CSCI 136
Data Structures &
Advanced Programming

Lecture 22

Fall 2017

Instructor: Bills

Announcement

Power outage (3-5am)

We’ll be shutting down systems at 10pm tonight

Rebooting at 9am tomorrow

Last Time

- Wrap up Binary Tree Iterators
- Breadth-First and Depth-First Search
- Array Representations of (Binary) Trees
- Application: Huffman Encoding

Today

Improving Huffman's Algorithm

- Priority Queues & Heaps
  - A "somewhat-ordered" data structure
    - Conceptual structure
    - Efficient implementations

Huffman Codes

- Example
  - AN_ANTARCTIC_PENGUIN
  - Compute letter frequencies

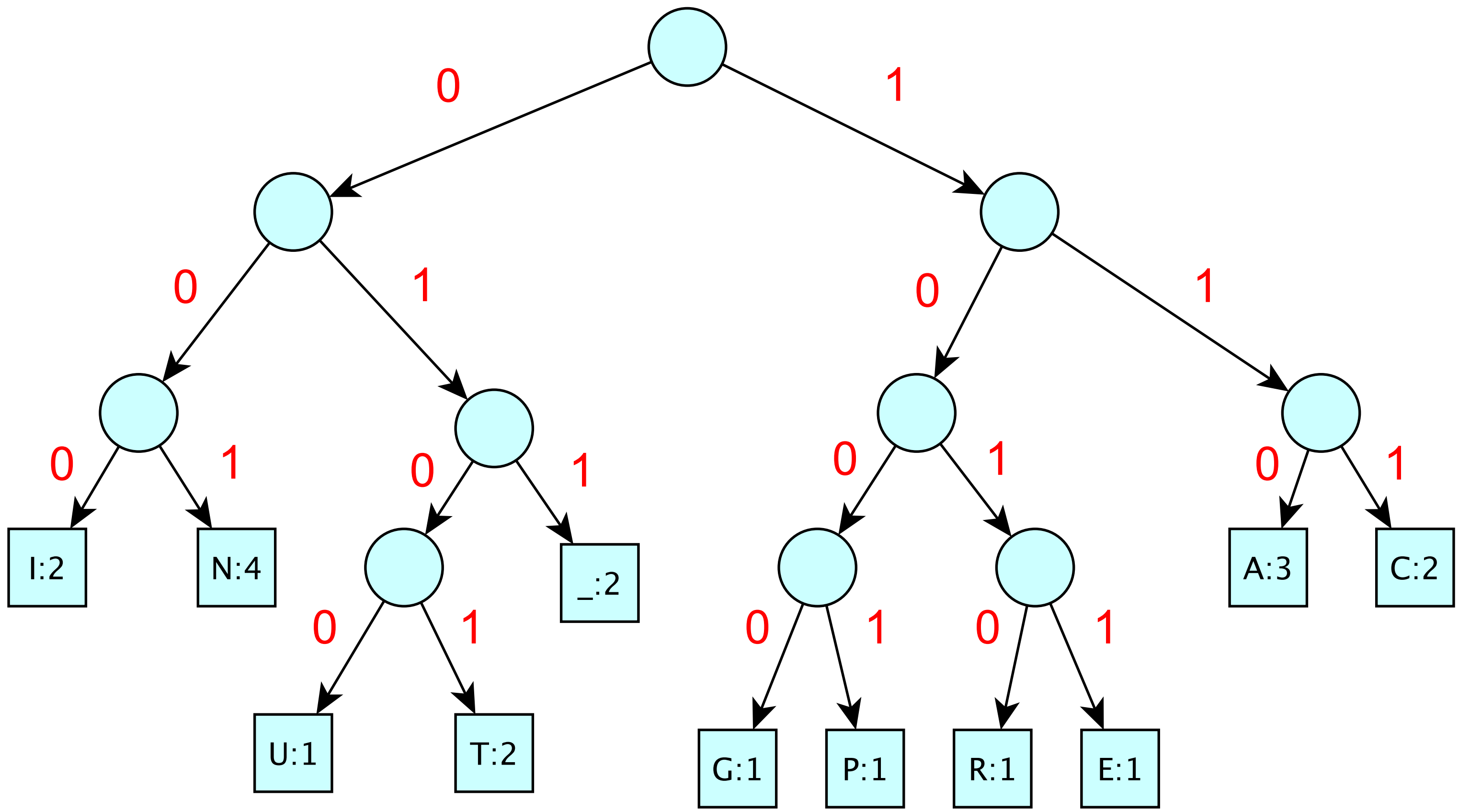
A	C	E	G	I	N	P	R	T	U	_
3	2	1	1	2	4	1	1	2	1	2

- Key Idea: Use fewer bits for most common letters

A	C	E	G	I	N	P	R	T	U	_
3	2	1	1	2	4	1	1	2	1	2
110	111	1011	1000	000	001	1001	1010	0101	0100	011

- Uses 67 bits to encode entire string

The Encoding Tree



Left = 0; Right = 1

Huffman Encoding Algorithm

Input: symbols of alphabet with frequencies

- Huffman encode as follows
  - Create a single-node tree for each symbol: key is frequency; weight is letter
  - while there is more than one tree
    - Find two trees T1 and T2 with lowest weights
    - Merge them into new tree T with:

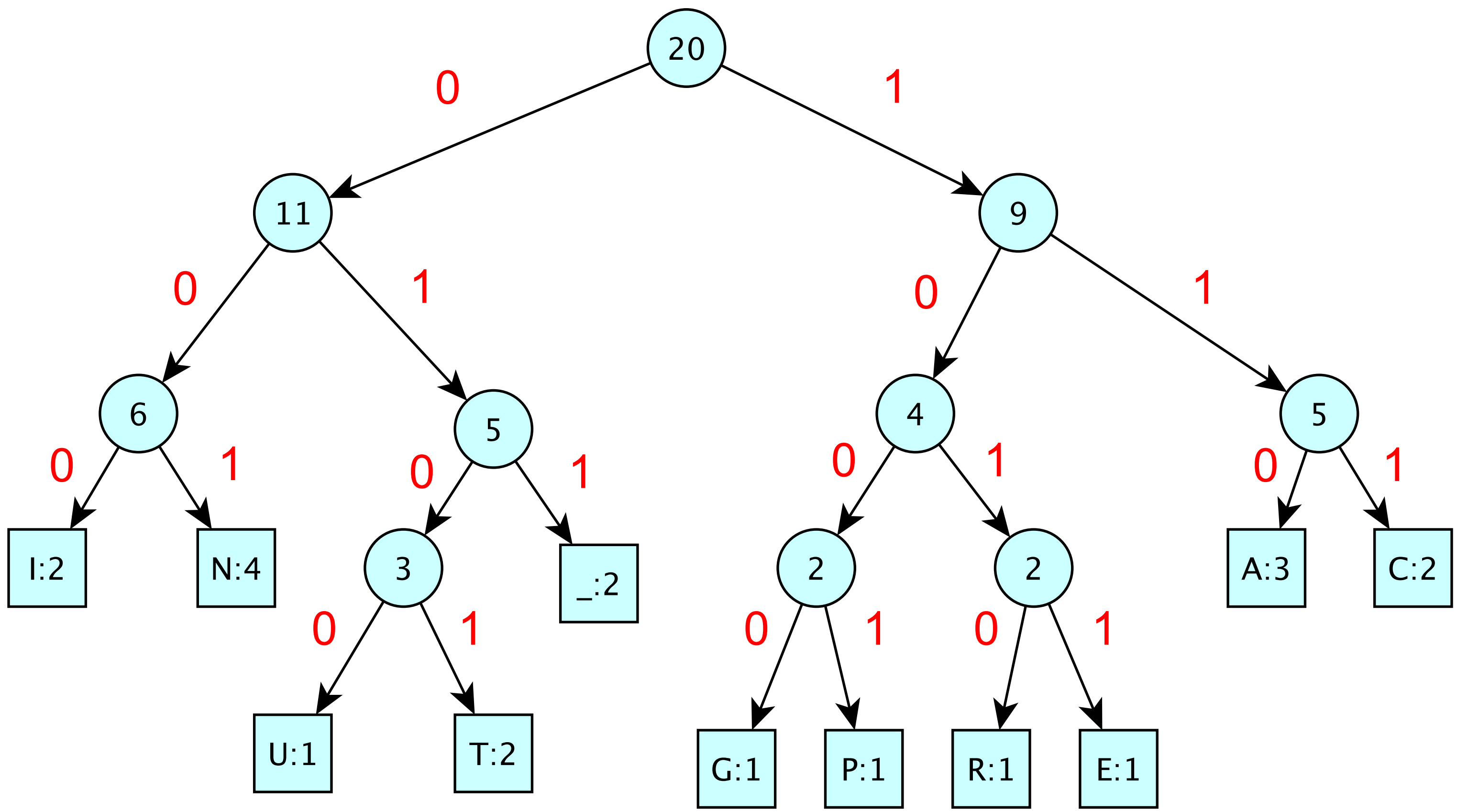
      weight = T1.weight+ T2.weigth
- Theorem: The tree computed by Huffman is an optimal encoding for given frequencies

Demo

- To run the Huffman code demo found on course webpage:

```
java -jar huffman.jar
```

The Encoding Tree (With Weights)



Left = 0; Right = 1

*Each node's value is the sum of the frequencies of all its children

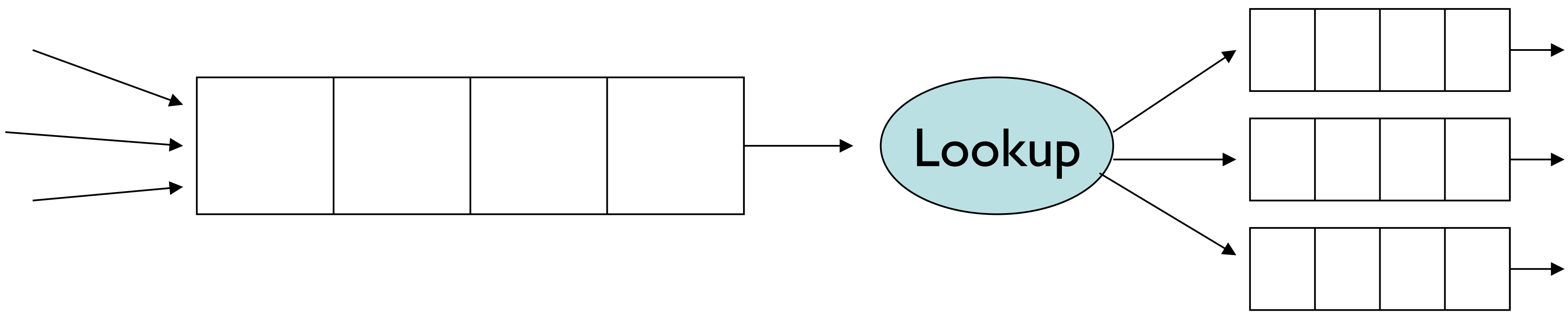
Implementing the Algorithm

- Keep a Vector of Binary Trees
- Sort them by decreasing frequency
  - Removing two smallest frequency trees is fast
- Insert merged tree into correct sorted location in Vector
- Running Time:
  - O(n log n) for initial sorting
  - $O(n^2)$ for while loop
- Can we do better...?

What Huffman Encoder Needs

- A structure S to hold items with *priorities*
- S should support operations
  - add(E item);   // add an item
  - E removeMin(); // remove min priority item
- S should be designed to make these two operations fast
- If, say, they both ran in O(log n) time, the Huffman while loop would take O(n log n) time instead of $O(n^2)$!
- We've seen this situation before….

Priority Queues



Packet Sources May Be Ordered by Sender

`sysnet.cs.williams.edu`	`priority = 1 (best)`
`bull.cs.williams.edu`	`2`
`yahoo.com`	`10`
`spammer.com`	`100 (worst)`

Priority Queues

- Priority queues are also used for:
  - Scheduling processes in an operating system
    - Priority is function of time lost + process priority
  - Order services on server
    - Backup is low priority, so don't do when high priority tasks need to happen
  - Scheduling future events in a simulation (lab next week!)
  - Medical waiting room
  - Huffman codes - order by tree size/weight
  - A variety of graph/network algorithms
  - To roughly rank choices that are generated out of order

Priority Queues

- Name is misleading: They are **not FIFO**
- Always dequeue object with **highest priority** (smallest rank) regardless of when it was enqueued
- Data can be received/inserted in any order, but it is always returned/removed according to priority
- Like ordered structures (i.e., OrderedVectors and OrderedLists), PQs require comparisons of values

An Apology

- On behalf of computer scientists everywhere, I'd like to apologize for the confusion that inevitably results from the fact that

  Higher Priority ↔Lower Rank

- The PQ removes the *lowest ranked* value in an ordering: that is, the *highest priority* value!

We're sorry!

PQ Interface

```
public interface PriorityQueue<E extends Comparable<E>> {
  public E getFirst(); // peeks at minimum element
  public E remove();   // removes minimum element
  public void add(E value); // adds an element
  public boolean isEmpty();
  public int size();
  public void clear();
}
```

Notes on PQ Interface

- Unlike previous structures, we do not extend any other interfaces
  - Many reasons: For example, it's not clear that there's an obvious iteration order
- PriorityQueue uses Comparables: methods *consume* Comparable parameters and *return* Comparable values
  - Could be made to use Comparators instead…

Implementing PQs

- Queue?
  - Wouldn’t work so well because we can’t insert and remove in the “right” way (i.e., keeping things ordered)
- OrderedVector?
  - Keep ordered vector of objects
  - O(n) to add/remove from vector
  - Details in book…
  - Can we do better than O(n)?
- Heap!
  - Partially ordered binary tree

Heap

- A heap is a special type of tree
  - Root holds smallest (highest priority) value
  - Subtrees are also heaps (this is important!)
- Values increase in priority (decrease in rank) from leaves to root (from descendant to ancestor)
- *Invariant for nodes:* For each child of each node
  - node.value() <= child.value()   // if child exists
- Several valid heaps for same data set (no unique representation)

Inserting into a PQ

- Add new value as a leaf
- "Percolate" it up the tree
  - while (value < parent's value) swap with parent
- This operation preserves the heap property since new value was the only one violating heap property
- Efficiency depends upon speed of
  - Finding a place to add new node
  - Finding parent
  - Tree height

Removing From a PQ

- Get value from root node (highest priority)
- Find a leaf, delete it, put its *data* in the root
- "Push" *data* down through the tree
  - while (*data.value* > value of (at least) one child)
    - Swap *data* with data of **smaller** child
- This operation preserves the heap property
- Efficiency depends upon speed of
  - Finding a leaf
  - Finding locations of children
  - Height of tree

Implementing Heaps

- VectorHeap
  - Use conceptual array representation of BT (ArrayTree)
  - But use extensible Vector instead of array (makes adding elements easier)
  - Note:
    - Root of tree is location 0 of Vector
    - Children of node in location i are in locations 2i+1 (left) and 2i+2 (right)
    - Parent of node i is in location (i-1)/2
      - Remember: dividing Integers truncates the result

Implementing Heaps

- Strategy: tree modifications that always preserve tree *completeness*, but may violate heap property. Then fix.
  - Add/remove never add gaps to array
    - We always add in next available array slot (left-most available spot in binary tree)
    - We always remove using "final" leaf
  - *Heap Invariant* becomes
    - data[i] <= data[2i+1]; data[i]<=data[2i+2] (or kids might be null)
  - When elements are added and removed, do small amount of work to "re-heapify"
    - How small? Note: finding a node's child or parent takes constant time, as does finding "final" leaf or next slot for adding
    - Since this heap corresponds to a full binary tree, the depth of the tree is O(log n), so percolate/pushDown takes O(log n) time!

VectorHeap Summary

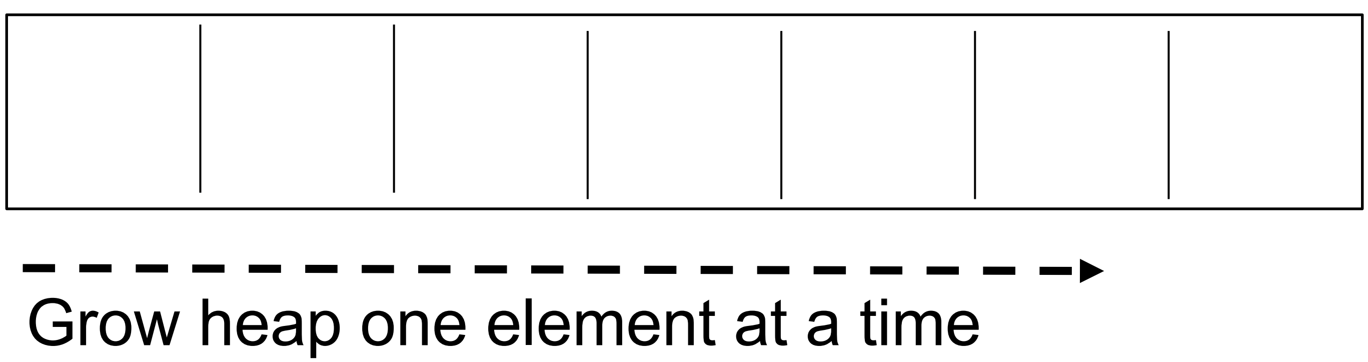
- Let's look at VectorHeap code....

- Add/remove are both O(log n)
- Data is not completely sorted
  - "Partial" order is maintained: all root-to-leaf paths
- Note: VectorHeap(Vector<E> v)
  - Takes an unordered Vector and uses it to construct a heap
  - How?

Heapifying A Vector (or array)

Problem: You are given a Vector V that is not a valid heap, and you want to "heapify" V

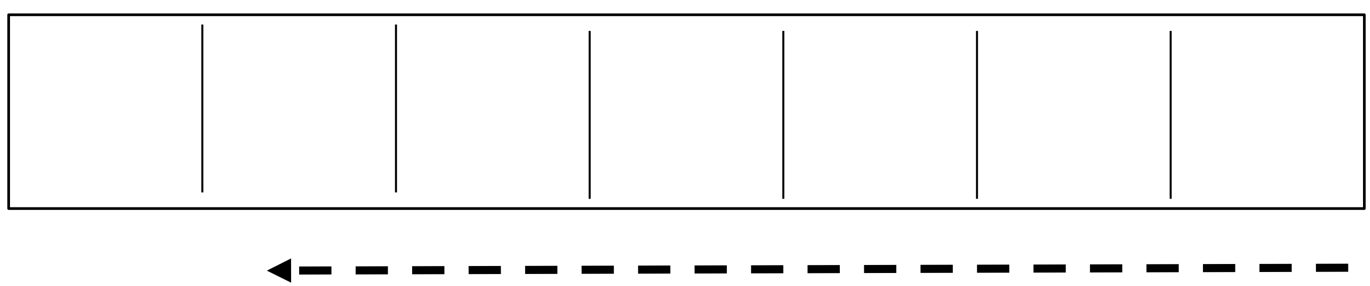
- Method 1: Top-Down
  - Assume V[0…k] satisfies the heap property
  - Now call percolateUp on item in location k+1
  - Then V[0..k+1] satisfies the heap property



Heapifying A Vector (or array)

Problem: You are given a Vector V that is not a valid heap, and you want to "heapify" V

- Method II: Bottom-up
  - Assume V[k..n] satisfies the heap property
  - Now call pushDown on item in location k-1
  - Then V[k-1..n] satisfies heap property



Top-Down vs Bottom-Up

- Top-down heapify: elements at depth d may be swapped d times: Total # of swaps is

$$\sum_{d=1}^{h} d2^d = (h-1)2^{h+1} = (\log n - 1)2n + 2$$

(recall: h = log n)

- This is O(n log n)
- Some intuition: most of the elements are in the lowest levels of the tree, so each of them might have to move to root: O(log n) swaps per element

Top-Down vs Bottom-Up

- Bottom-up heapify: elements at depth d may be swapped h-d times: Total # of swaps is

$$\sum_{d=1}^{h}(h-d)2^d = 2^{h+1} - h - 2 = 2n - \log n + 2$$

  - This is O(n) --- beats top-down!
  - Some intuition: most of the elements are in the lowest levels of the tree, so each of them will only be pushed down (swapped) a small number of times SO COOL!!!

Some Sums (for your toolbox)

$$\sum_{d=0}^{d=k} 2^d = 2^{k+1} - 1$$

$$\sum_{d=0}^{d=k} r^d = (r^{k+1} - 1)/(r - 1)$$

$$\sum_{d=1}^{d=k} d * 2^d = (k-1) * 2^{k+1} + 2$$

$$\sum_{d=1}^{d=k} (k-d) * 2^d = 2^{k+1} - k - 2$$

All of these can be proven by (weak) induction.

Try these to hone your skills

The second sum is called a geometric series. It works for any $r \neq 0$