CSCI 136
Data Structures &
Advanced Programming

Lecture 26

Fall 2017

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Administrative Details

- Lab 10: Two Towers is online
  - Individual lab again this week
- Final Exam location: TBL 112
  - It's on Dec. 14th, 9:30--noon

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Last Time

- Binary Seach Tree Implementation details
- *Balanced* Binary search trees
  - AVL Trees
    - Height is O(log n), so all operations are O(log n)
  - Red-Black Trees
    - Different height-balancing idea: height is O(log n)
    - All operations are O(log n)
  - Splay Trees
    - No guaranteed balance; good *amortized* performance
    - Any sequence of m operations take O(m log n) time

Today's Outline

Much less esoteric…

- Bit operations
  - Useful in general, but required for Lab 10
- Introduction To Graphs
  - Basic Definitions and Properties
  - Applications and Problems

Representing Numbers

- Humans usually think of numbers in base 10
- But even though we write `int x = 23;` the computer stores `x` as a sequence of `1`s and `0`s
- Recall Lab 3:

```
public static String printInBinary(int n) {
        if (n <= 1)
            return "" + n%2;

        return printInBinary(n/2)+n%2;
}
```

- 00000000 00000000 00000000 00010111

Bitwise Operations

- We can use *bitwise* operations to manipulate the `1`s and `0`s in the binary representation
  - Bitwise 'and': `&`
  - Bitwise 'or': `|`
- Also useful: bit shifts
  - Bit shift left: `<<`
  - Bit shift right: `>>`

& and |

- Given two integers `a` and `b`, the bitwise *or* expression `a | b` returns an integer s.t.
  - At each bit position, the result has a `1` if that bit position had a `1` in EITHER `a` OR `b`
  - `3 | 6 = ?`
- Given two integers `a` and `b`, the bitwise *and* expression `a & b` returns an integer s.t.
  - At each bit position, the result has a `1` if that bit position had a `1` in BOTH `a` AND `b`
  - `3 & 6 = ?`

>> and <<

- Given two integers `a` and `i`, the expression `(a << i)` returns `(a * 2`^{i}`)`
  - Why? It shifts all bits left by `i` positions
  - `1 << 4 = ?`
- Given two integers `a` and `i`, the expression `(a >> i)` returns `(a / 2`^{i}`)`
  - Why? It shifts all bits right by `i` positions
  - `1 >> 4 = ?`
  - `97 >> 3 = ?` `(97 = 1100001)`
- Be careful about shifting left and "overflow"!!!

Revisiting printInBinary(int n)

- How would we rewrite a recursive `printInBinary` using bit shifts and bitwise operations?

```
public static String printInBinary(int n) {
    if (n <= 1) {
        return "" + n;
    return printInBinary(n >> 1) + (n & 1);
}
```

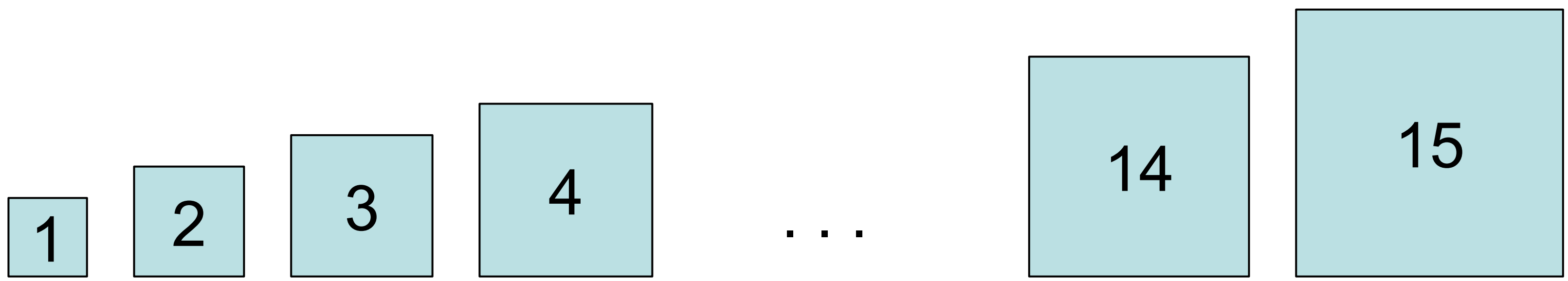
Revisiting printInBinary(int n)

- How would we write an iterative `printInBinary` using bit shifts and bitwise operations?

```
public static String printInBinary(int n,
                                   int width) {
    String result = "";
    for(int i = 0; i < width; i++)
        if ((n & (1<<i)) == 0)
            result = 0 + result;
        else
            result = 1 + result;
    return result;
}
```

Lab 8: Two Towers

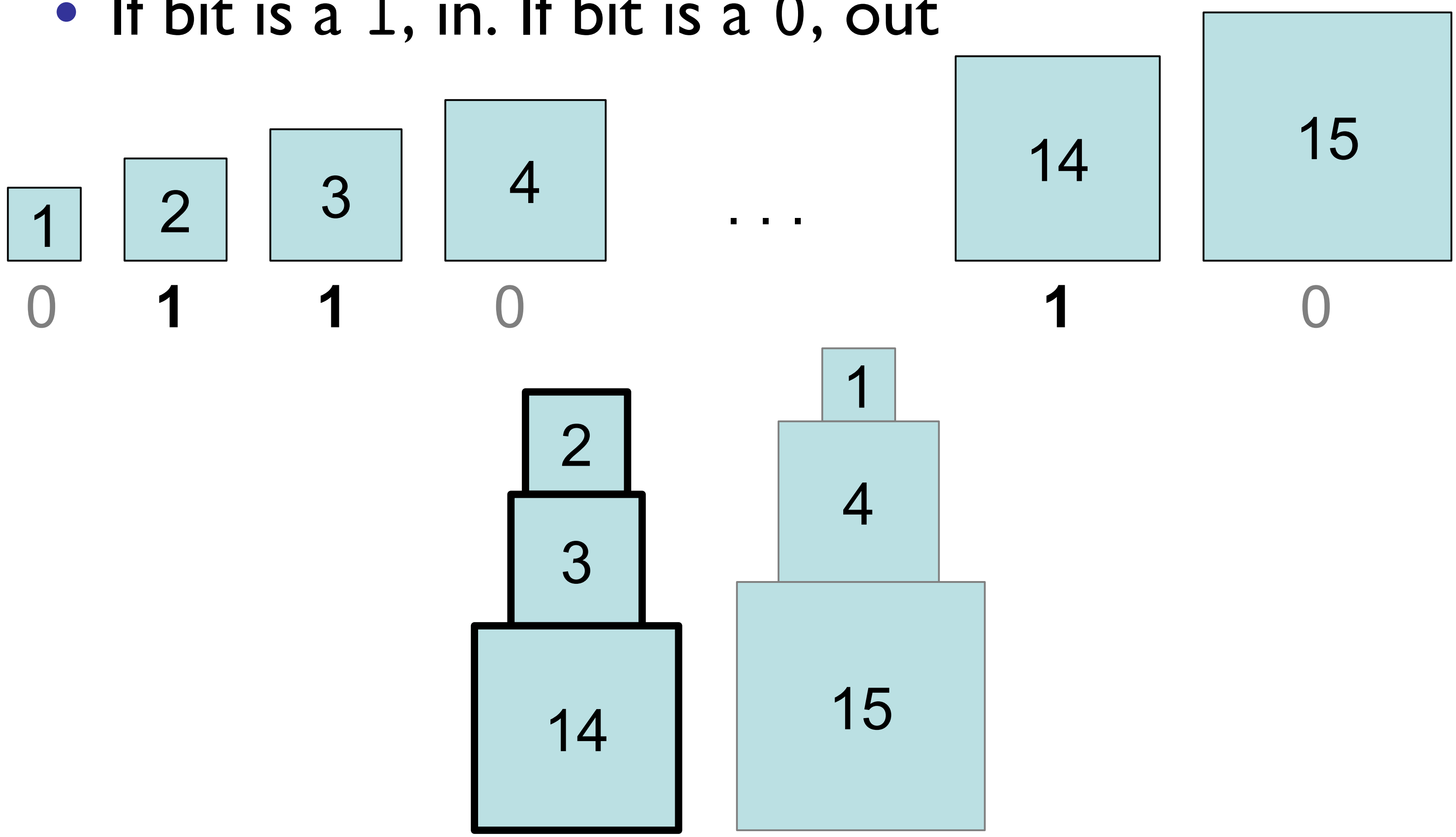
- Goal: given a set of blocks, iterate through all possible subsets to find the *best* set



- "Best" set produces the most balanced towers
- Strategy: create an iterator that uses the bits in a binary number to represent subsets

Lab 8: Two Towers

- A block can either be in the set or out
  - If bit is a 1, in. If bit is a 0, out



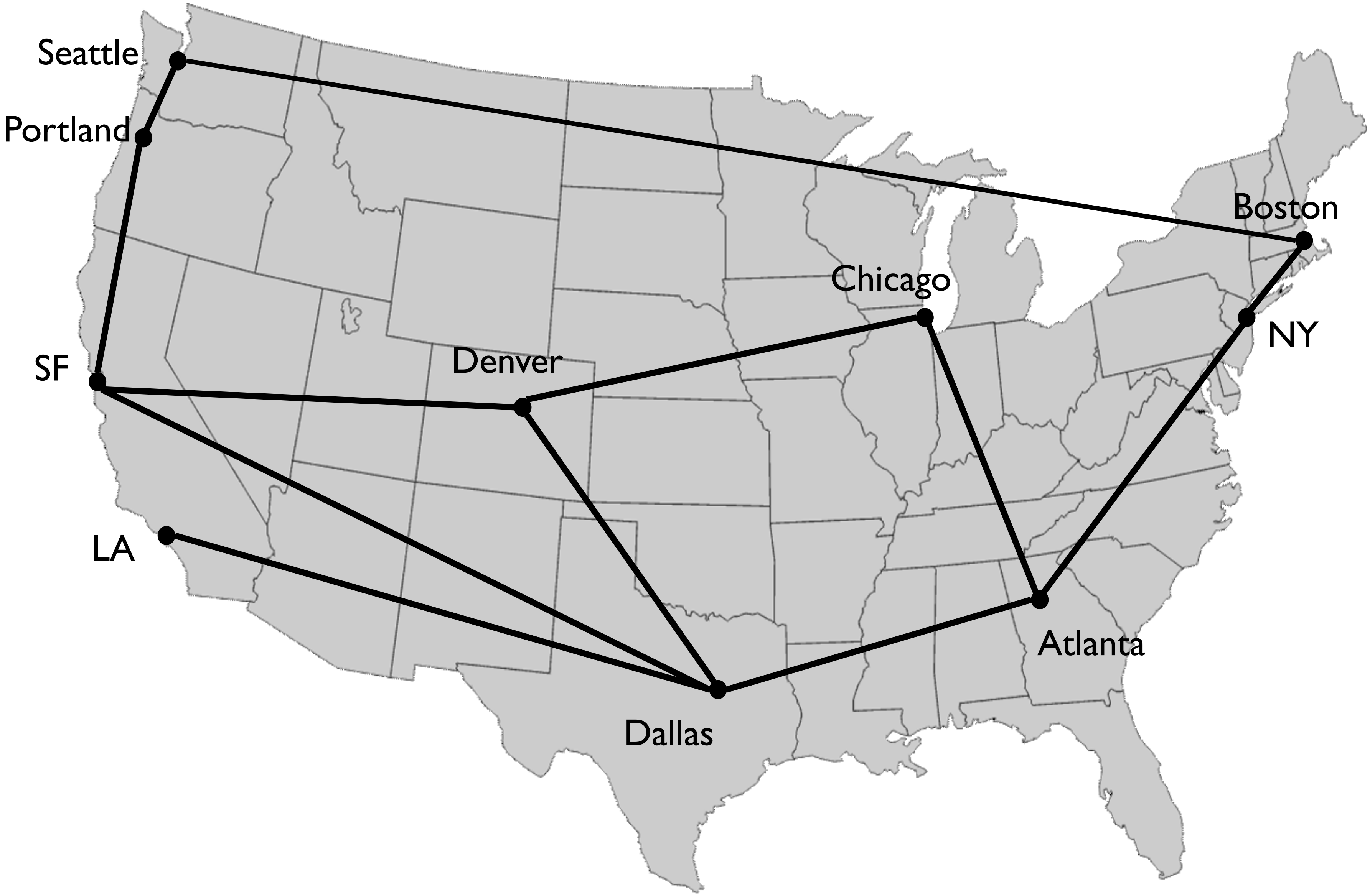
Questions?

- We will write a "SubsetIterator" to enumerate all possible subsets of a Vector<E>
- We will use SubsetIterator to solve two problems
  - Two Towers
  - Identify all Subsequences of a String that are words
    - Use your LexiconTrie! (or an OrderedStructure)

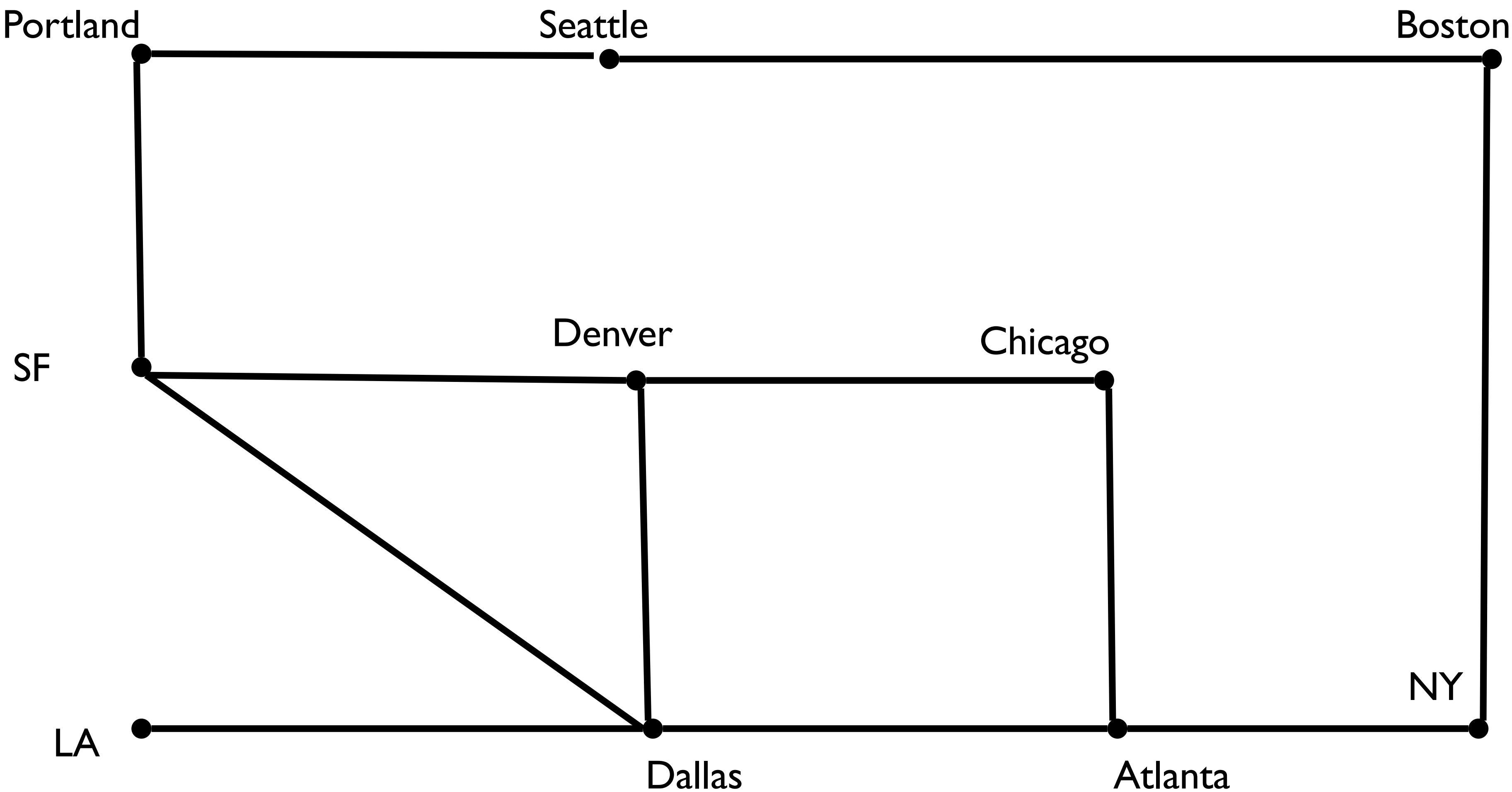
Graphs Describe the World[1]

- Transportation Networks
- Communication Networks
- Social Networks
- Molecular structures
- Dependency structures
- Scheduling
- Matching
- Graphics Modeling
- ....

Nodes = subway stops; Edges = subway lines

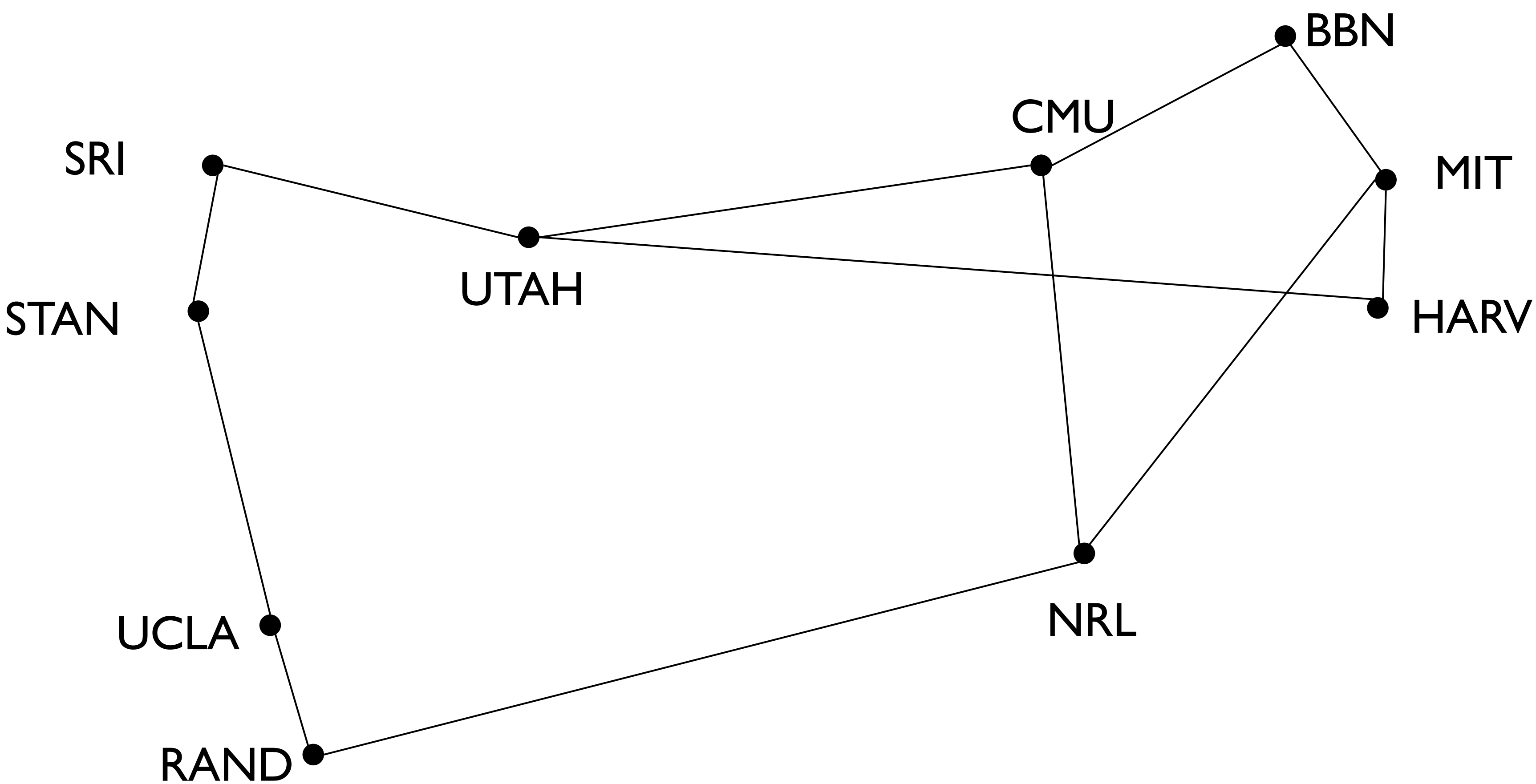


Nodes = cities; Edges = rail lines connecting cities

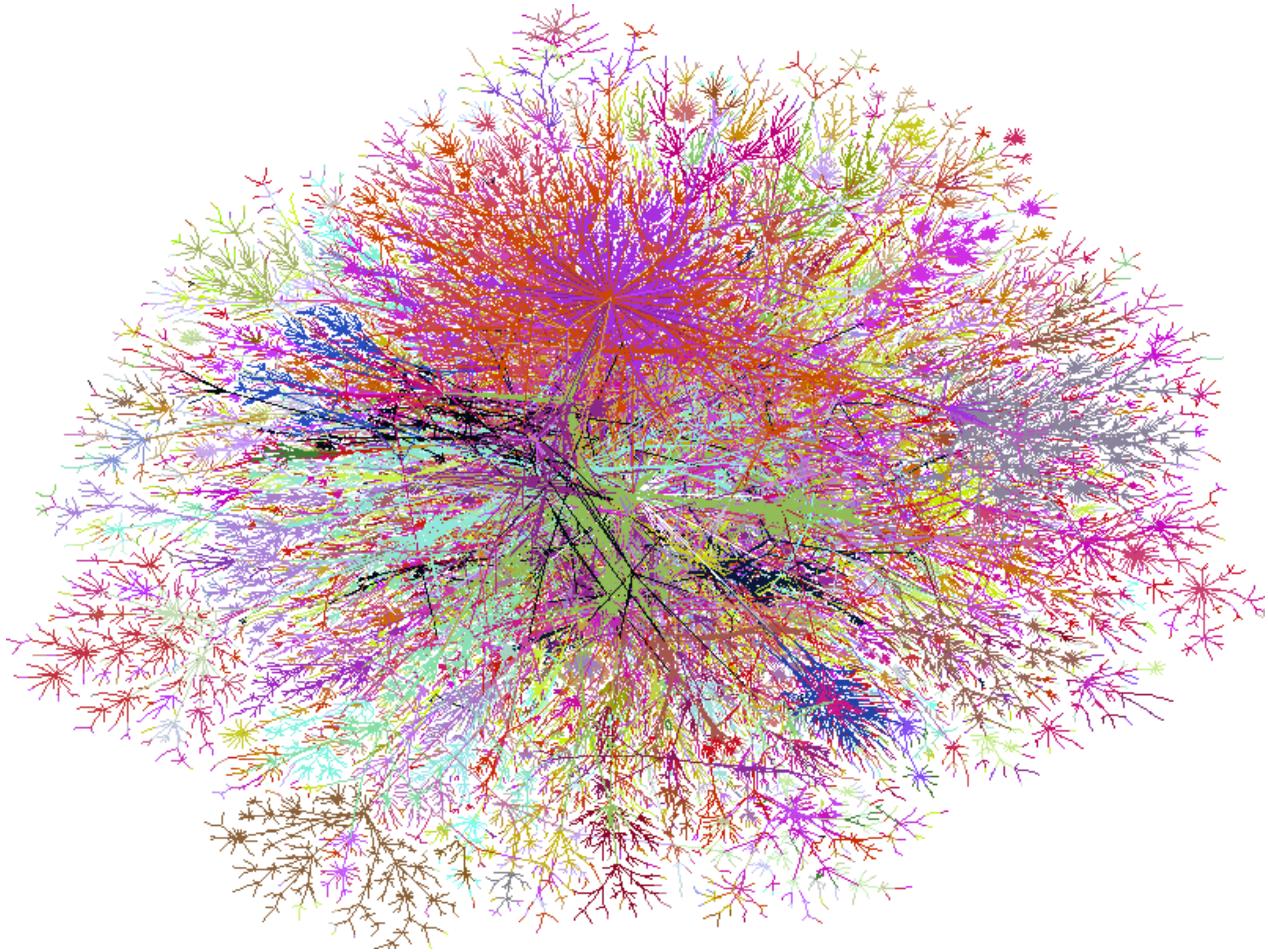


Note: Connections in graph matter, not precise locations of nodes

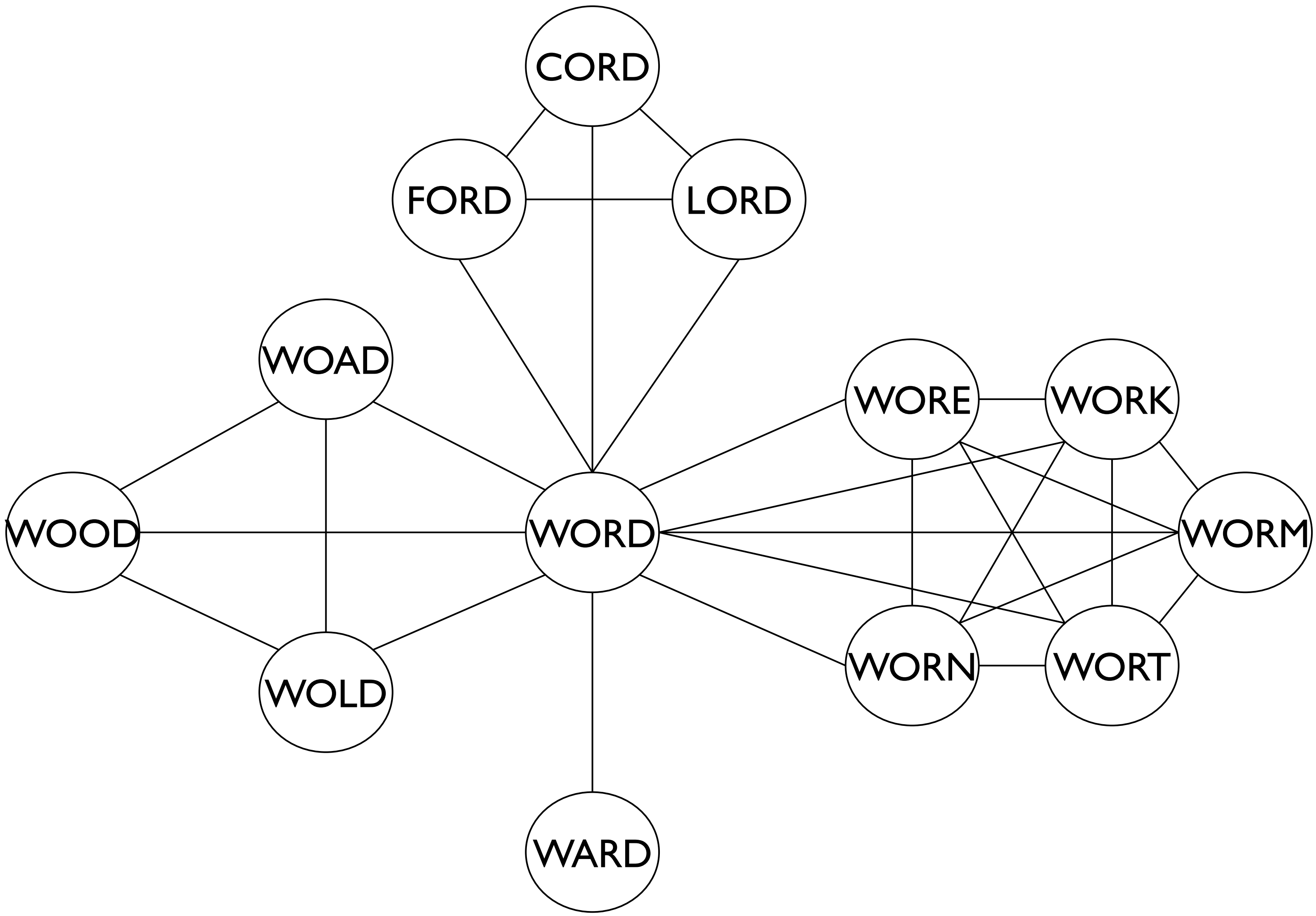
Internet (~1972)



Internet (~1998)

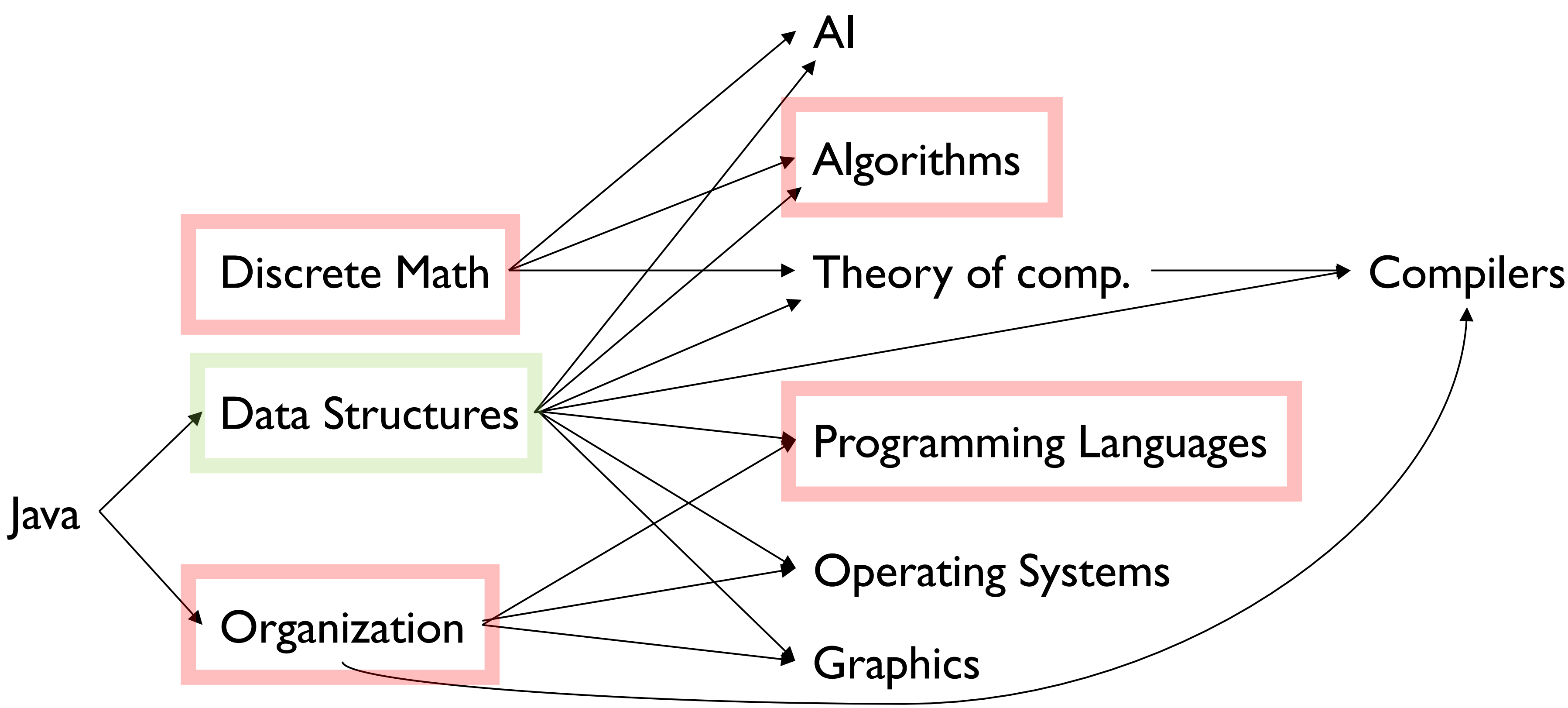


Word Game



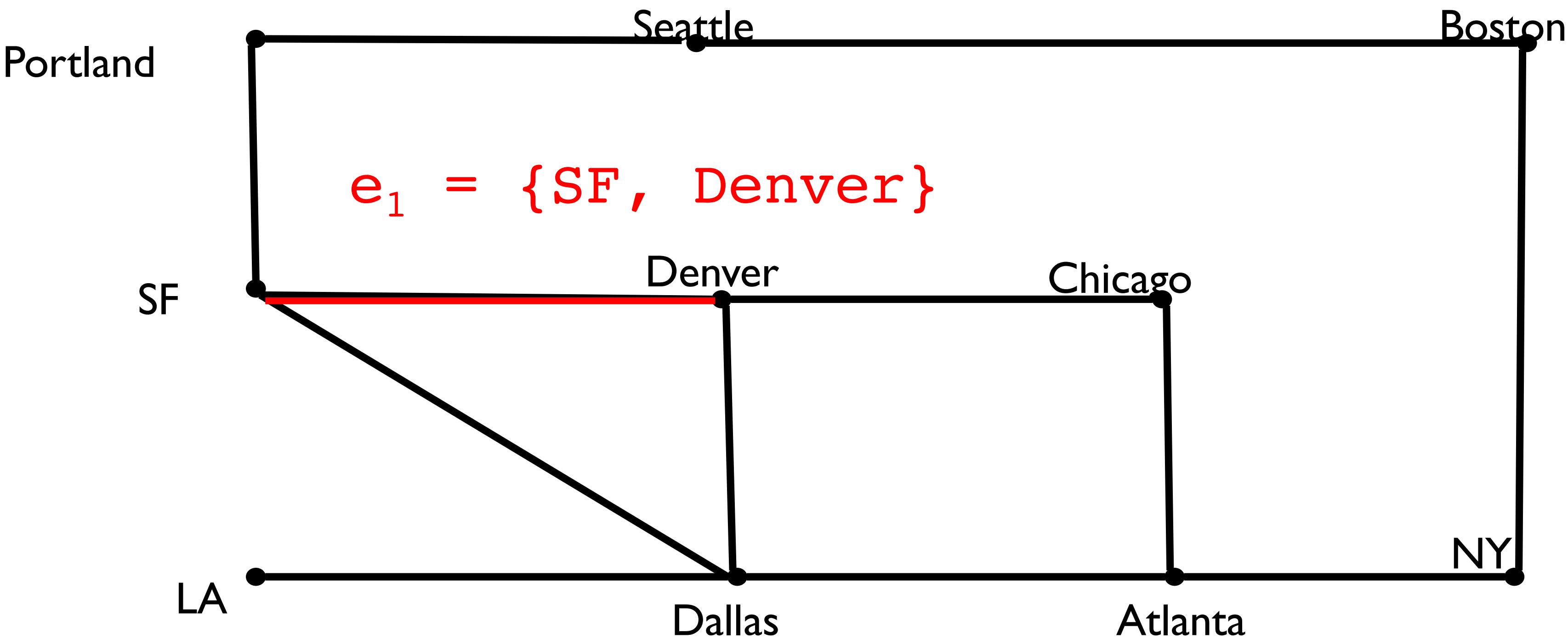
Nodes = words; Edges = words that differ by exactly one letter

CS Pre-requisite Structure (subset)



Nodes = courses; Edges = prerequisites ***

Basic Definitions & Concepts



Definition: An *undirected graph* `G = (V,E)` consists of two sets

- `V` : the *vertices* of `G`, and `E` : the *edges* of `G`
- Each edge `e` in `E` is defined by a set of two vertices: its *incident vertices*.
- We write `e = {u,v}` and say that `u` and `v` are *adjacent*.

Walking Along a Graph

- A *walk from u to v* in a graph G = (V,E) is an *alternating* sequence of vertices and edges

  $$u = v_0, e_1, v_1, e_2, v_2, \ldots, v_{k-1}, e_k, v_k = v$$

  such that each $e_i = \{v_i, v_{i+1}\}$ for $i = 1, \ldots, k$
- Note a walk starts and ends on a vertex
- If no *edge* appears more than once then the walk is called a *path*
- If no *vertex* appears more than once then the walk is a *simple path*

Walking In Circles

- A *closed walk* in a graph G = (V,E) is a *walk*

  $v_0, e_1, v_1, e_2, v_2, \ldots, v_{k-1}, e_k, v_k$

  such that $v_0 = v_k$ (it ends at the starting v)

- A *circuit* is a *path* where $v_0 = v_k$
  - Circuit vs. closed walk? Circuit has no repeat edges
- A *cycle* is a *simple* path where $v_0 = v_k$
  - Circuit vs. cycle? Cycle has no repeated vertices.
- The length of any of these is the number of *edges* in the sequence