not empty
    pick a vertex v in G; create an empty list L; add v to L
    for each vertex $u \neq v$ in G
        if u is not adjacent to any vertex of L
            add u to L
    remove all vertices of L from G
    add L to C
Return C as the coloring
```

Greedy Coloring







Greedy Coloring

Some observations

- Each list (color class) L is a set of vertices no two of which are adjacent (an *independent set*)
- Each color class is maximal: cannot be made any larger
  - The hope is that this results in fewer colors being needed
  - But the solution is not always optimum!
  - This is a *very hard problem*
- The coloring problem is the same as finding a *partition* of the vertex set into independent sets
  - Partition means union of disjoint sets

Lab 11 : Exam Scheduling

Find a schedule (set of time slots) for exams so that

- No student has two exams in the same slot
- Every course is in a slot
- The number of slots is as small as possible

This is just the graph coloring problem in disguise!

- Each course is a vertex
- Two vertices are adjacent if the courses share students
- A slot must be an independent set of vertices (that is, a color class)

Lab 11 Notes: Using Graphs

- Create a new graph in structure5
  - GraphListDirected, GraphListUndirected,
  - GraphMatrixDirected, GraphMatrixUndirected

- Graph<V,E> conflictGraph = new GraphListUndirected<V,E>();

Lab 11 : Useful Graph Methods

- `void add(V label)`
  - add vertex to graph
- `void addEdge(V vtx1, V vtx2, E label)`
  - add edge between vtx1 and vtx2
- `Iterator<V> neighbors(V vtx1)`
  - Get iterator for all neighbors to vtx1
- `boolean isEmpty()`
  - Returns true iff graph is empty
- `Iterator<V> iterator()`
  - Get vertex iterator
- `V remove(V label)`
  - Remove a vertex from the graph
- `E removeEdge(V vLabel1, V vLabel2)`
  - Remove an edge from graph