CSCI 136
Data Structures & Advanced Programming

Lecture 24

Fall 2016

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Administrative Details

- Lab 9: Simulations
  - You will simulate two queuing strategies
  - You can work with a partner
  - Time spent on lab before Wed. is time well-spent!

Last Time

- Finishing up with heaps
    - More on implementation
    - “Heapifying” constructor for VectorHeap
    - Alternate heapify approach

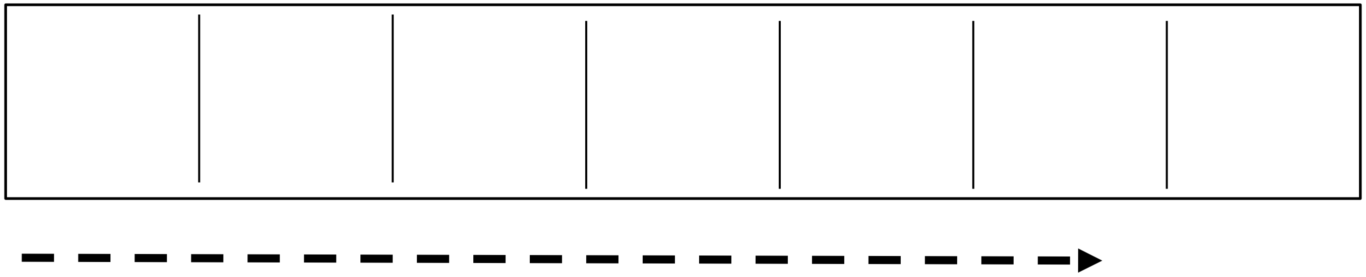
Today

- Finishing up with heaps
  - Review “Heapify” (rushed at end of last lecture)
  - HeapSort
  - Alternative Heap Structures
- Binary Search Tree: A New Ordered Structure
  - Definitions
  - Implementation

Heapifying A Vector (or array)

Problem: You are given a Vector V that is not a valid heap, and you want to "heapify" V

- Method 1: Top-Down
  - Assume V[0...k] satisfies the heap property
  - Now call percolateUp on item in location k+1
  - Then V[0..k+1] satisfies the heap property



Grow heap one element at a time

Practice Top-Down

Input:

- `int a[6] = {7,5,9,1,2,5,4}`

```
              0 1 2 3 4 5 6
```

```
for (int i = 0; i < a.length; i++)
    percolateUp(a, i);
```

Result: a is a valid heap!

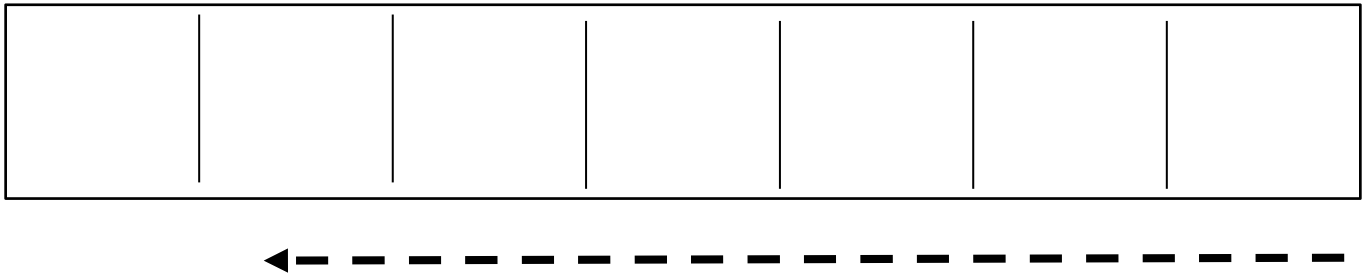
- `a = [1|2|4|7|5|9|5]`

```
       0 1 2 3 4 5 6
```

Heapifying A Vector (or array)

Problem: You are given a Vector V that is not a valid heap, and you want to "heapify" V

- Method II: Bottom-up
  - Assume V[k..n] satisfies the heap property
  - Now call pushDown on item in location k-1
  - Then V[k-1..n] satisfies heap property


Grow heap one element at a time

Practice Bottom-Up

Input:

- `int a[6] = {7,5,9,1,2,5,4}`

```
int a[6] = {7,5,9,1,2,5,4}
            0 1 2 3 4 5 6
```

```
for (int i = a.length-1; i > 0; i++)
    pushDownRoot(a, i);
```

Result: a is a valid heap!

- `a = [1|2|4|5|7|5|9]`

```
a = [1|2|4|5|7|5|9]
     0 1 2 3 4 5 6
```

Top-Down vs Bottom-Up

- Top-down heapify: elements at depth d may be swapped d times: Total # of swaps is

$$\sum_{d=1}^{h} d2^{d} = (h-1)2^{h+1} = (\log n - 1)2n + 2$$

(recall: h = log n)

- This is O(n log n)
- Some intuition: most of the elements are in the lowest levels of the tree, so each of them might have to move to root: O(log n) swaps per element

Top-Down vs Bottom-Up

- Bottom-up heapify: elements at depth d may be swapped h-d times: Total # of swaps is

$$\sum_{d=1}^{h}(h-d)2^{d} = 2^{h+1} - h - 2 = 2n - \log n + 2$$

  - This is O(n) --- beats top-down!
  - Some intuition: most of the elements are in the lowest levels of the tree, so each of them will only be pushed down (swapped) a small number of times SO COOL!!!

Some Sums (for your toolbox)

$$\sum_{d=0}^{d=k} 2^d = 2^{k+1} - 1$$

$$\sum_{d=0}^{d=k} r^d = (r^{k+1} - 1)/(r - 1)$$

$$\sum_{d=1}^{d=k} d * 2^d = (k-1) * 2^{k+1} + 2$$

$$\sum_{d=1}^{d=k} (k-d) * 2^d = 2^{k+1} - k - 2$$

All of these can be proven by (weak) induction.

Try these to hone your skills

The second sum is called a geometric series. It works for any $r \neq 0$

HeapSort

- The “niftiest” sort so far
- Strategy:
  - Make a *max-heap*: array[0…n]
    - array[0] is largest value
    - array[n] is rightmost leaf
  - Take the largest value (array[0]) and swap it with the rightmost leaf (array[n])
  - Call pushDownRoot(0) on array[0…n-1]
    - Now our heap is one element smaller, but largest element is at end of array.
  - Repeat until array is sorted

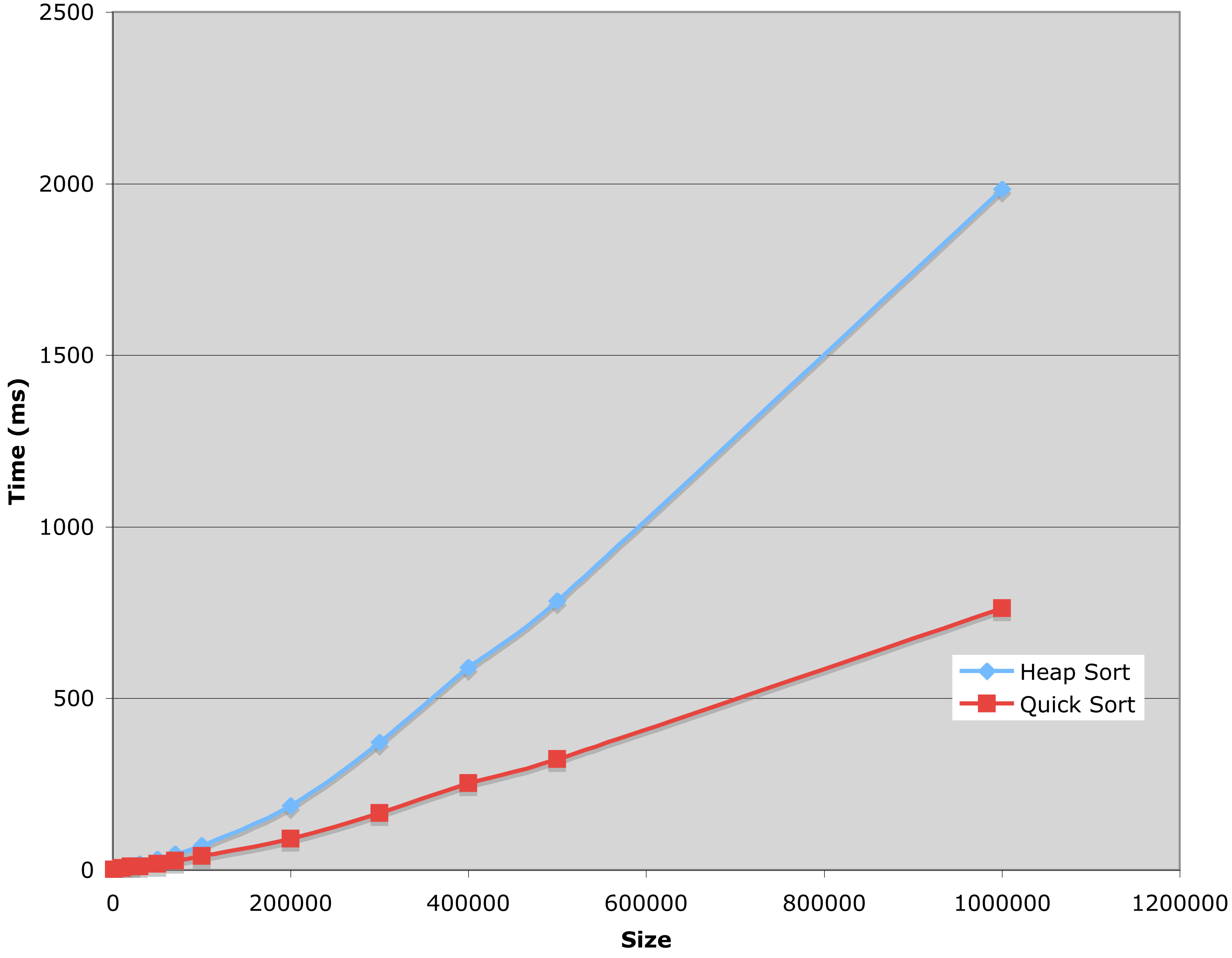
HeapSort

- Another O(n log n) sort method
- Heapsort is not *stable*
  - The relative ordering of elements is not preserved in the final sort
    - Why?
      - There are multiple valid heaps given the same data
- Heapsort can be done *in-place*
  - No extra memory required!!!
  - Great for resource-constrained environments

HeapSort

- HeapSort pseudocode for unsorted vector V:
  - Perform bottom-up heapify on the reverse ordering: that is: highest rank/lowest priority elements are near the root (low end of Vector)
  - Repeatedly remove elements to fill in Vector from tail to head
    - for(int i = v.size() – 1; i > 0; i--)
      - removeMin from v[0..i] // v[i] is now not in heap
      - Put removed value in location v[i] // v[0..i-1] is a valid heap
        // v[i..n] is sorted

Heap Sort vs QuickSort



Why Heapsort?

- Heapsort is slower than Quicksort in general
- Any benefits to heapsort?
    - *Guaranteed* O(n log n) runtime
- Works well on mostly sorted data, unlike quicksort
- Good for incremental sorting

More on Heaps

- Set-up: We want to build a *large* heap. We have several processors available.
- We'd like to use them to build smaller heaps and then merge them together
- Suppose we can share the array holding the elements among the processors.
  - How long to merge two heaps?
  - How complicated is it?
- What if we use BinaryTrees for our heaps?

Mergeable Heaps

- We now want to support the additional *destructive* operation merge(heap1, heap2)
- Basic idea: heap with larger root somehow points into heap with smaller root
- Challenges
  - Points how? Where?
  - How much reheapifying is needed
  - How deep do trees get after many merges?

Skew Heap

- Don’t force heaps to be complete BTs?
- Develop recursive merge algorithm that keeps tree shallow over time
- Theorem: Any set of m SkewHeap operations can be performed in O(m log n) time, where n is the total number of items in the SkewHeaps
- Let’s sketch out merge operation....

Skew Heap: Merge Pseudocode

```
SkewHeap merge(SkewHeap S, SkewHeap T)
    if either S or T is empty, return the other          Case 1
    if T.minValue < S.minValue
        swap S and T        (S now has minValue)
    if S has no left subtree, T becomes its left subtree
                                                         Case 2
    else
        let temp point to right subtree of S
        left subtree of S becomes right subtree of S
        merge(temp, T) becomes left subtree of S
                                                         Case 3
    return S                                             (recurse)
```

Tree Summary

- Trees
    - Express hierarchical relationships
    - Level ordering captures the relationship
        - i.e., ancestry, game boards, decisions, etc.
- Heap
    - Partially ordered tree based on item priority
    - Node invariants: parent has higher priority than each child
    - Provides efficient PriorityQueue implementation