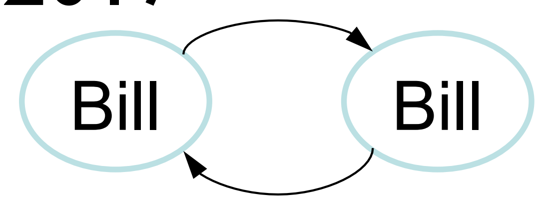
CSCI 136
Data Structures & Advanced Programming

Lecture 28

Fall 2017

Instructors:



Announcements

- Dzung will be moving his TA hours from 2-4pm Saturday to 2-4pm Sunday this week.

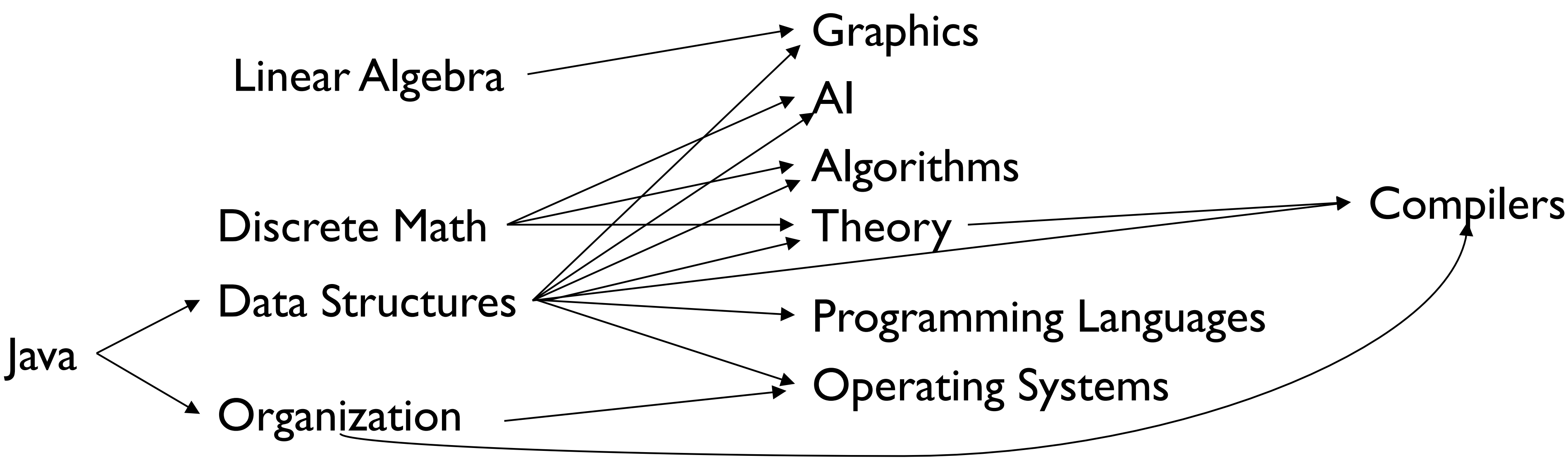
Last Time

- More on Graphs
  - Applications and Problems
    - Testing connectedness
    - Counting connected components
    - Breadth-first search
    - Depth-first search
      - And recursive depth-first search
  - Directed Graphs : Introduction

Today's Outline

- Directed Graphs
  - Definition and Properties
  - Reachability and (Strong) Connectedness
- Graph Data Structures: Implementation
  - Graph Interface
  - Adjacency Array Implementation Basic Concepts
  - Adjacency List Implementation Basic Concepts
  - Adjacency Array Implementation Details

Directed Graphs

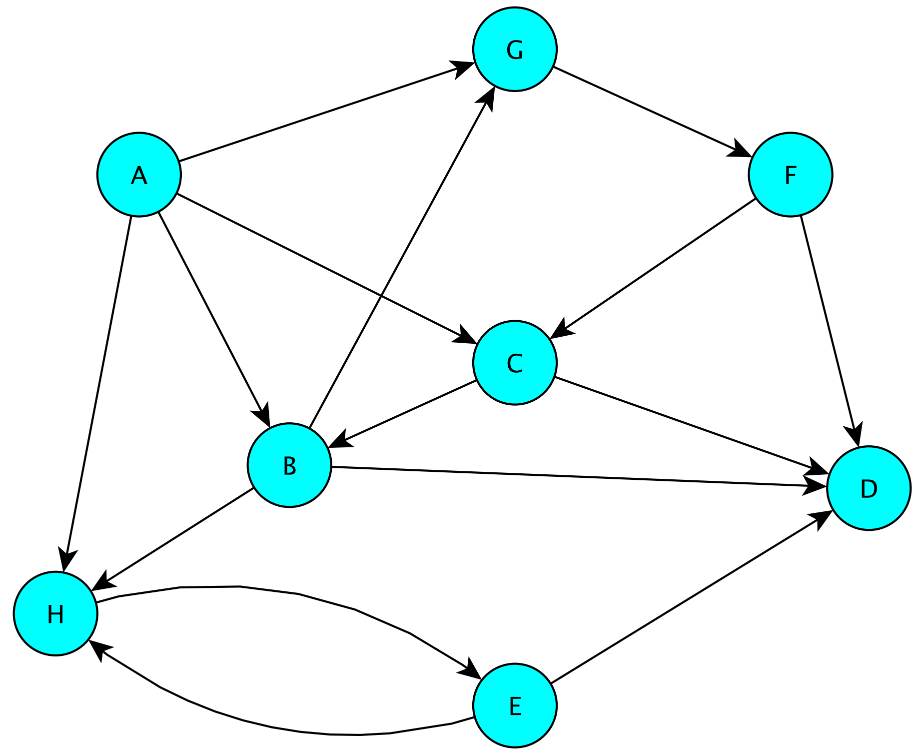


Def'n: In a *directed graph G = (V,E)*, each edge e in E is an *ordered* pair: e = (u,v) vertices: its *incident vertices*. The *source* of e is u; the *destination/target* is v.

Note: (u,v) ≠ (v,u)

Directed Graphs

- The (out) neighbors of B are D, G, H: B has out-degree 3
- The in neighbors of B are A, C: B has in-degree 2
- A has in-degree 0: it is a *source* in G; D has out-degree 0: it is a *sink* in G



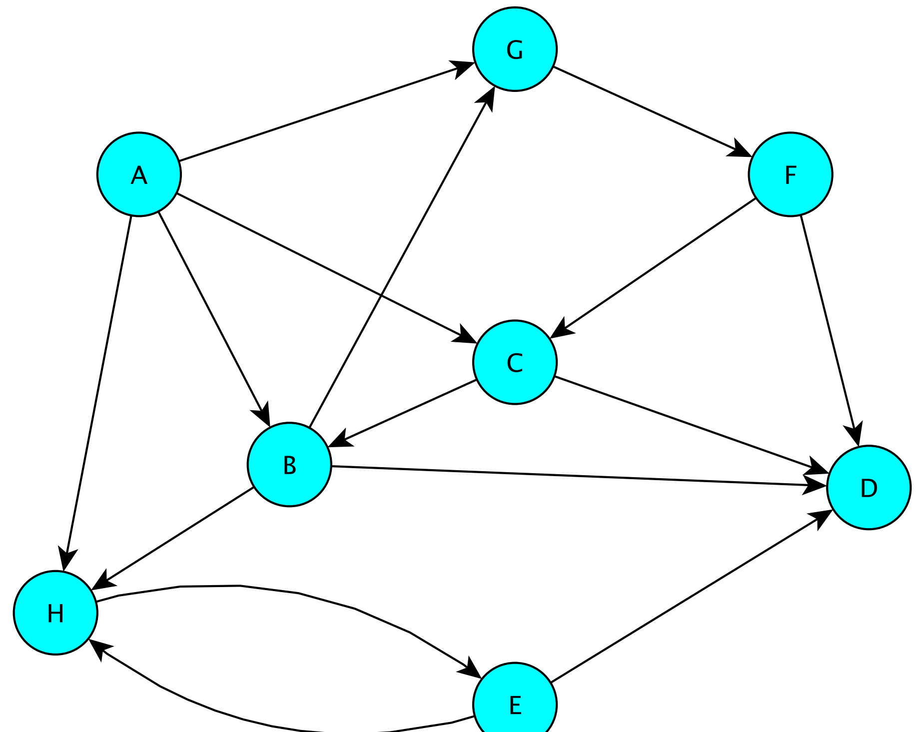
A walk is still an alternating sequence of vertices and edges

$$u = v_0, e_1, v_1, e_2, v_2, \ldots, v_{k-1}, e_k, v_k = v$$

but now $e_i = (v_{i-1}, v_i)$: all edges *point along direction* of walk

Directed Graphs

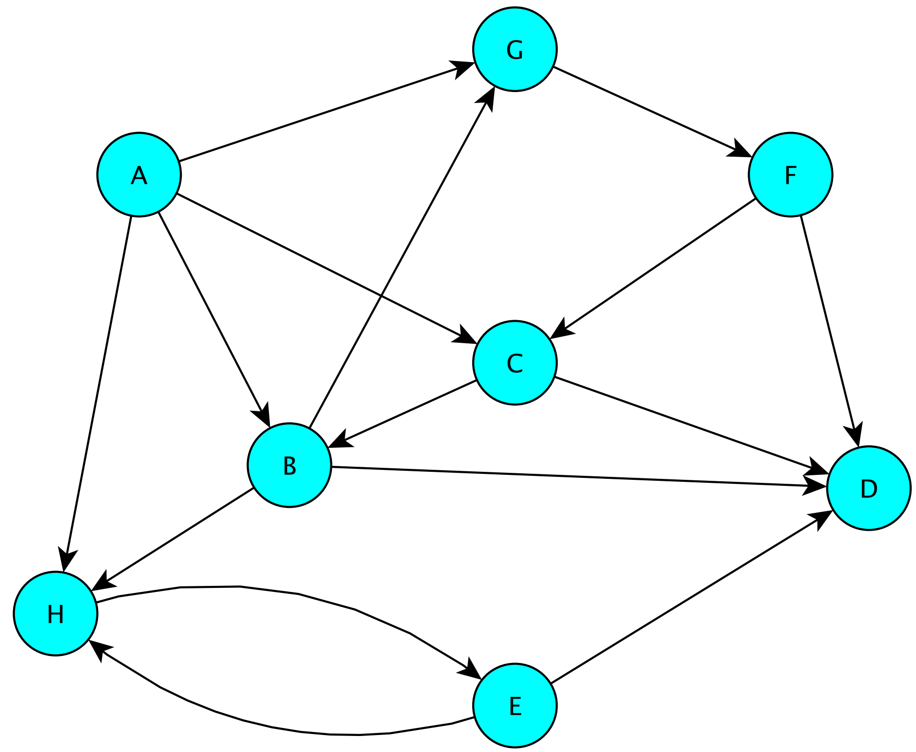
- A, B, H, E, D is a walk from A to D
- It's also a (simple) path
- D, E, H, B, A is *not* a walk from D to A
- B, G, F, C, B is a (directed) cycle (it's a 4-cycle)
- So is H, E, H (a 2-cycle)



- D is reachable from A (via path A, B, D), but A is not reachable from D
- In fact, every vertex is reachable from A

Directed Graphs

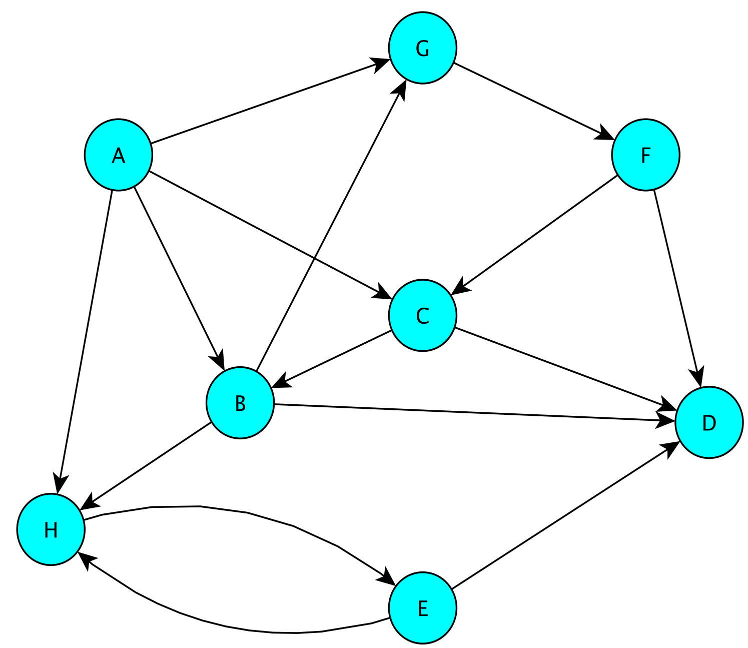
- A BFS of *G* from A visits every vertex
- A BFS of *G* from F visits all vertices but A
- A BFS of *G* from E visits only E, H, D

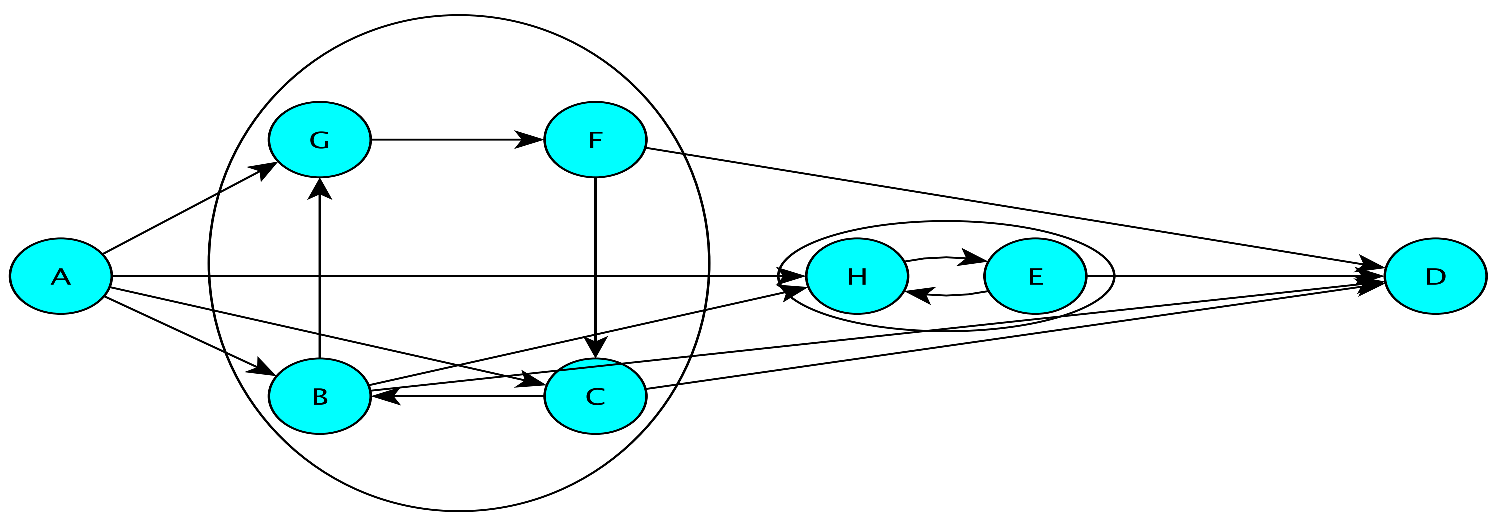


- Connectivity in directed graphs is more subtle than in undirected graphs!

Directed Graphs

- Vertices u and v are *mutually reachable* vertices if there are paths from u to v and v to u
- *Maximal* sets of mutually reachable vertices form *the strongly connected components* of G





Implementing Graphs

- Involves a number of implementation decisions, depending on intended uses
  - What kinds of graphs will be availabe?
    - Undirected, directed, mixed
  - What underlying data structures will be used?
  - What functionality will be provided
  - What aspects will be public/protected/private
- We'll focus on popular implementations for undirected and directed graphs (separately)

Graphs in structure5

- We want to store information at vertices and at edges, but we favor vertices
  - Let V and E represent the types of information held by vertices and edges respectively
  - Interface Graph<V,E> extends Structure<V>
    - Vertices are the building blocks; edges depend on them
- Type V holds a *label* for a (hidden) vertex
- Type E holds a *label* for an (available) edge
  - Label: Application-specific data for a vertex/edge

Graphs in structure5

- The methods described in the Structure interface deal wih *vertices*
  - but also impact edges: e.g., clear()
- We'll want to add a number of similar methods to provide information about edges, and the graph itself

Recall: Desired Functionality

- What are the basic operations we need to describe algorithms on graphs?
  - Given vertices u and v: are they adjacent?
  - Given vertex v and edge e, are they incident?
  - Given an edge e, get its incident vertices (*ends*)
  - How many vertices are adjacent to v? (*degree* of v)
    - The vertices adjacent to v are called its *neighbors*
  - Get a list of the neighbors of v (or the edges incident with v)

Graph Interface Methods

- void add(V vLabel), V remove(V vLabel)
  - Add/remove vertex to graph
- void addEdge(V vLabel1, V vLabel2, E edgeLabel), E removeEdge(V vLabel1, V vLabel2)
  - Add/remove edge between vLabel1 and vLabel2
- boolean containsEdge(V vLabel1, V vLabel2)
  - Returns true iff there is an edge between vLabel1 and vLabel2
- Edge<V,E> getEdge(V vLabel1, V vLabel2)
  - Returns edge between vLabel1 and vLabel2
- void clear()
  - Remove all nodes (and edges) from graph

Graph Interface Methods

- boolean visit(V vLabel)
  - Mark vertex as "visited" and return *previous* value of visited flag
- boolean visitEdge(Edge<V,E> e)
  - Mark edge as "visited"
- boolean isVisited(V vLabel), boolean isVisitedEdge(Edge<V,E> e)
  - Returns true iff vertex/edge has been visited
- Iterator<V> neighbors(V vLabel)
  - Get iterator for all neighbors of vLabel
  - For directed graphs, out-edges only
- Iterator<V> iterator()
  - Get vertex iterator
- void reset()
  - Remove visited flags for all nodes/edges

Edge Class

- Graph *edges* are defined in their own public class
  - ```
    Edge<V,E>(    V vLabel1, V vLabel2,
           E label, boolean directed)
    ```
  - Construct a (possibly directed) edge between two labeled vertices (`vLabel1` → `vLabel2`)
  - vLabel1 : here; vLabel2 : there
- Useful methods:

```
label(), here(), there()
setLabel(), isVisited(), isDirected()
```
```

Reachability: Breadth-First Search

```
BFS(G, v)        // Do a breadth-first search of G starting at v
// pre: all vertices are marked as unvisited
// post: return number of visited vertices
count ← 0;
Create empty queue Q; enqueue v; mark v as visited; count++
While Q isn't empty
        current ← Q.dequeue();
        for each unvisited neighbor u  of current :
                add u to Q; mark u as visited; count++
return count;
```

Breadth-First Search

```
int BFS(Graph<V,E> g, V src) {
  Queue<V> todo = new QueueList<V>(); int count = 0;
  g.visit(src); count++;
  todo.enqueue(src);
  while (!todo.isEmpty()) {
    V node = todo.dequeue();
    Iterator<V> neighbors = g.neighbors(node);
    while (neighbors.hasNext()) {
       V next = neighbors.next();
       if (!g.isVisited(next)) {
          g.visit(next); count++;
          todo.enqueue(next);
       }
    }
  }
  return count;
}
```

Breadth-First Search of Edges

```
int BFS(Graph<V,E> g, V src) {
  Queue<V> todo = new QueueList<V>(); int count = 0;
  g.visit(src); count++;
  todo.enqueue(src);
  while (!todo.isEmpty()) {
    V node = todo.dequeue();
    Iterator<V> neighbors = g.neighbors(node);
    while (neighbors.hasNext()) {
       V next = neighbors.next();
       if (!g.isVisitedEdge(node,next)) g.visitEdge(next,node);
       if (!g.isVisited(next)) {
          g.visit(next); count++;
          todo.enqueue(next);
       }
    }
  }
  return count;
}
```

Recursive Depth-First Search

```
// Before first call to DFS, set all vertices to unvisited
//Then call DFS(G,v)
DFS(G, v)
        Mark v as visited; count=1;
        for each unvisited neighbor u of v:
                count += DFS(G,u);
        return count;
```

Recursive Depth-First Search

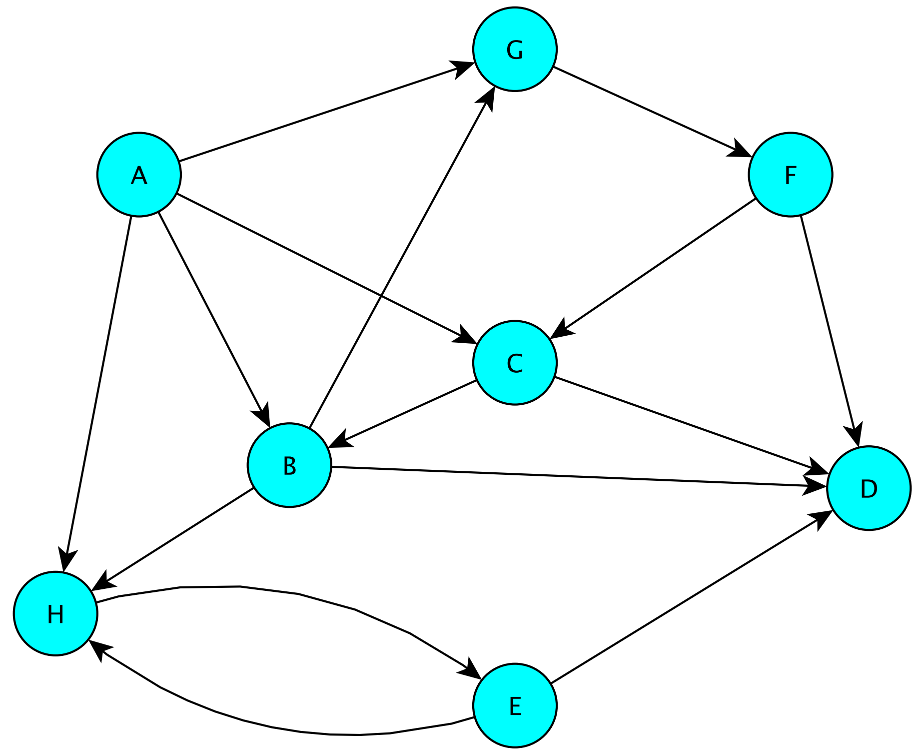
```
int DFS(Graph<V,E> g, V src) {
    g.visit(src);
    int count = 1;
    Iterator<V> neighbors = g.neighbors(src);
    while (neighbors.hasNext()) {
       V next = neighbors.next();
       if (!g.isVisited(next))
             count += DFS(g, next);
    }
  }
  return count;
}
```

Representing Graphs

- Two standard approaches
  - Option 1: Array-based (directed and undirected)
  - Option 2: List-based (directed and undirected)
- We'll look at both
  - Array-based graphs store the edge information in a 2-dimensional array indexed by the vertices
  - List-based graphs store the edge information in a (1-dimensional) array of lists
    - The array is indexed by the vertices
    - Each array element is a list of edges incident with that vertex

Adjacency Array: Directed Graph

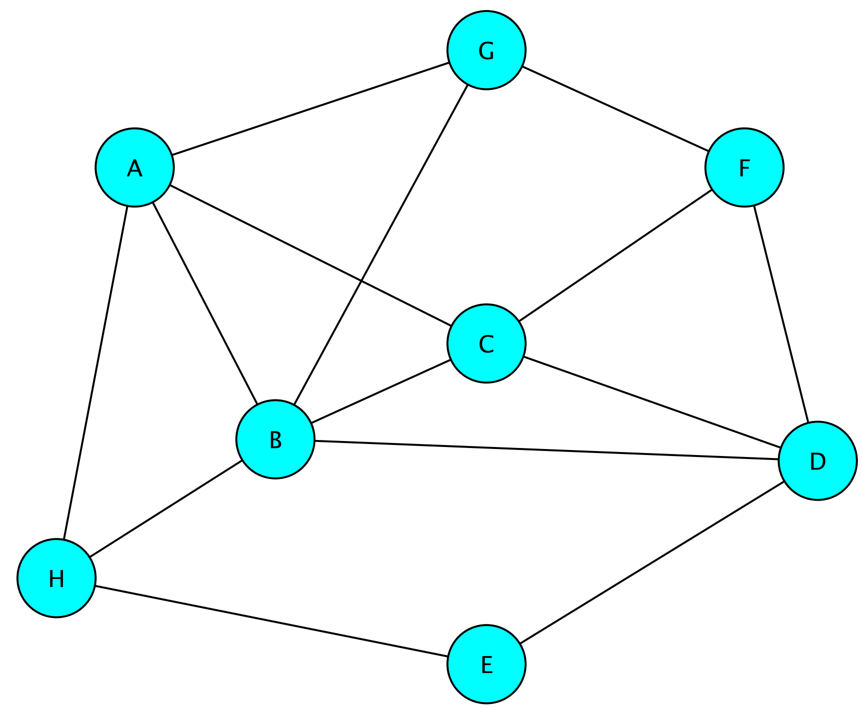
	A	B	C	D	E	F	G	H
A	0	1	1	0	0	0	1	1
B	0	0	0	1	0	0	1	1
C	0	1	0	1	0	0	0	0
D	0	0	0	0	0	0	0	0
E	0	0	0	1	0	0	0	1
F	0	0	1	1	0	0	0	0
G	0	0	0	0	0	1	0	0
H	0	0	0	0	1	0	0	0



Entry (i,j) stores 1 if there is an edge from i to j; 0 otherwise
E.G.: edges(B,C) = 1 but edges(C,B) = 0

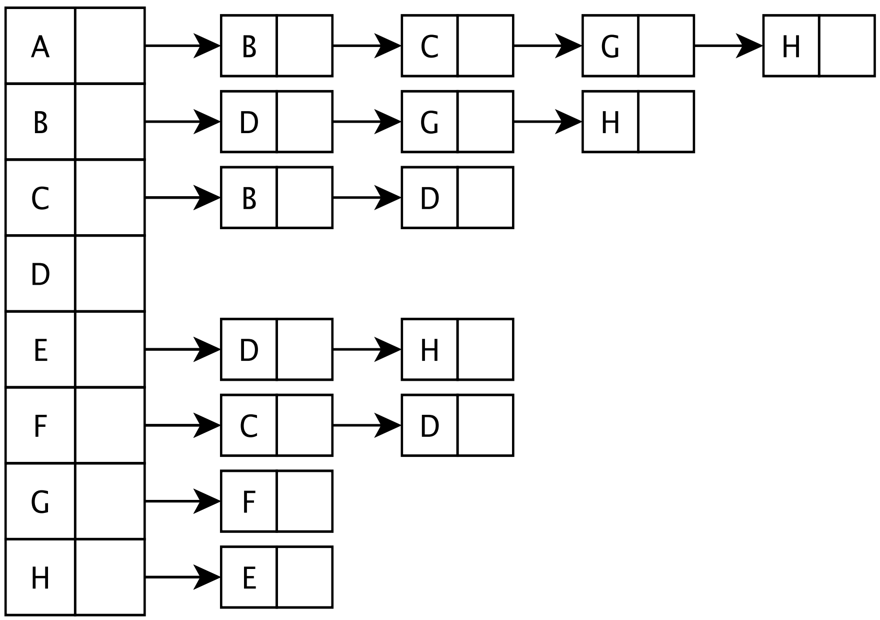
Adjacency Array: Undirected Graph

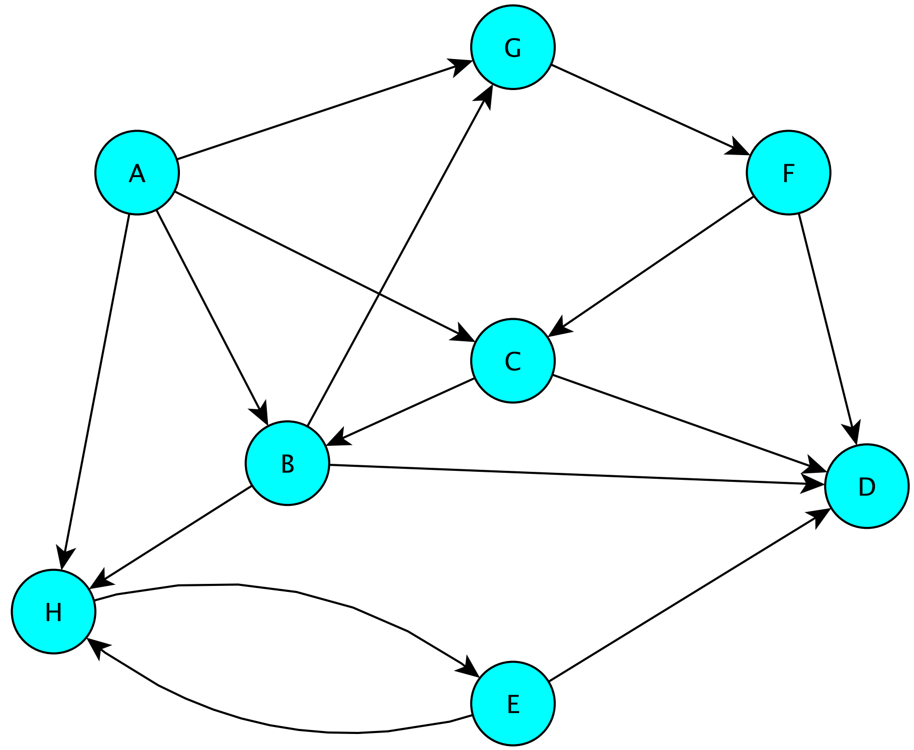
	A	B	C	D	E	F	G	H
A	0	1	1	0	0	0	1	1
B	1	0	1	1	0	0	1	1
C	1	1	0	1	0	1	0	0
D	0	1	1	0	1	1	0	0
E	0	0	0	1	0	0	0	1
F	0	0	1	1	0	0	1	0
G	1	1	0	0	0	1	0	0
H	1	1	0	0	1	0	0	0



Entry (i,j) store 1 if there is an edge between i and j; else 0
E.G.: edges(B,C) = 1 = edges(C,B)

Adjacency List : Directed Graph

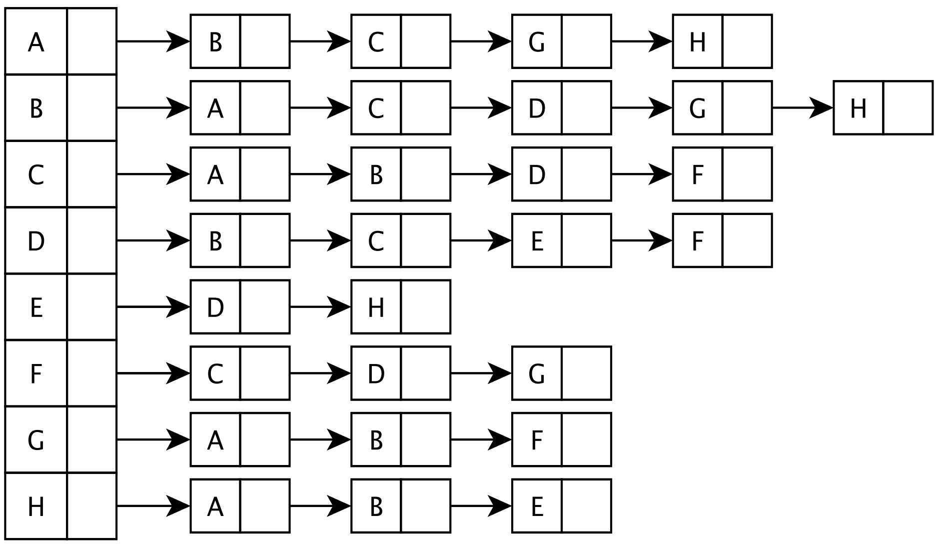


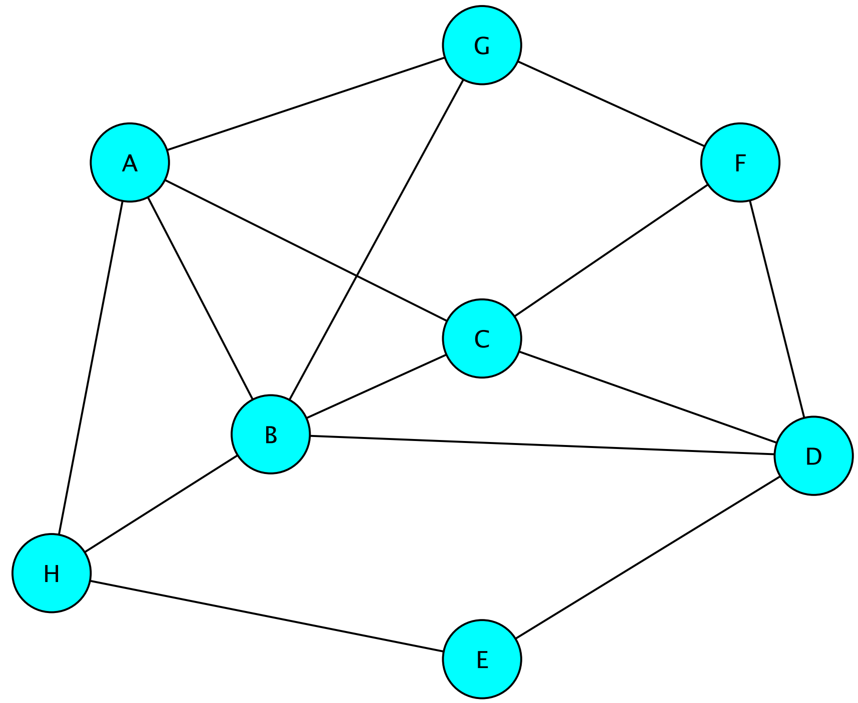


The vertices are stored in an array V[]

V[] contains a linked list of edges having a given source

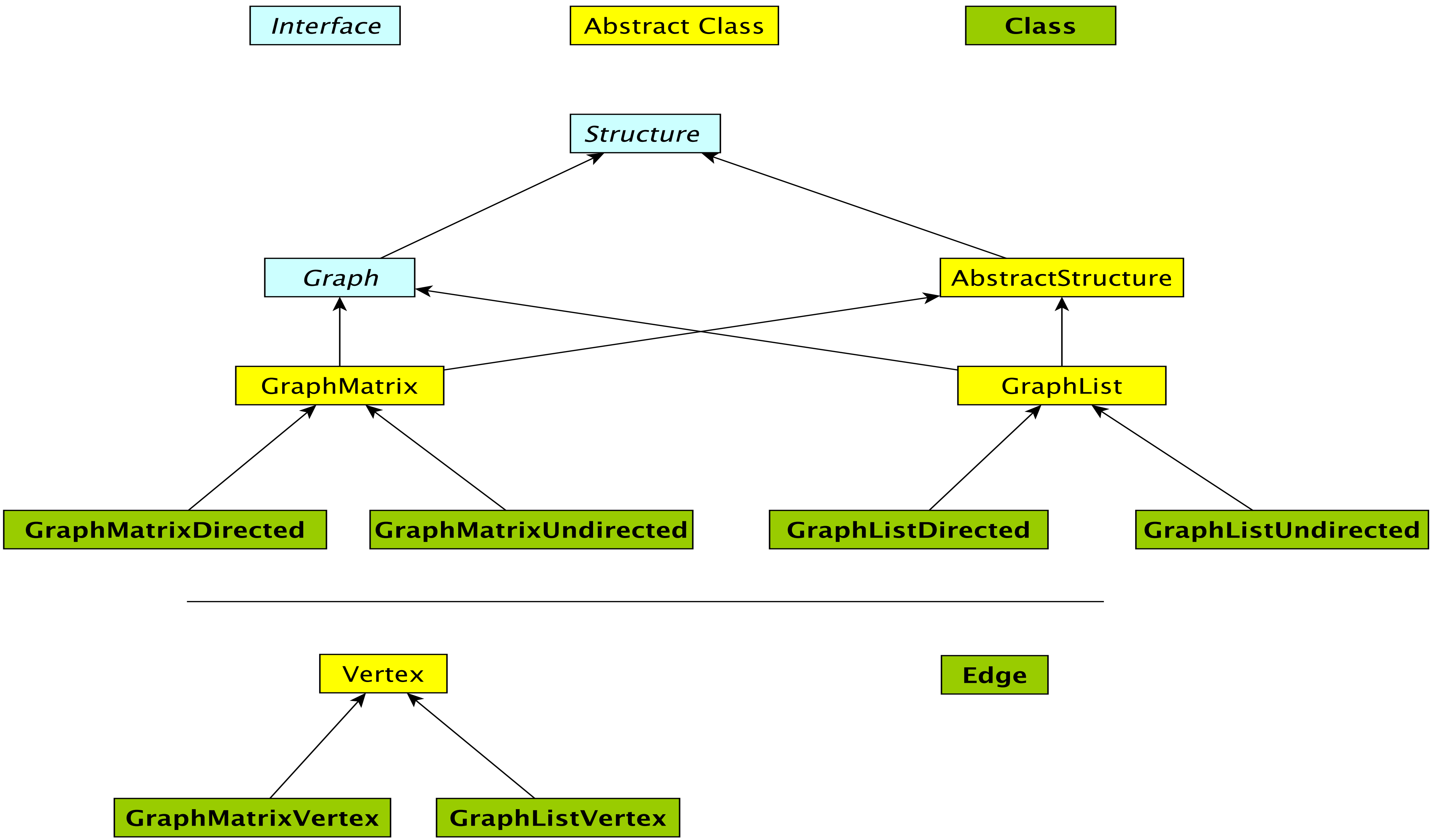
Adjacency List : Undirected Graph





The vertices are stored in an array V[]
V[] contains a linked list of edges incident to a given vertex

Graph Classes in structure5



Graph Classes in structure5

Why so many?!

- There are two types of graphs: undirected & directed
- There are two implementations: arrays and lists
- We want to be able to avoid large amounts of identical code in multiple classes
- We abstract out features of implementation common to both directed and undirected graphs

We'll tackle array-based graphs first....