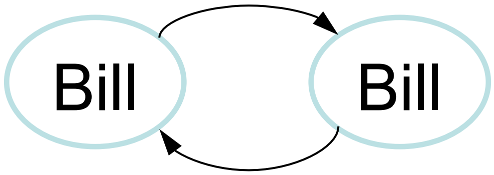
CSCI 136
Data Structures & Advanced Programming

Lecture 28

Fall 2017

Instructors:



Bill

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Last Time

- More on Graphs
  - Applications and Problems
    - Testing connectedness
    - Counting connected components
    - Breadth-first
    - Depth-first search
      - And recursive depth-first search
  - Directed Graphs : Introduction

Today

- Graph Data Structures: Implementation
  - Using the Graph Interface
  - Implementing the Graph Interface
    - Adjacency Array
    - Adjacency List

Implementing Graphs

- Involves a number of implementation decisions, depending on intended uses
    - What kinds of graphs will be availabe?
        - Undirected, directed, mixed
    - What underlying data structures will be used?
    - What functionality will be provided
    - What aspects will be public/protected/private
- We'll focus on popular implementations for undirected and directed graphs (separately)

Graphs in structure5

- We want to store information at vertices and at edges, but we favor vertices
  - Let V and E represent the types of information held by vertices and edges respectively
  - Interface Graph<V,E> extends Structure<V>
    - Vertices are the building blocks; edges depend on them
- Type V holds a *label* for a (hidden) vertex
- Type E holds a *label* for an (available) edge
  - label: Application-specific data for a vertex/edge

Graphs in structure5

- So, the methods described in the Structure interface are about vertices (but also impact edges: e.g., `clear()`)
- We'll want to add a number of similar methods to provide information about edges, and the graph itself

Recall: Desired Functionality

- What are the basic operations we need in order to describe algorithms on graphs?
  - Given vertices `u` and `v`: are they adjacent?
  - Given vertex `v` and edge `e`, are they incident?
  - Given an edge `e`, get its incident vertices (*ends*)
  - How many vertices are adjacent to `v`? (`deg(v)`)
    - The vertices adjacent to `v` are called its *neighbors*
  - Get a list of the neighbors of `v` (or the edges incident with `v`)

Graph Interface Methods

- void add(V vLabel), V remove(V vLabel)
  - Add/remove vertex to graph
- void addEdge(V vLabel1, V vLabel2, E edgeLabel),

  E removeEdge(V vLabel1, V vLabel2)
  - Add/remove edge between vLabel1 and vLabel2
- boolean containsEdge(V vLabel1, V vLabel2)
  - Returns true iff there is an edge between vLabel1 and vLabel2
- Edge<V,E> getEdge(V vLabel1, V vLabel2)
  - Returns edge between vLabel1 and vLabel2
- void clear()
  - Remove all nodes (and edges) from graph

Graph Interface Methods

- boolean visit(V vLabel)
  - Mark vertex as "visited" and return *previous* value of visited flag
- boolean visitEdge(Edge<V,E> e)
  - Mark edge as "visited"
- boolean isVisited(V vLabel), boolean isVisitedEdge(Edge<V,E> e)
  - Returns true iff vertex/edge has been visited
- Iterator<V> neighbors(V vLabel)
  - Get iterator for all neighbors of vLabel
  - For directed graphs, out-edges only
- Iterator<V> iterator()
  - Get vertex iterator
- void reset()
  - Remove visited flags for all nodes/edges

Edge Class

- Graph *edges* are defined in their own public class

  -
    ```
    Edge<V,E>(V vLabel1, V vLabel2,
                   E label, boolean directed)
    ```
  - Construct a (possibly directed) edge between two labeled vertices (`vLabel1` → `vLabel2`)
  - vLabel1 : here; vLabel2 : there
- Useful methods (getters and setters):

```
label(), here(), there()
setLabel(), isVisited(), isDirected()
```

Reachability: Breadth-First Search

```
BFS(G, v)        // Do a breadth-first search of G starting at v
// pre: all vertices are marked as unvisited
// post: return number of visited vertices
count ←0;
Create empty queue Q;
add v to Q, mark v as visited, add 'v' to count
While Q isn't empty
        current ←Q.dequeue();
        for each unvisited neighbor u  of current :
                add u to Q, mark u as visited, add 'u' to count
return count;
```

How does this translate to code?

Breadth-First Search

```
int BFS(Graph<V,E> g, V src) {
  int count = 0; Queue<V> todo = new QueueList<V>();
  todo.enqueue(src);
  g.visit(src); count++;
  while (!todo.isEmpty()) {
    V vertex = todo.dequeue();
    Iterator<V> neighbors = g.neighbors(vertex);
    while (neighbors.hasNext()) {
       V next = neighbors.next();
       if (!g.isVisited(next)) {
          todo.enqueue(next);
          g.visit(next); count++;
       }
    }
  }
  return count;
}
```

Breadth-First Search of Edges

```
int BFS(Graph<V,E> g, V src) {
  int count = 0; Queue<V> todo = new QueueList<V>();
  todo.enqueue(src);
  g.visit(src); count++;
  while (!todo.isEmpty()) {
    V vertex = todo.dequeue();
    Iterator<V> neighbors = g.neighbors(vertex);
    while (neighbors.hasNext()) {
       V next = neighbors.next();
       if (!g.isVisitedEdge(vertex, next))
              g.visitEdge(vertex, next);
       if (!g.isVisited(next)) {
          todo.enqueue(next);
          g.visit(next); count++;
       }
    }
  }
  return count;
}
```

Recursive Depth-First Search

```
// Before first call to DFS, set all vertices to unvisited
//Then call DFS(G,v)
DFS(G, v)
    Mark v as visited; count=1;
    for each unvisited neighbor u of v:
        count += DFS(G,u);
    return count;
```

How does this translate to code?

Recursive Depth-First Search

```
int depthFirstSearch(Graph<V,E> g, V src) {
    g.visit(src);
    int count = 1;
    Iterator<V> neighbors = g.neighbors(src);
    while (neighbors.hasNext()) {
       V next = neighbors.next();
       if (!g.isVisited(next))
              count += depthFirstSearch(g, next);
    }
  }
  return count;
}
```

Beyond the API

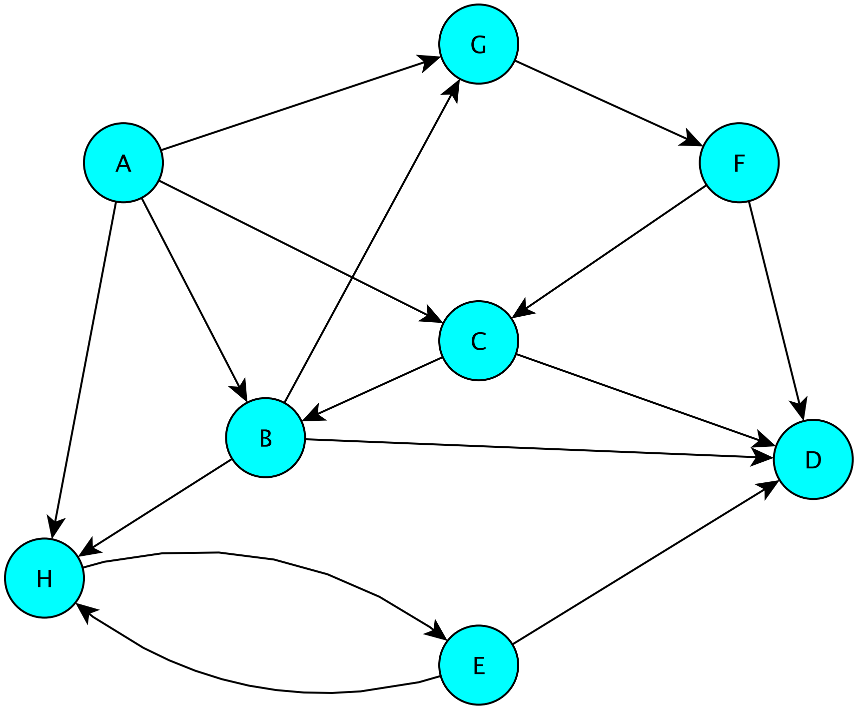
- So far we have *used* the structure5 graph interface methods in graph traversal algorithms
- How would we design classes that *implement* the interface?
  - What data structures should store the vertices?
  - What data structures should store the edges?

Representing Graphs

- Two standard approaches
  - Option 1: Array-based (directed and undirected)
  - Option 2: List-based (directed and undirected)
- We'll look at both
  - Array-based graphs store the edge information in a 2-dimensional array indexed by the vertices
  - List-based graphs store the edge information in a (1-dimensional) array of lists
    - The array is indexed by the vertices
    - Each array element is a list of edges incident with that vertex

Adjacency Array: Directed Graph

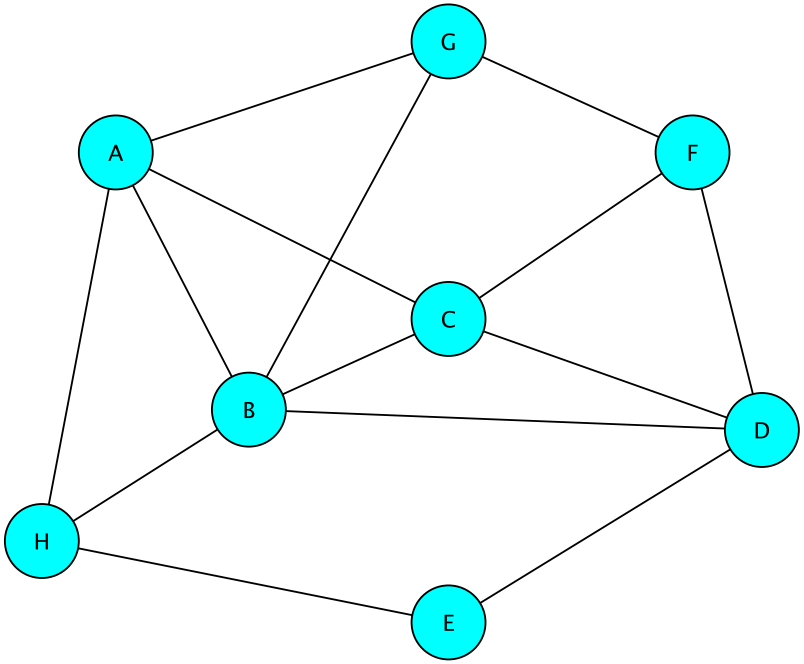
	A	B	C	D	E	F	G	H
A	0	1	1	0	0	0	1	1
B	0	0	0	1	0	0	1	1
C	0	1	0	1	0	0	0	0
D	0	0	0	0	0	0	0	0
E	0	0	0	1	0	0	0	1
F	0	0	1	1	0	0	0	0
G	0	0	0	0	0	1	0	0
H	0	0	0	0	1	0	0	0



Entry (i,j) stores 1 if there is an edge from i to j; 0 otherwise
E.G.: edges(B,C) = 1 but edges(C,B) = 0

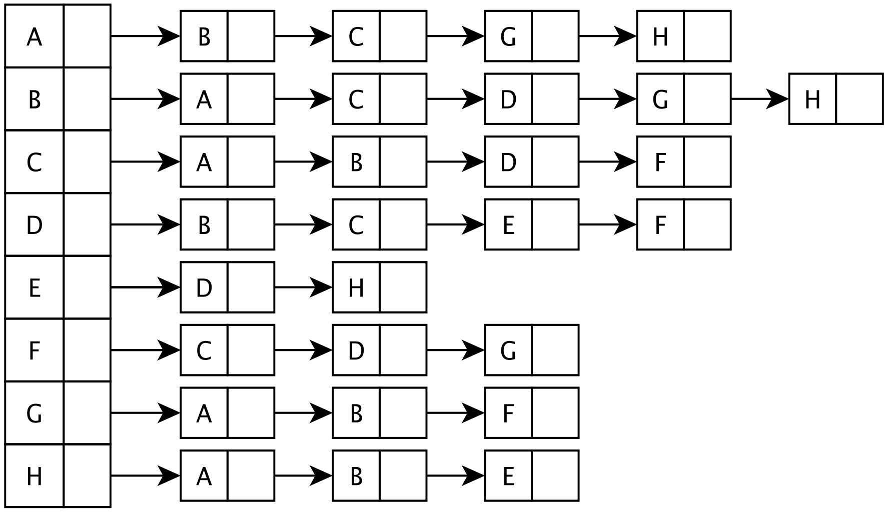
Adjacency Array: Undirected Graph

	A	B	C	D	E	F	G	H
A	0	1	1	0	0	0	1	1
B	1	0	1	1	0	0	1	1
C	1	1	0	1	0	1	0	0
D	0	1	1	0	1	1	0	0
E	0	0	0	1	0	0	0	1
F	0	0	1	1	0	0	1	0
G	1	1	0	0	0	1	0	0
H	1	1	0	0	1	0	0	0

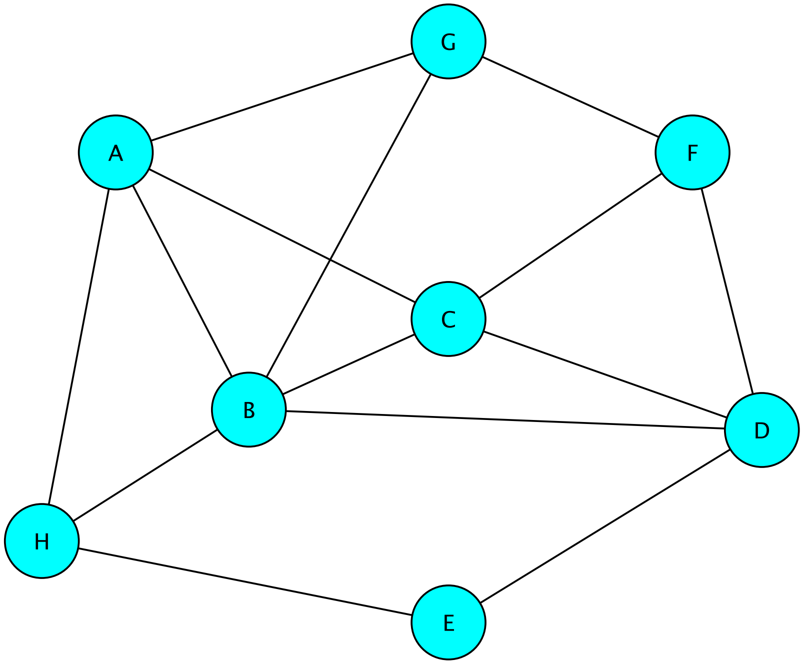


Entry (i,j) store 1 if there is an edge between i and j; else 0
E.G.: edges(B,C) = 1 = edges(C,B)

Adjacency List : Undirected Graph

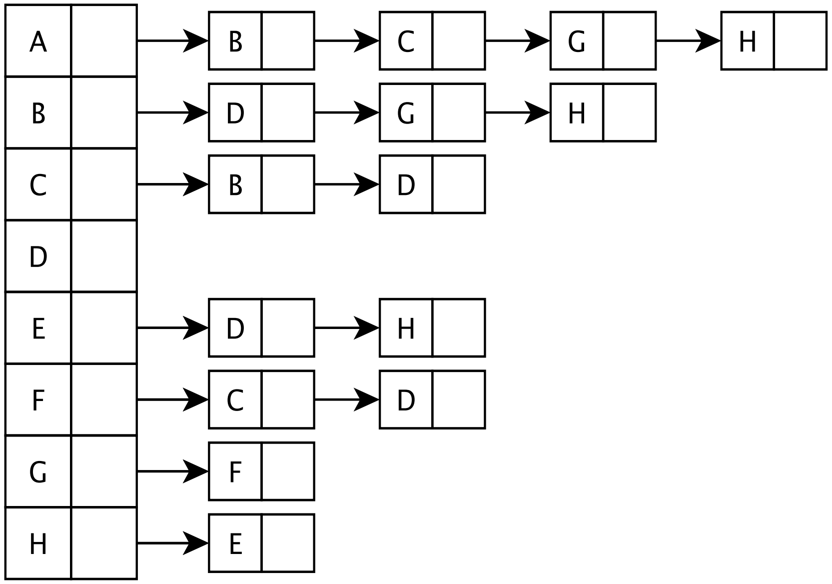


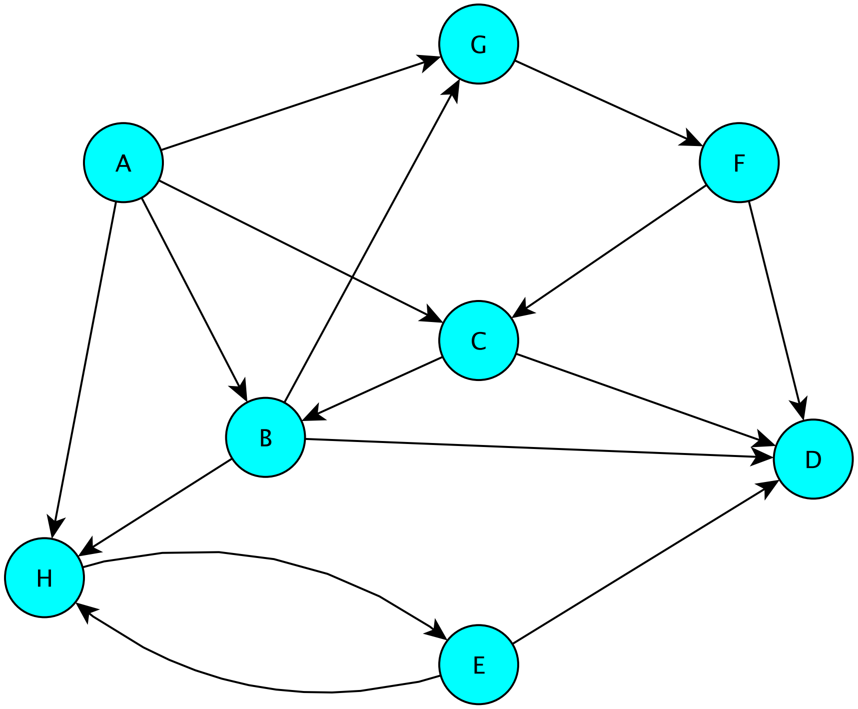
A → B → C → G → H
B → A → C → D → G → H
C → A → B → D → F
D → B → C → E → F
E → D → H
F → C → D → G
G → A → B → F
H → A → B → E



The vertices are stored in an array V[]
V[] contains a linked list of edges incident to a given vertex

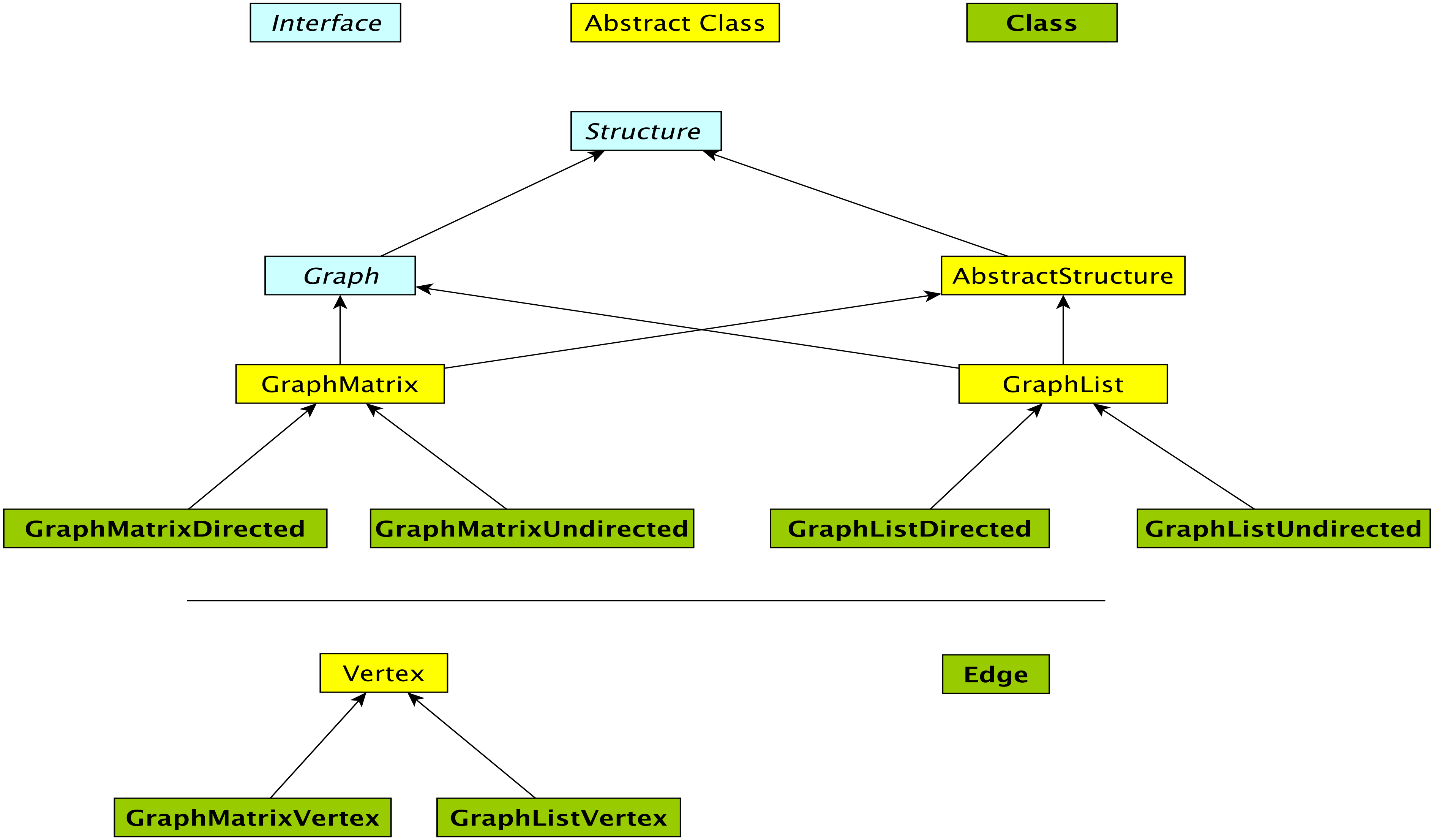
Adjacency List : Directed Graph





The vertices are stored in an array V[]
V[] contains a linked list of edges having a given source

Graph Classes in structure5



Graph Classes in structure5

Why so many?!

- There are two types of graphs: undirected & directed
- There are two implementations: arrays and lists
- We want to be able to avoid large amounts of identical code in multiple classes
- We abstract out features of implementation common to both directed and undirected graphs

We'll tackle array-based graphs first....