CSCI 136
Data Structures &
Advanced Programming

Lecture 20

Fall 2017

Instructor: Bills

Administrative Details

- Lab 8 is available online
  - No partners this week
  - Review before lab; come to lab with design doc
  - We'll give an overview shortly

Tonight

Computer Science Preregistration Info Session

- Learn about Computer Science courses offered Spring 2017.
- Talk to professors about their classes.
- Meet other Computer Science students.
- Most importantly... **EAT PIZZA!**

Monday, October 30
at 9:00 pm
Biology Lounge
TBL 211

Last Time

- Trees
  - Implementation
  - Recursion/Induction on Trees
  - Applications: Decision Trees

Today

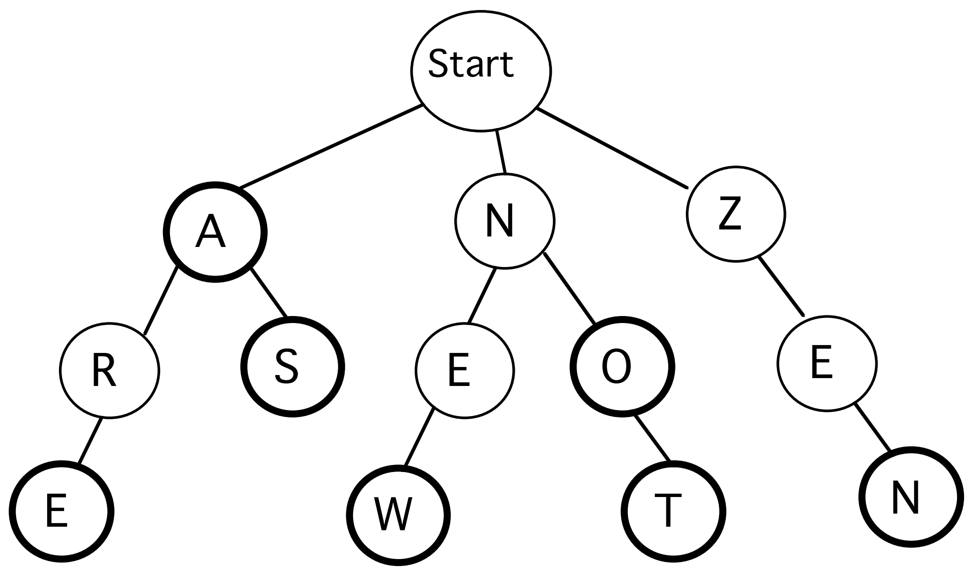
- Trees with more than 2 children
  - Representations
  - Application: Lab 8: Lexicon!
- Binary Trees
  - Traversals
    - As methods taking a BinaryTree parameter
    - With Iterators

Representing Arbitrary Trees

- What if nodes can have many children?
  - Example: Game trees
- Replace left/right node references with a list of children (Vector, SLL, etc)
  - Allows getting "i^{th}" child
- Should provide method for getting degree of a node
- Degree 0 ↔Empty list ↔No children ↔Leaf

Lab 8 : Lexicon

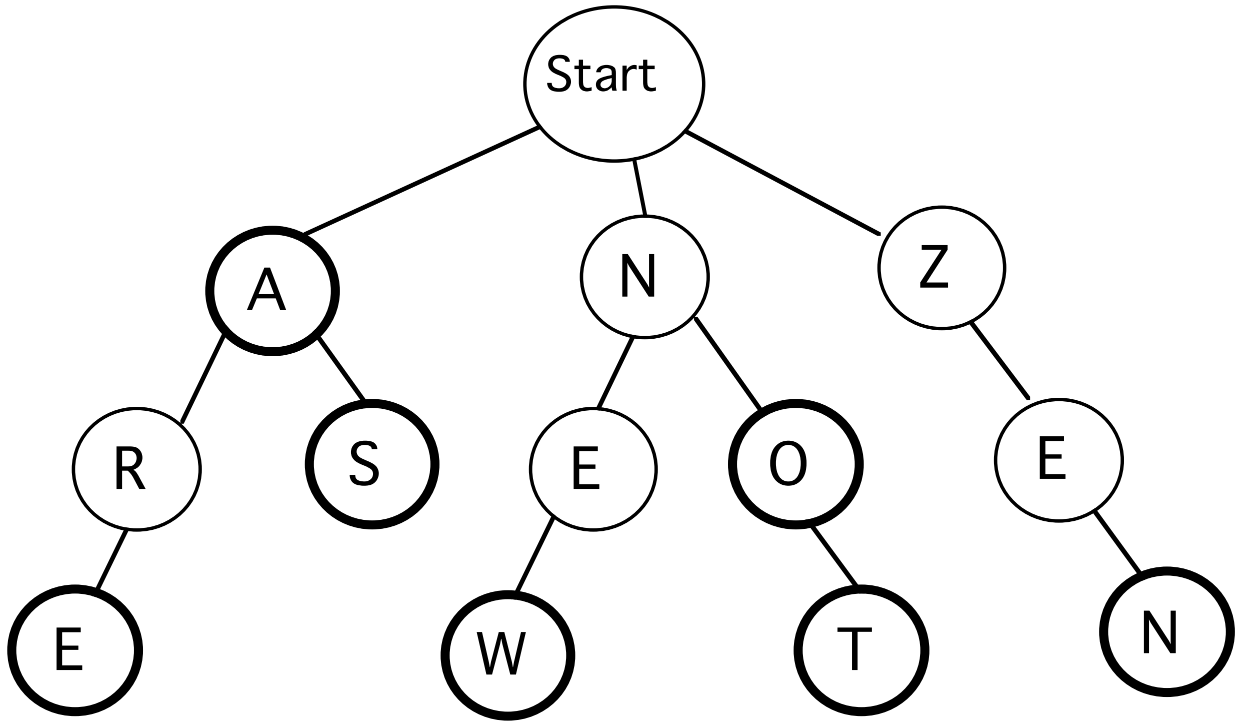
- Goal: Build a data structure that can efficiently store and search a large set of words
- A special kind of tree called a *trie*



Lab 8 : Tries

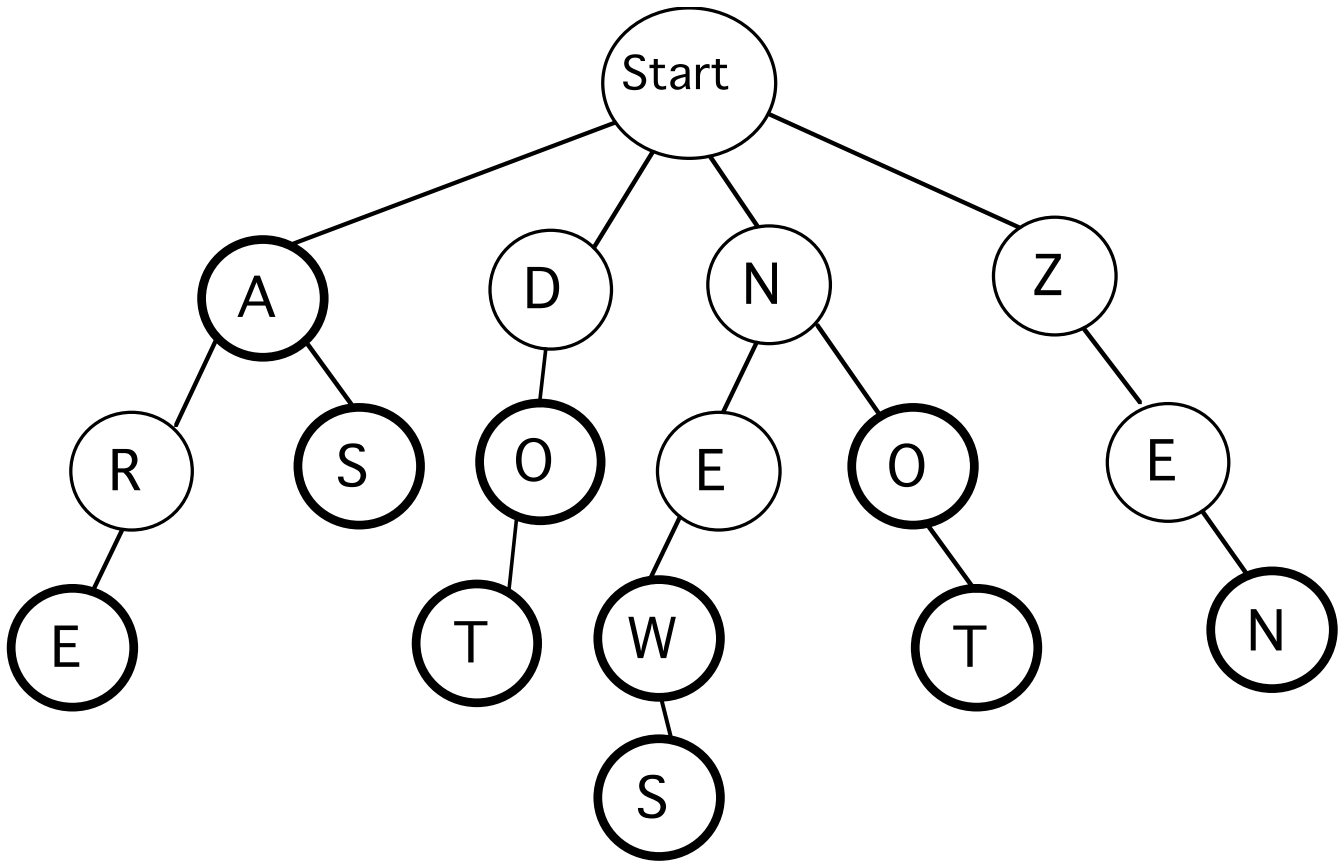
- A trie is a tree that stores words where
  - Each node holds a letter
  - Some nodes are "word" nodes (dark circles)
  - Any path from the root to a word node describes one of the stored words
  - All paths from the root form prefixes of stored words (a word is considered a prefix of itself)

Tries



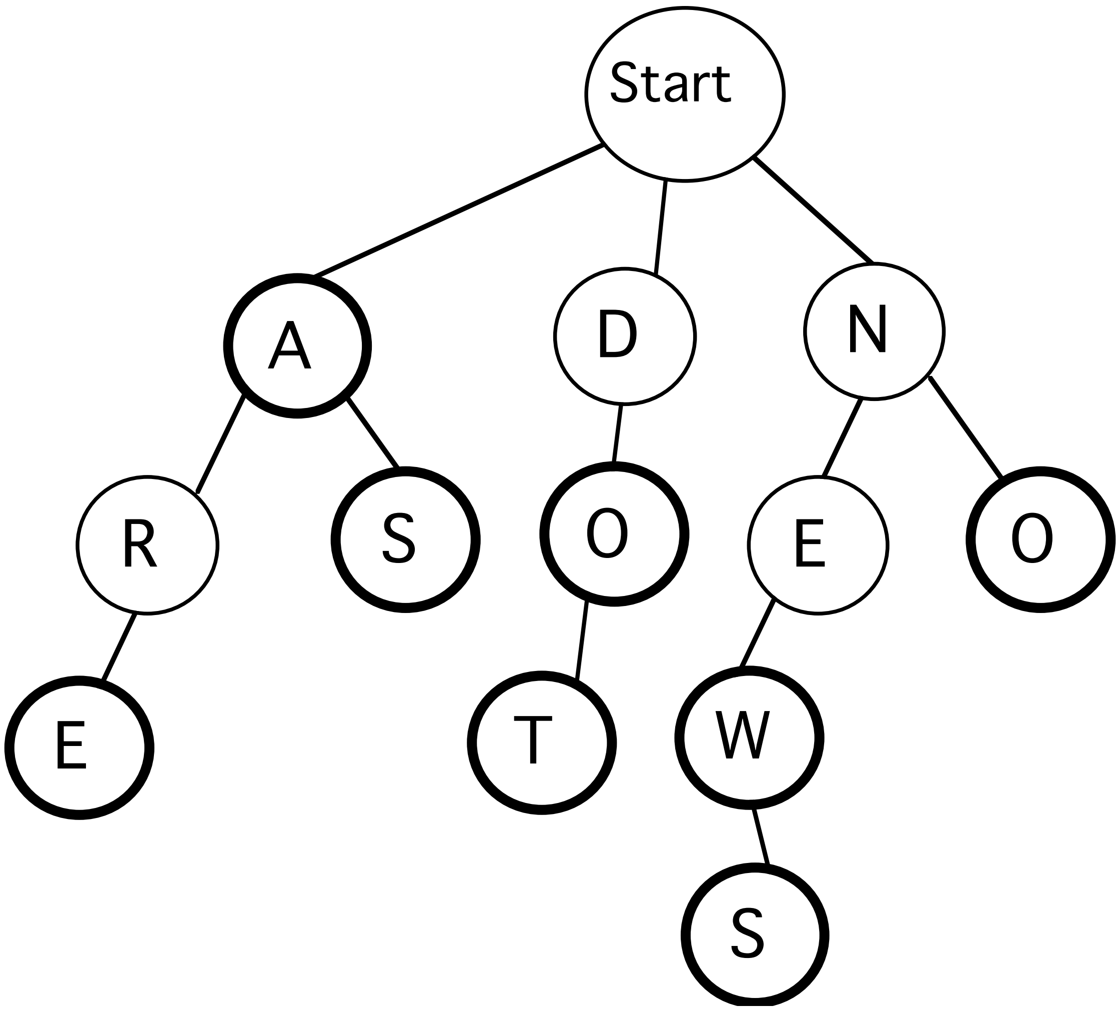
Now add “dot” and “news”

Tries



Now remove “not” and “zen”

Tries



Lab 8 : Lexicon

An inteface that provides the methods

```
public interface Lexicon {
    public boolean addWord(String word);
    public int addWordsFromFile(String filename);
    public boolean removeWord(String word);
    public int numWords();
    public boolean containsWord(String word);
    public boolean containsPrefix(String prefix);
    public Iterator<String> iterator();
    public Set<String> suggestCorrections(String
            target, int maxDistance);
    public Set<String> matchRegex(String pattern);
}
```

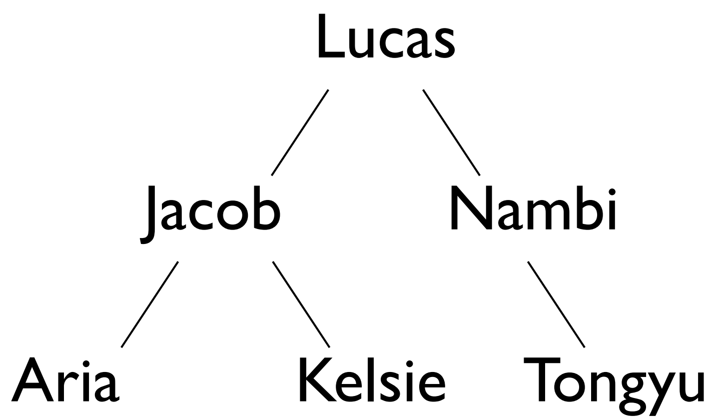
Lab 8

- Implement a program that creates, updates, and searches a Lexicon
  - Based on a LexiconTrie class
    - Each node of the Trie is a LexiconNode
    - Analogous to a SLL consisting of SLLNodes
  - LexiconTrie implements the Lexicon Interface
  - Supports
    - adding/removing words
    - searching for words and prefixes
    - reading words from files
    - Iterating over all words

Tree Traversals

- In linear structures, there are only a few basic ways to traverse the data structure
  - Start at one end and visit each element
  - Start at the other end and visit each element
- How do we traverse binary trees?
  - (At least) four reasonable mechanisms

Tree Traversals



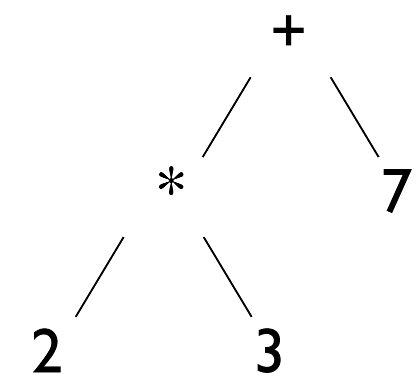
In-order: Aria, Jacob, Kelsie, Lucas, Nambi, Tongyu

Pre-order: Lucas, Jacob, Aria, Kelsie, Nambi, Tongyu

Post-order: Aria, Kelsie, Jacob, Tongyu, Nambi, Lucas,

Level-order: Lucas, Jacob, Nambi, Aria, Kelsie, Tongyu

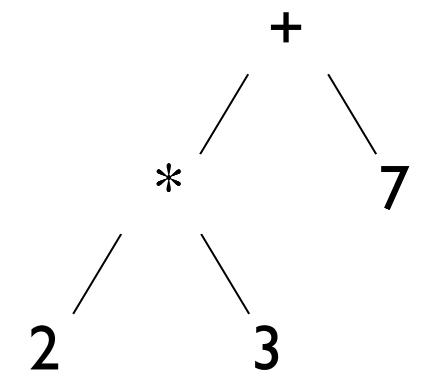
Tree Traversals



- Pre-order
  - Each node is visited before any children. Visit node, then each node in left subtree, then each node in right subtree. (node, left, right)
    - +*237
- In-order
  - Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree. (left, node, right)
    - 2*3+7

("pseudocode")

Tree Traversals

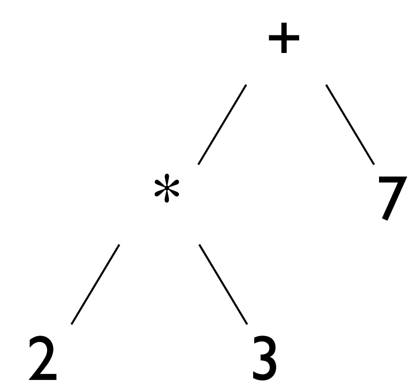


- Post-order
  - Each node is visited after its children are visited. Visit all nodes in left subtree, then all nodes in right subtree, then node itself. (left, right, node)
    - 23*7+
- Level-order (not obviously recursive!)
  - All nodes of level i are visited before nodes of level i+1. (visit nodes left to right on each level)
    - +*723

("pseudocode")

Tree Traversals

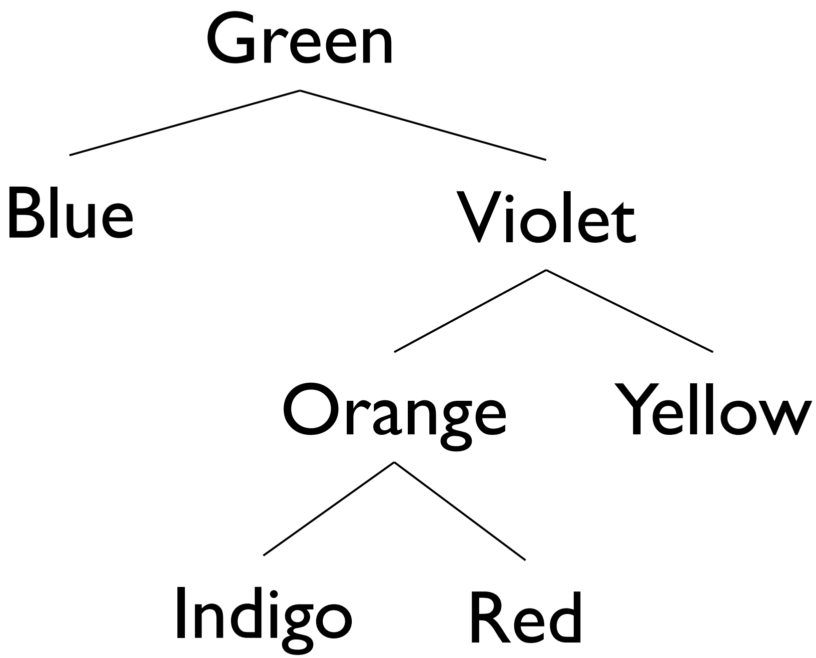
```
public void pre-order(BinaryTree t) {
      if(t.isEmpty()) return;
      touch(t); // some method
      preOrder(t.left());
      preOrder(t.right());
}
```



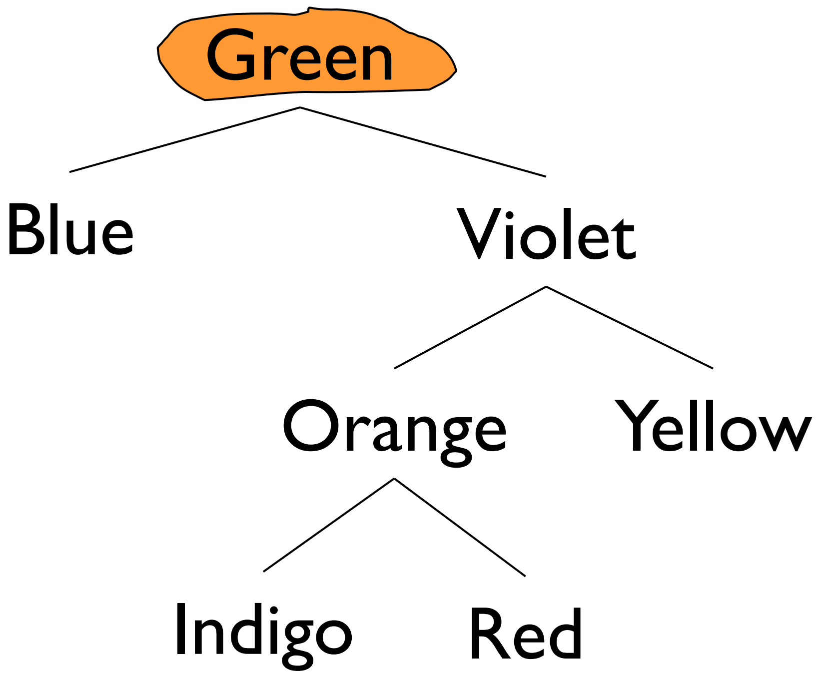
For in-order and post-order: just move touch(t)!

But what about level-order???

Level-Order Traversal

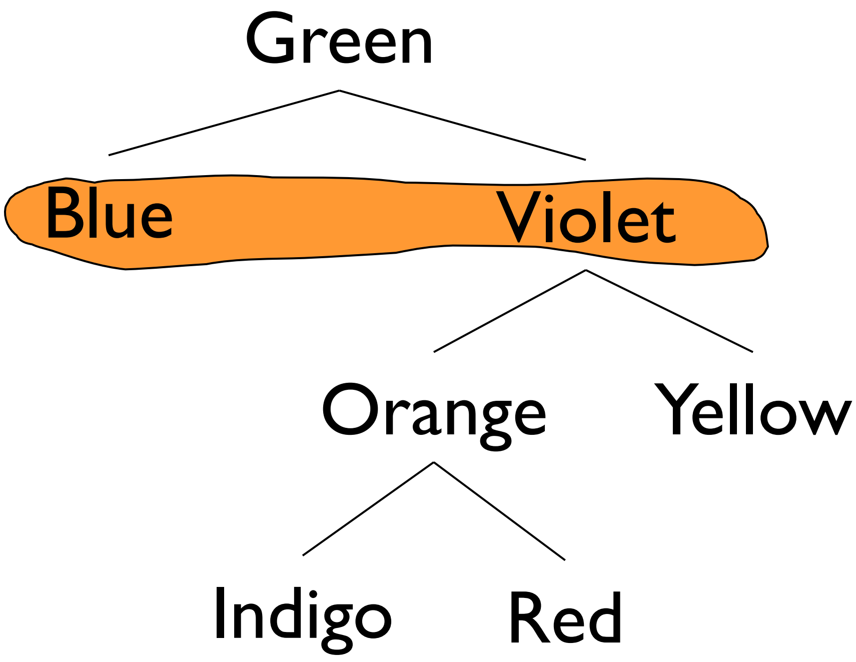


Level-Order Traversal



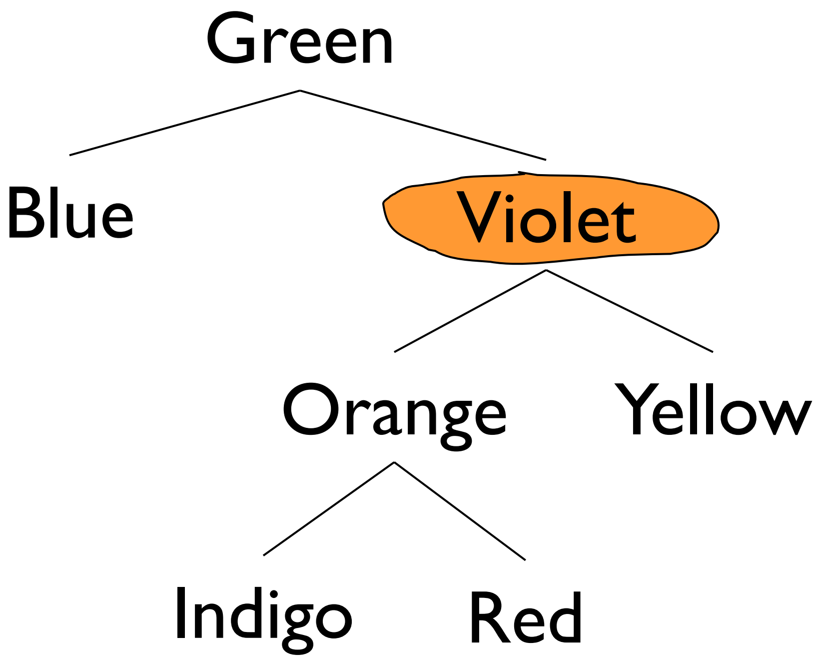
G

Level-Order Traversal



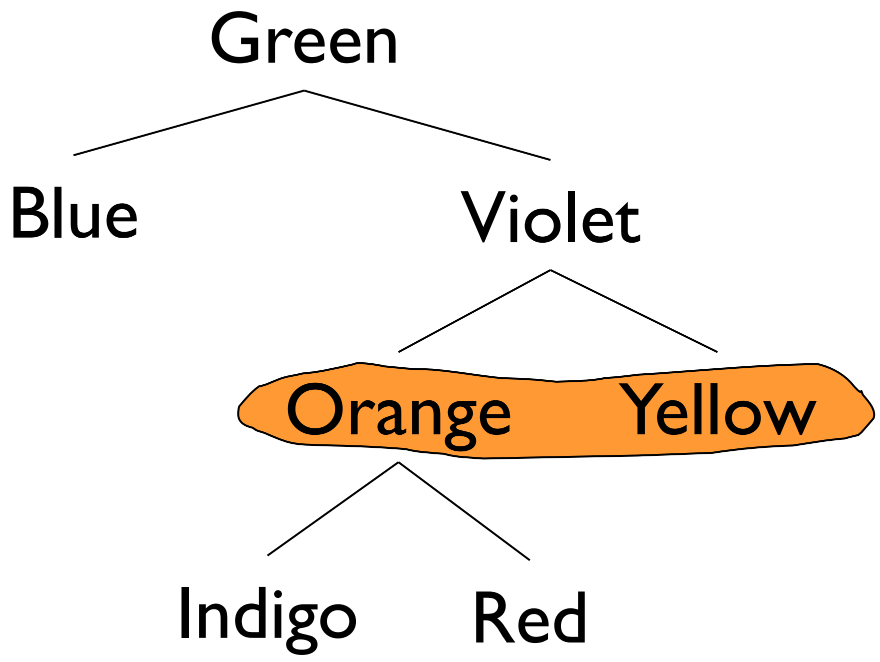
G

Level-Order Traversal



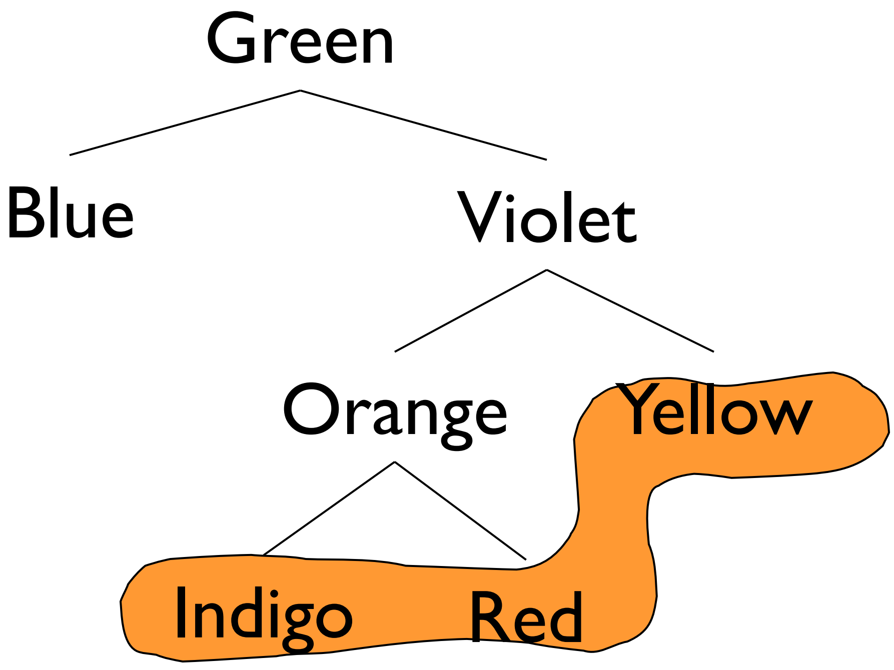
G B

Level-Order Traversal



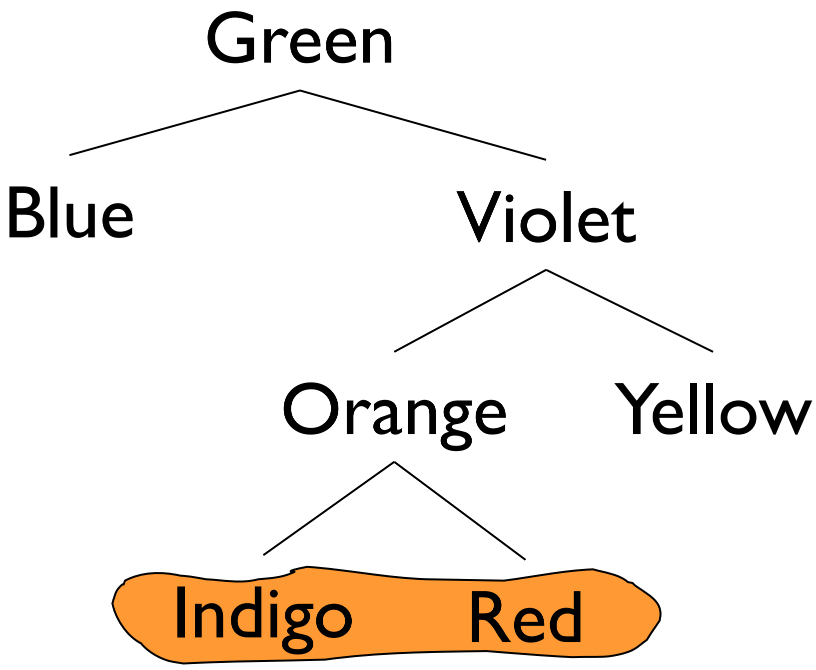
G B V

Level-Order Traversal



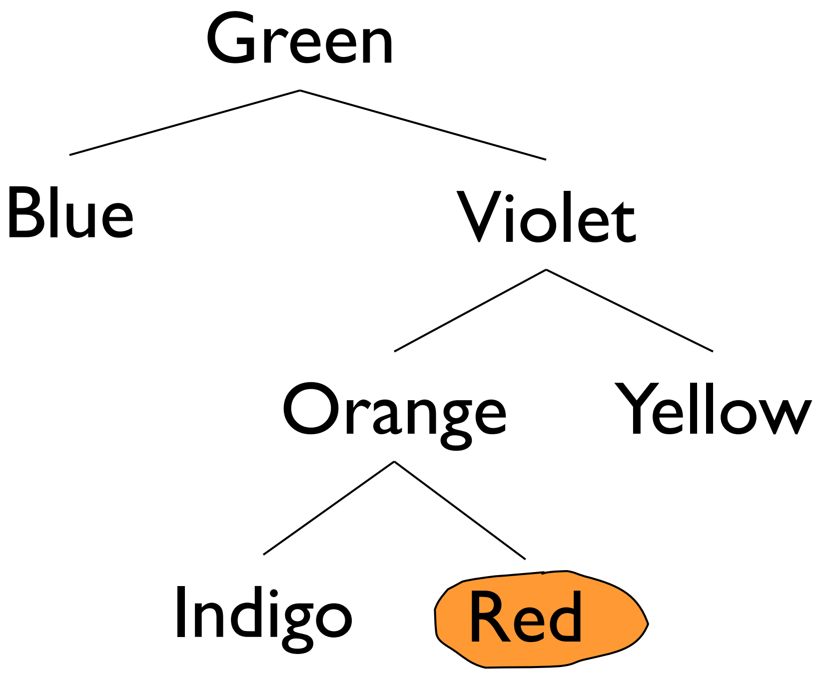
G B V O

Level-Order Traversal



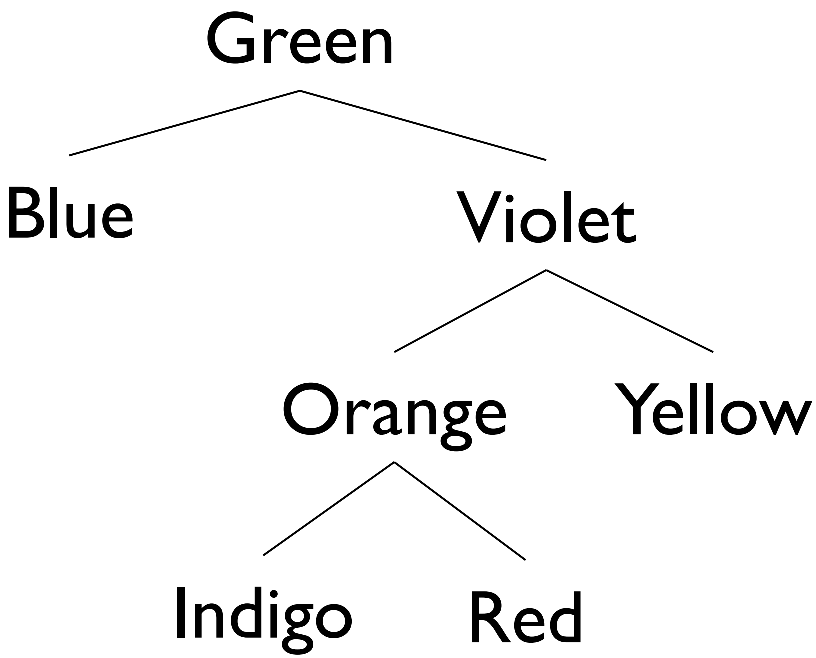
G B V O Y

Level-Order Traversal



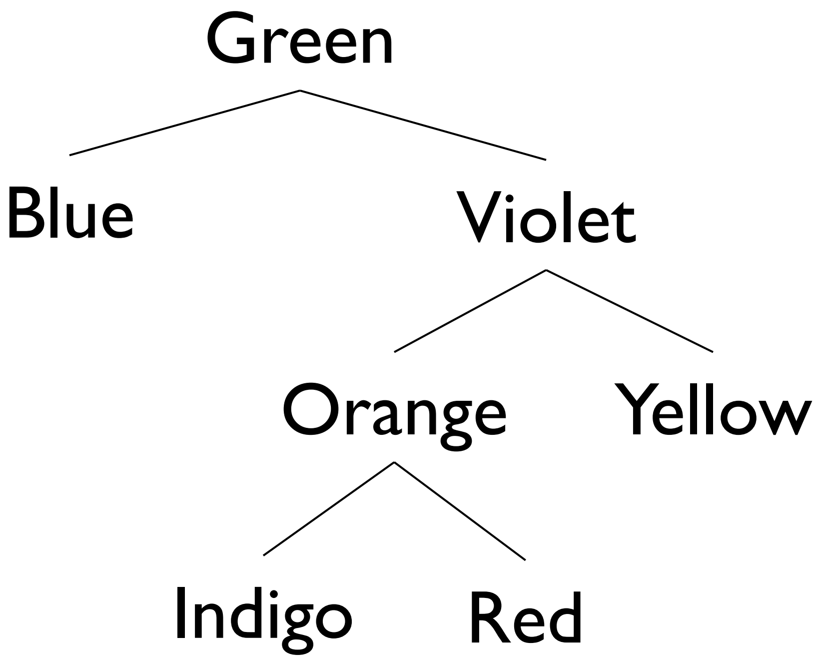
G B V O Y I

Level-Order Traversal

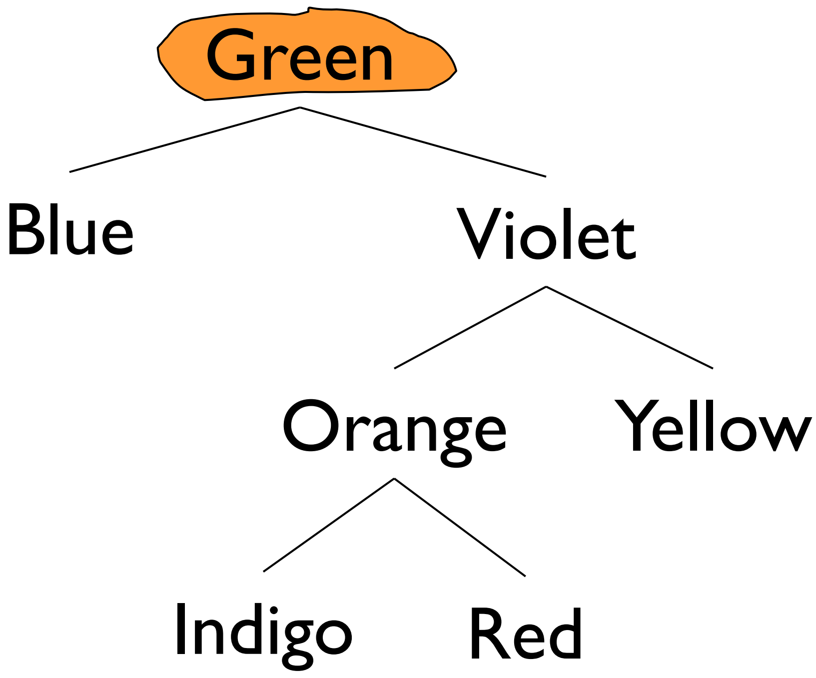


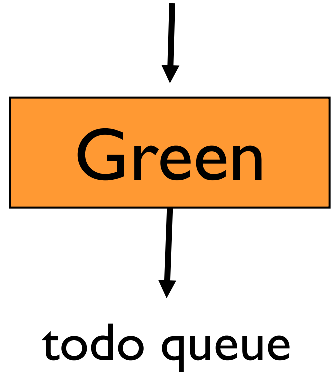
G B V O Y I R

Level-Order Traversal

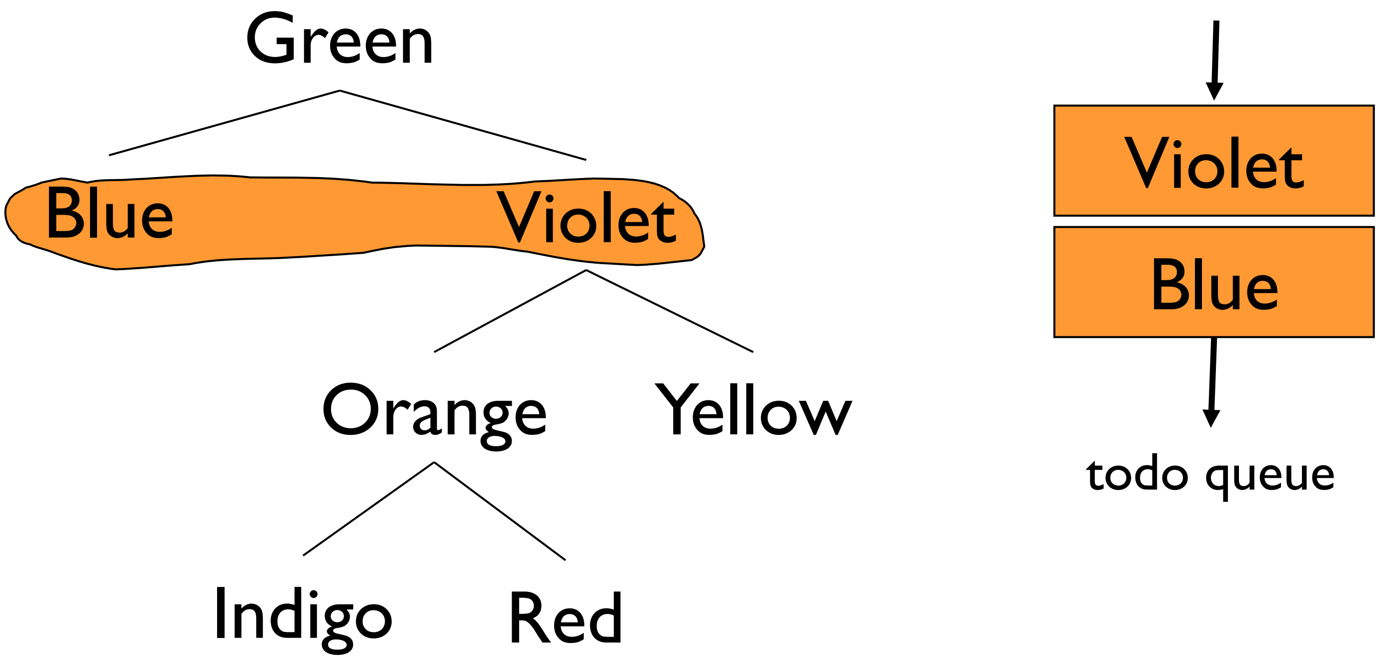


Level-Order Traversal



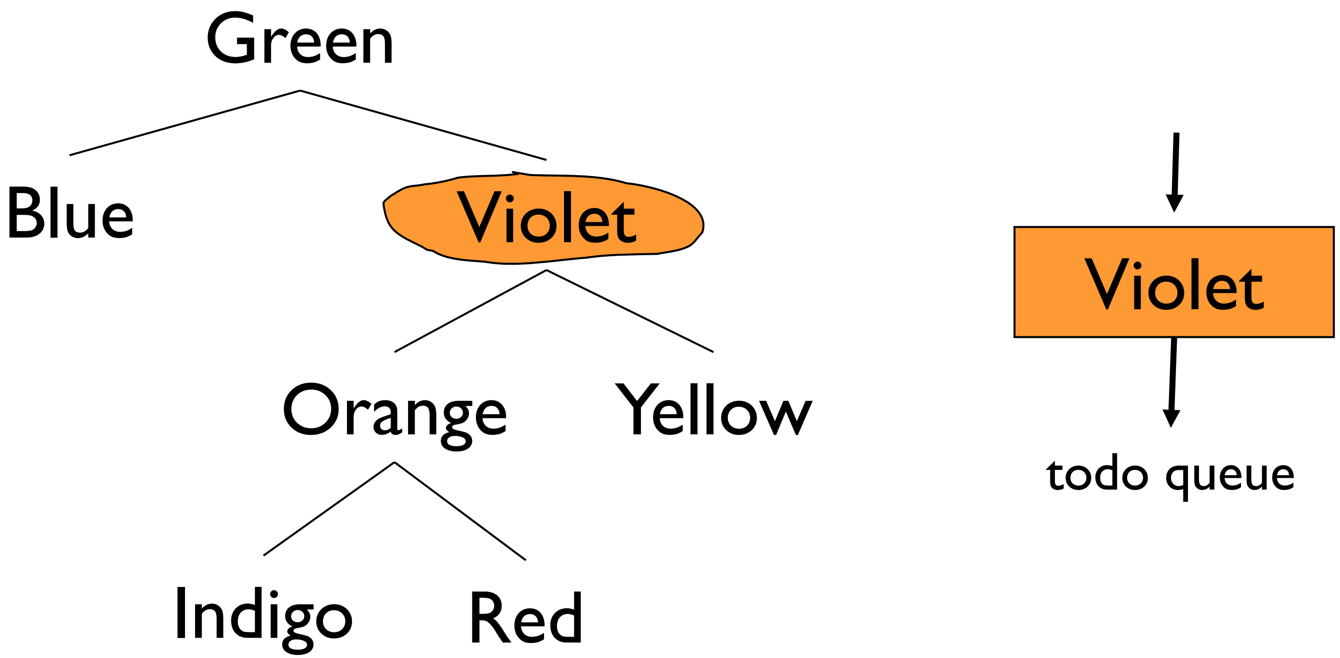


Level-Order Traversal



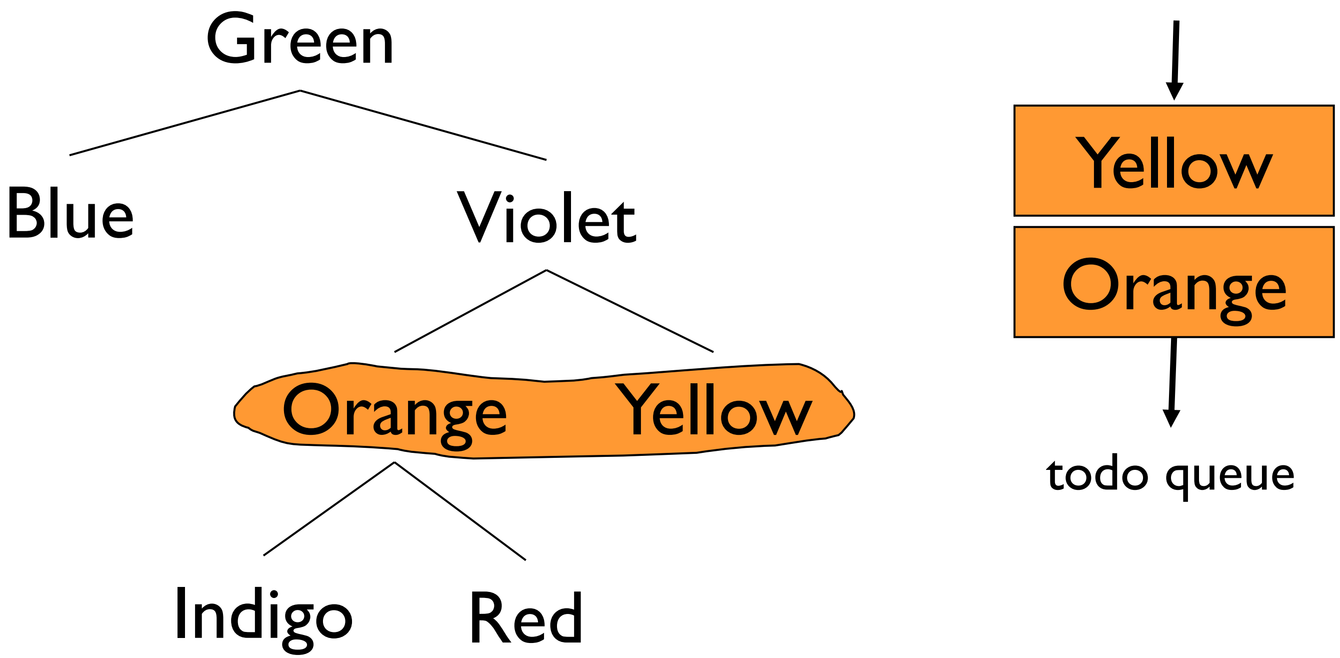
G

Level-Order Traversal



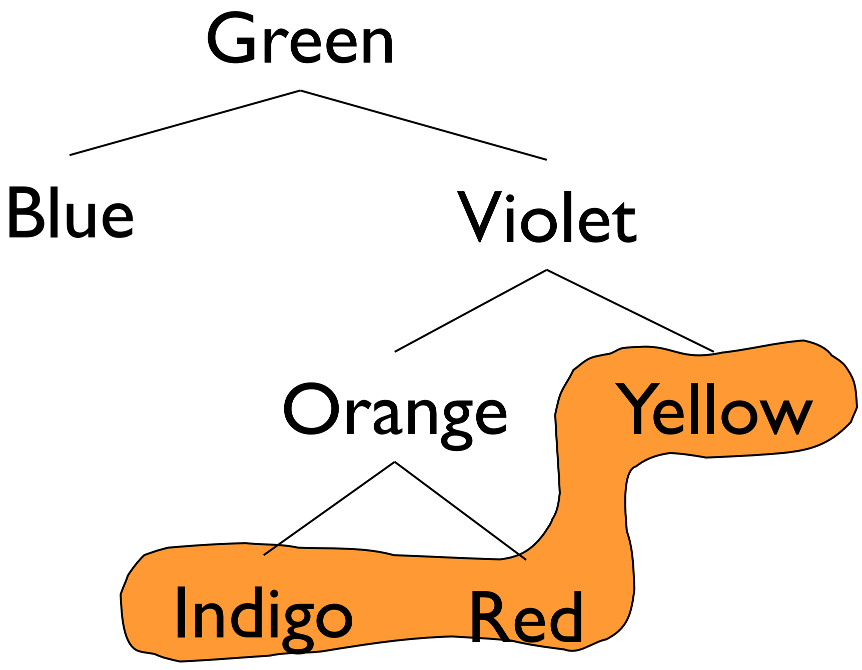
G B

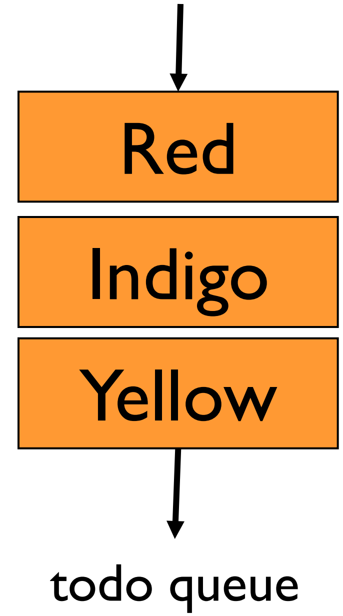
Level-Order Traversal



G B V

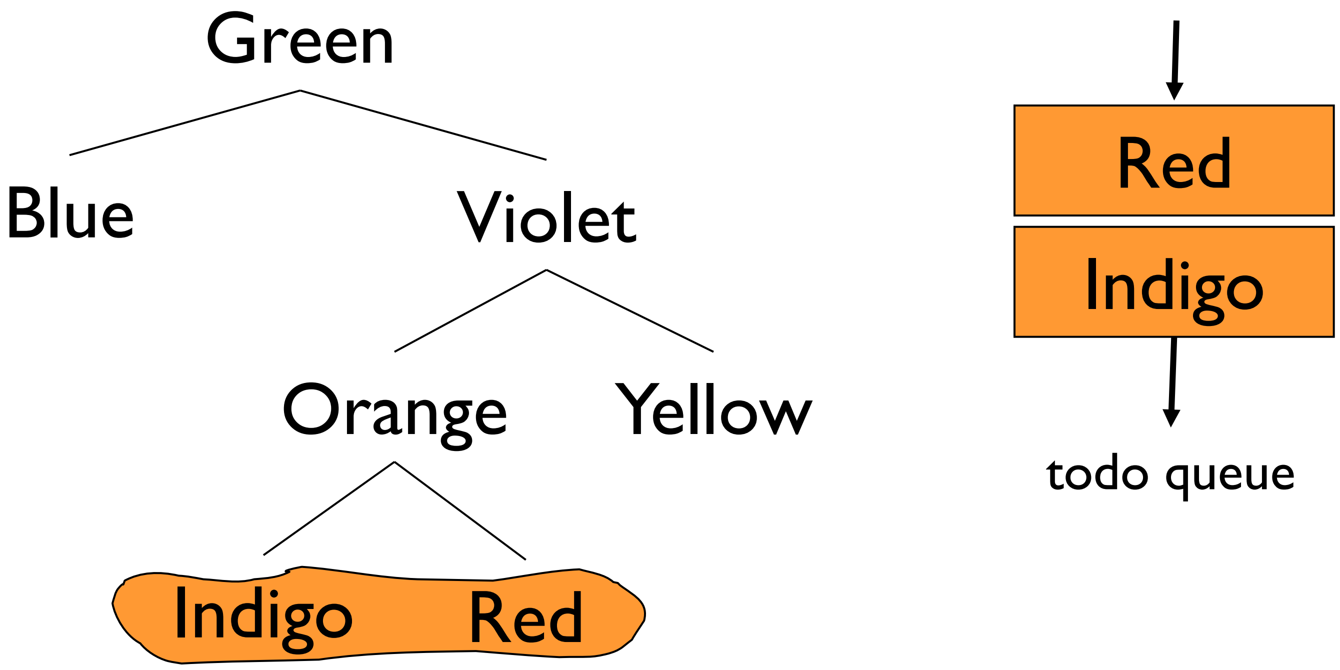
Level-Order Traversal





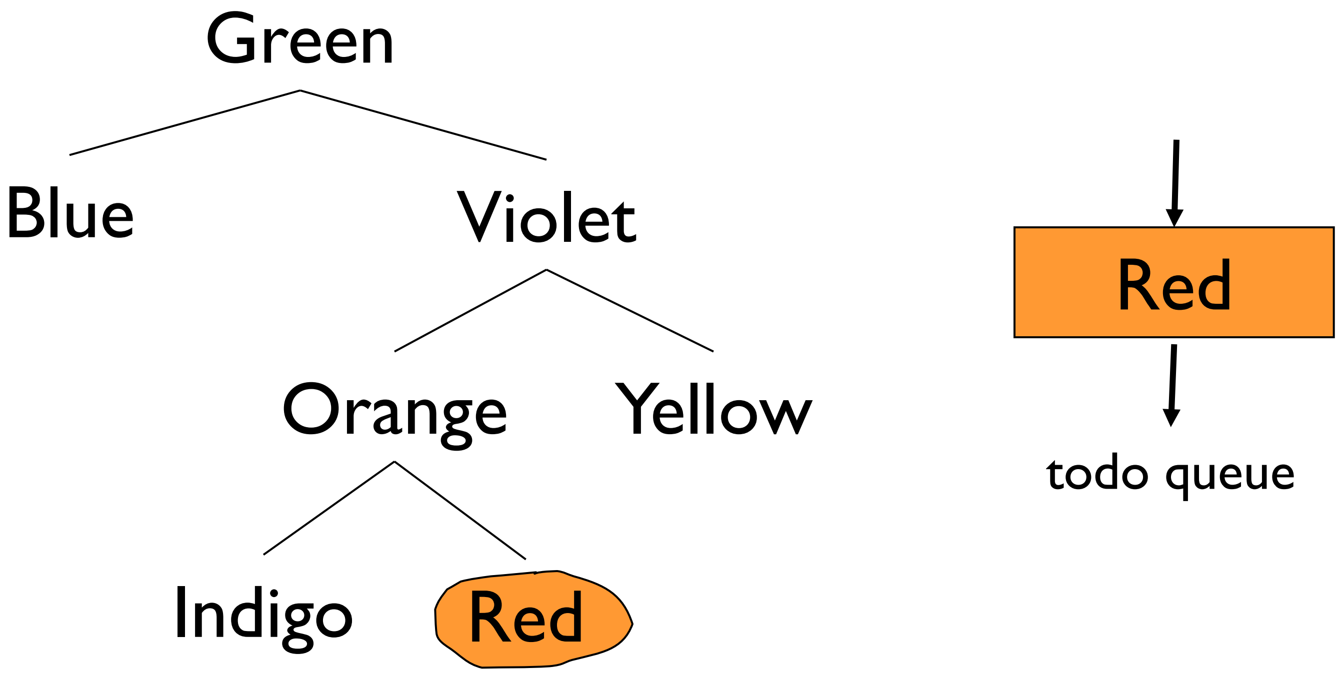
G B V O

Level-Order Traversal



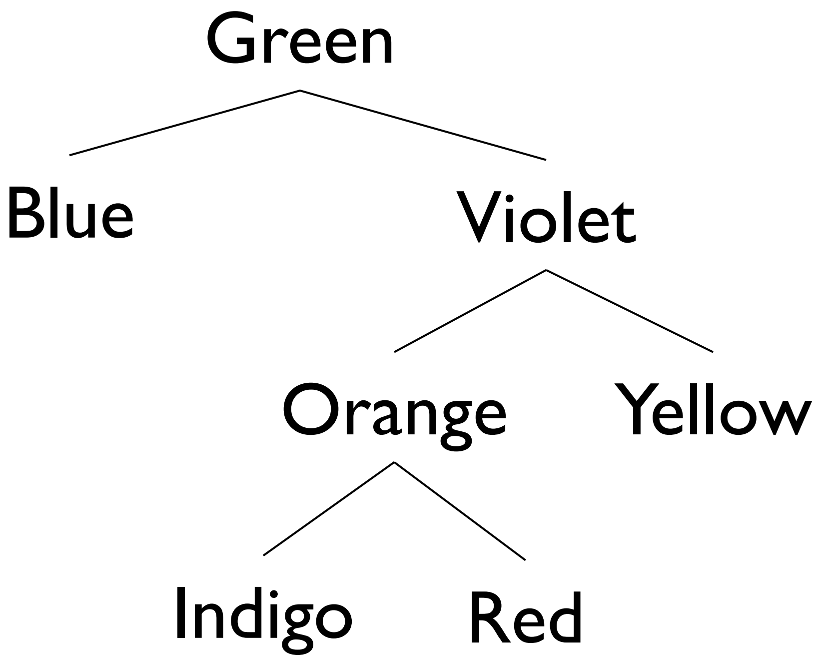
G B V O Y

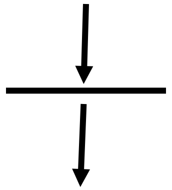
Level-Order Traversal



G B V O Y I

Level-Order Traversal





todo queue

G B V O Y I R

Level-Order Tree Traversal

```
public static <E> void levelOrder(BinaryTree<E> t) {
   if (t.isEmpty()) return;

   // The queue holds nodes for in-order processing
   Queue<BinaryTree<E>> q = new QueueList<BinaryTree<E>>();
   q.enqueue(t); // put root of tree in queue

   while(!q.isEmpty()) {
      BinaryTree<E> next = q.dequeue();
      touch(next);
      if(!next.left().isEmpty()  ) q.enqueue( next.left() );
      if(!next.right().isEmpty() ) q.enqueue(next.right());
   }
}
```

Iterators

- Provide iterators that implement the different tree traversal algorithms

- Methods provided by BinaryTree class:
  - preorderIterator()
  - inorderIterator()
  - postorderIterator()
  - levelorderIterator()

Implementing the Iterators

- Basic idea
  - Should return elements in same order as corresponding traversal method shown
  - Recursive methods don't convert as easily: must phrase in terms of next() and hasNext()
  - So, let's start with levelOrder!

Level-Order Iterator

```
public BTLevelorderIterator(BinaryTree<E> root)
   {
       todo = new QueueList<BinaryTree<E>>();
       this.root = root; // needed for reset
       reset();
   }

public void reset()
   {
       todo.clear();
       // empty queue, add root
       if (!root.isEmpty()) todo.enqueue(root);
   }
```

Level-Order Iterator

```
public boolean hasNext() {
       return !todo.isEmpty();
}

public E next() {
       BinaryTree<E> current = todo.dequeue();
       E result = current.value();
       if (!current.left().isEmpty())
           todo.enqueue(current.left());
       if (!current.right().isEmpty())
           todo.enqueue(current.right());
       return result;
}
```

Pre-Order Iterator

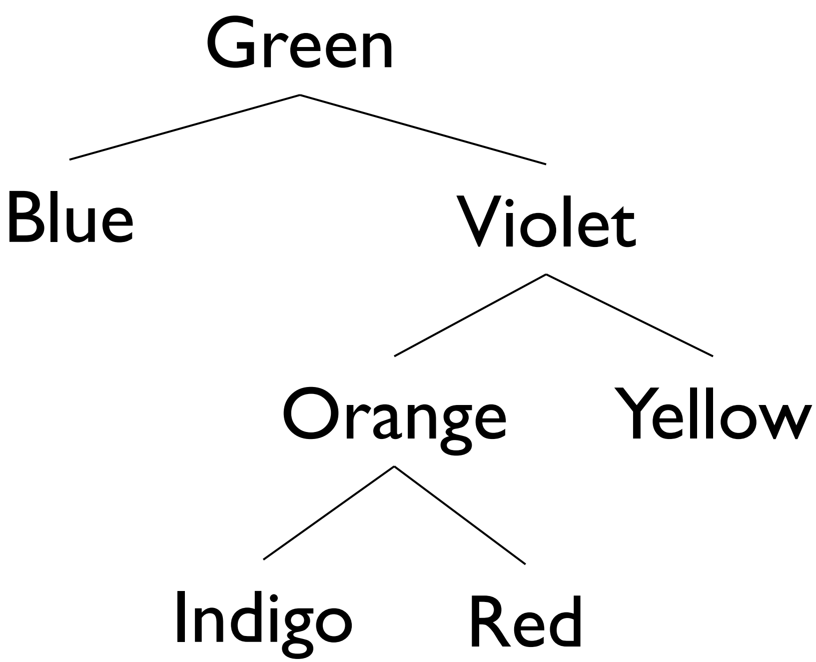
- Basic idea
  - Should return elements in same order as processed by pre-order traversal method
  - Must phrase in terms of next() and hasNext()
  - We "simulate recursion" with stack
    - The stack holds "partially processed" nodes

Pre-Order Iterator

- Outline: node - left tree – right tree
  1. Constructor: Push root onto todo stack
  2. On call to next():
     - Pop node from stack
     - Push right and then left nodes of popped node onto stack
     - Return node's value
  3. On call to hasNext():
     - return !stack.isEmpty()

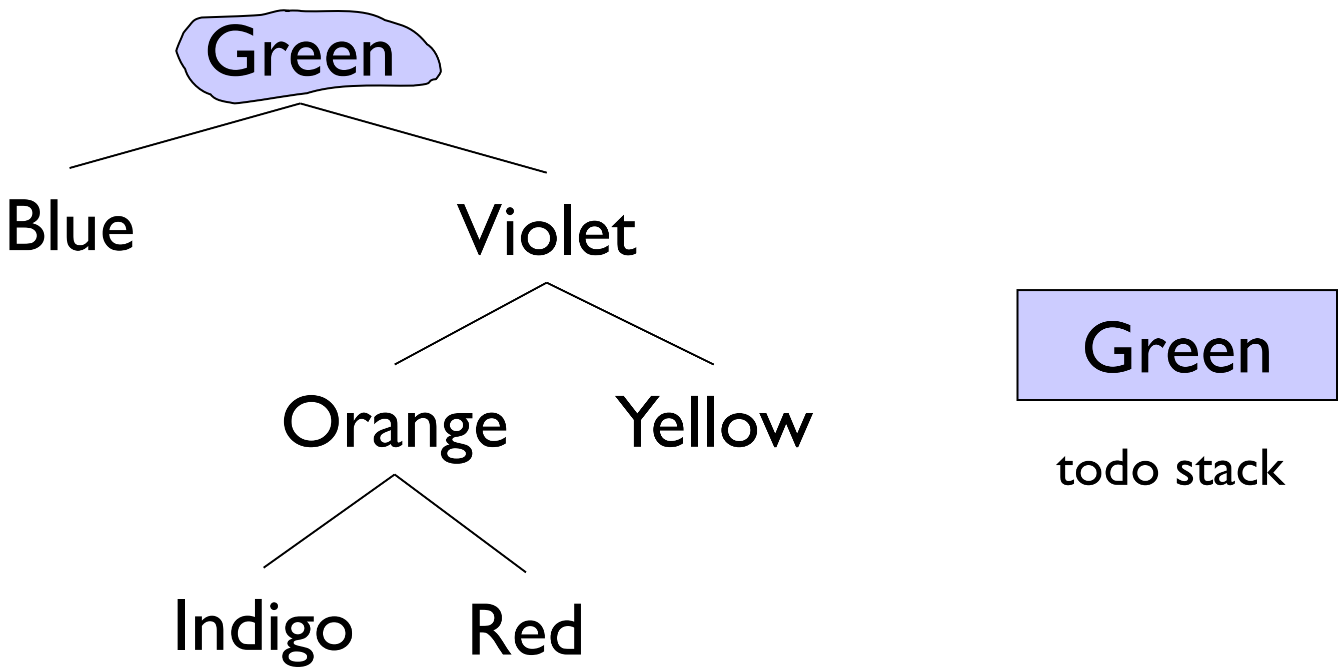
Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



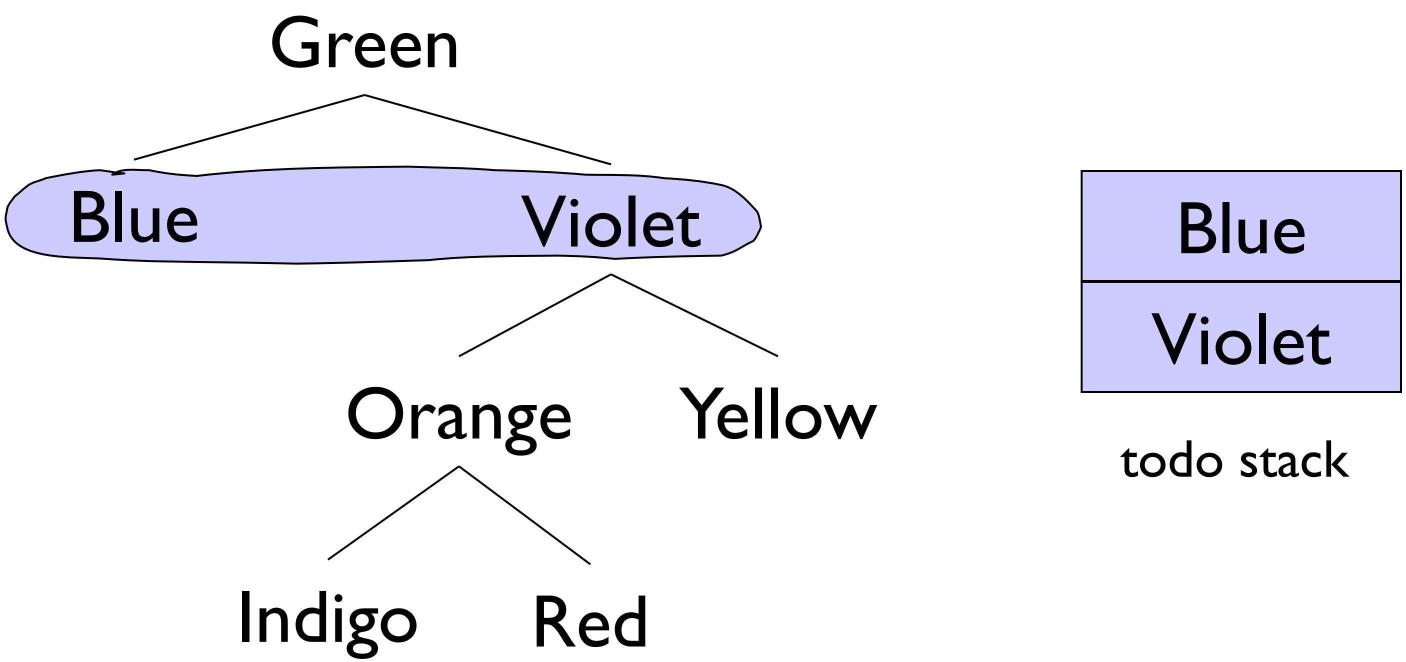
Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



Pre-Order Iterator

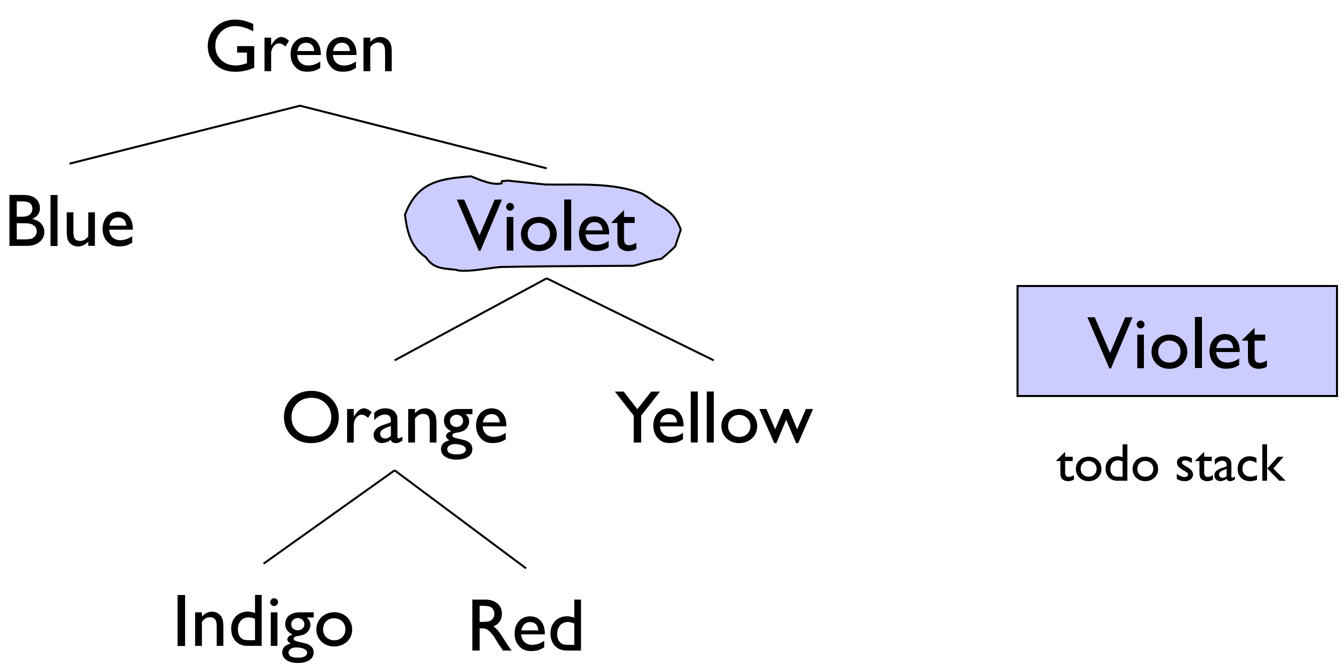
Visit node, then each node in left subtree, then each node in right subtree.



G

Pre-Order Iterator

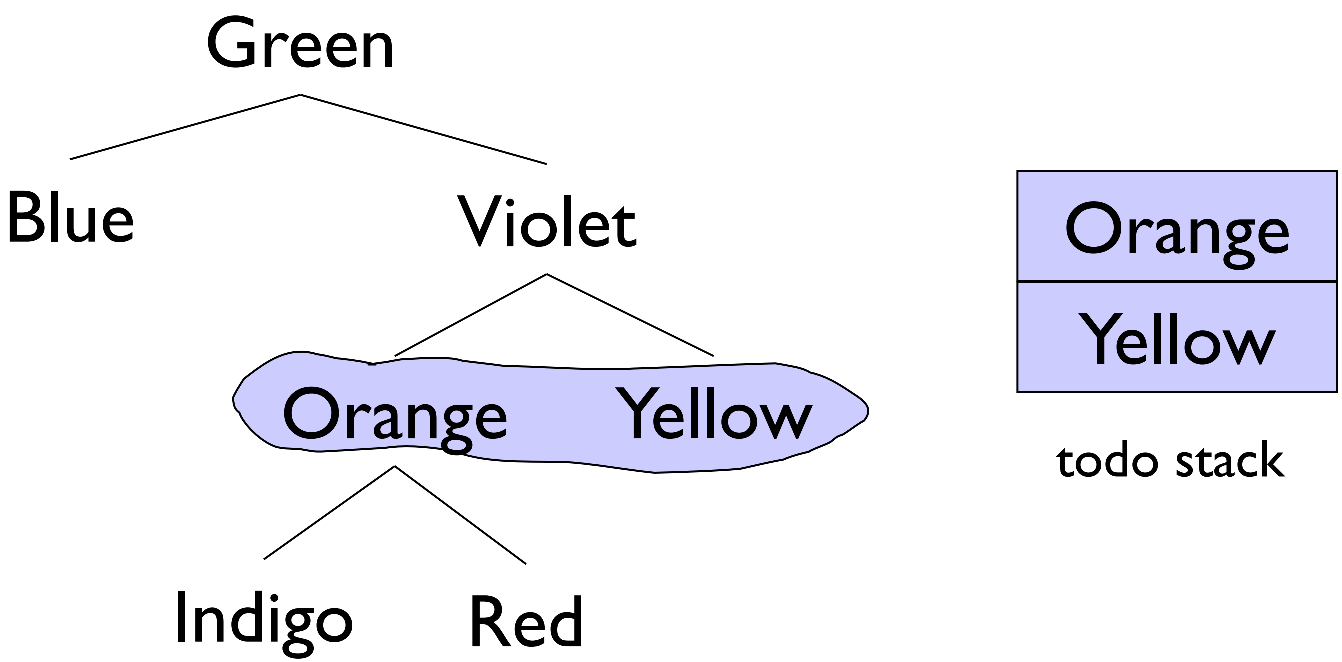
Visit node, then each node in left subtree, then each node in right subtree.



G B

Pre-Order Iterator

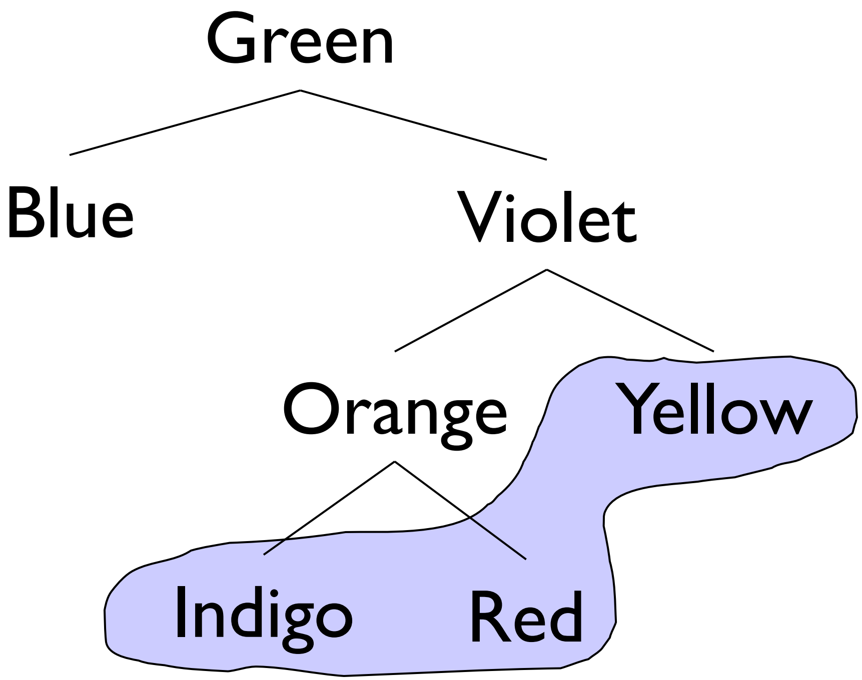
Visit node, then each node in left subtree, then each node in right subtree.

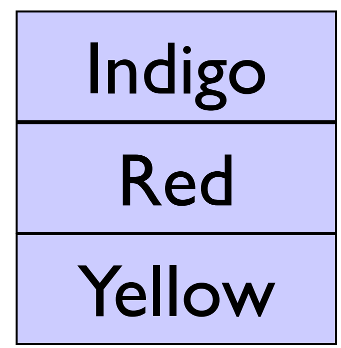


G B V

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



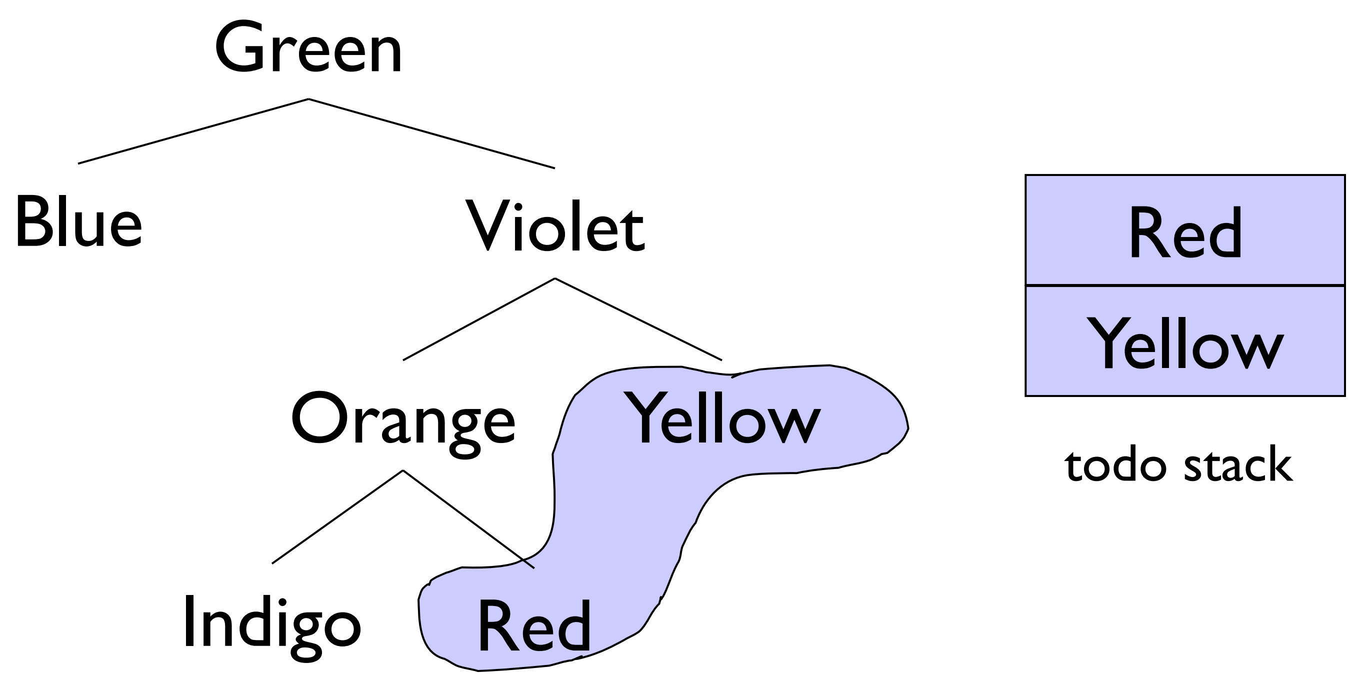


todo stack

G B V O

Pre-Order Iterator

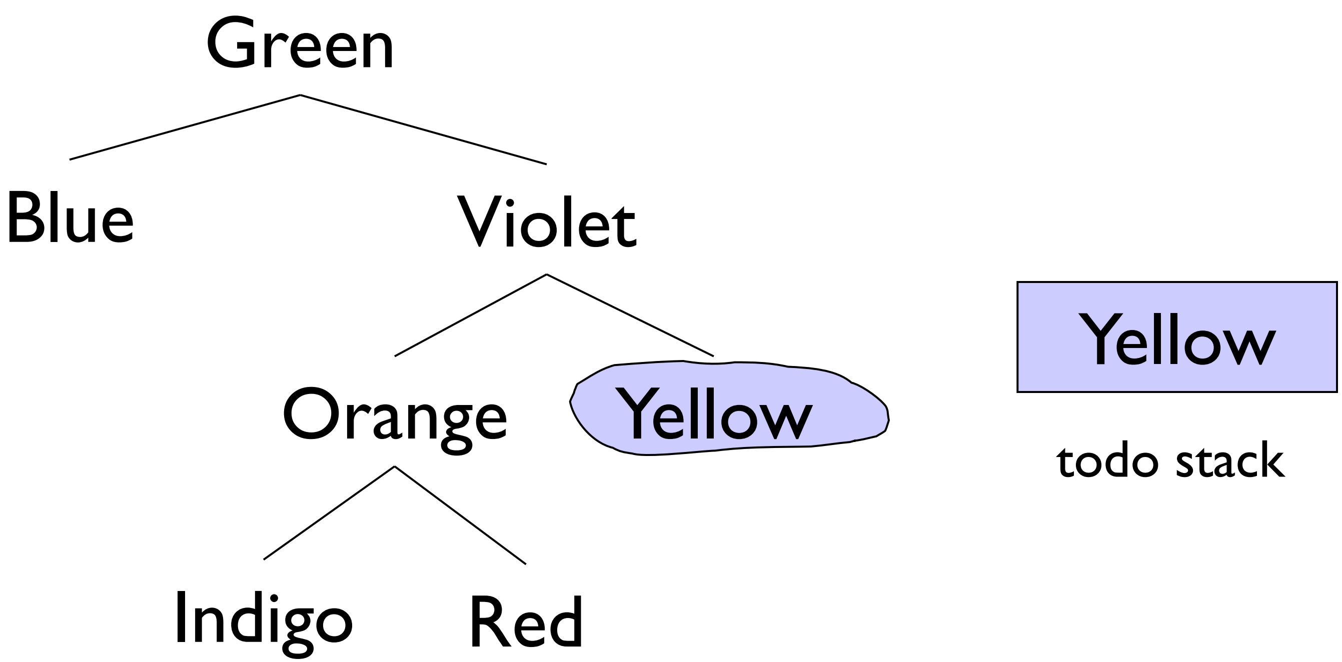
Visit node, then each node in left subtree, then each node in right subtree.



G B V O I

Pre-Order Iterator

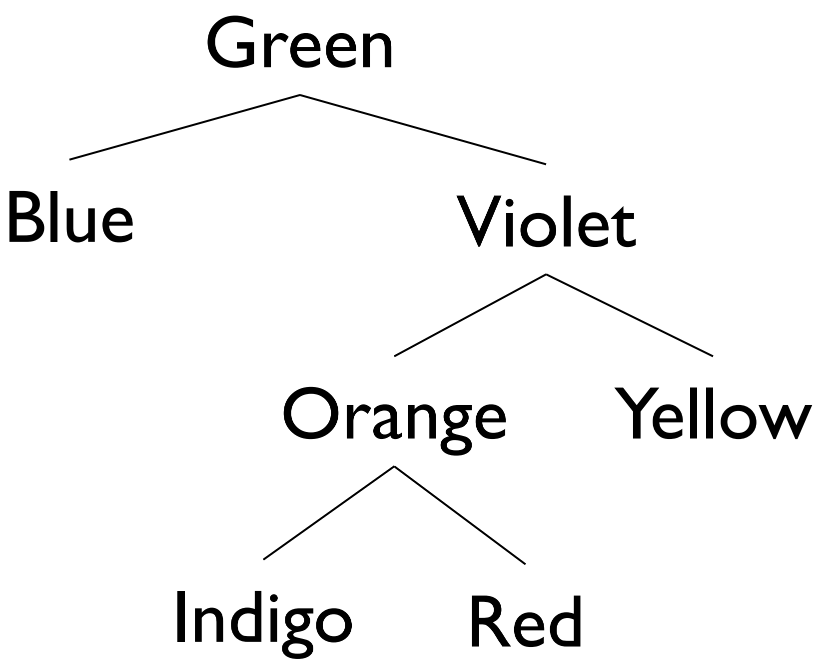
Visit node, then each node in left subtree, then each node in right subtree.



G B V O I R

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



todo stack

G B V O I R Y

Pre-Order Iterator

```
public BTPreorderIterator(BinaryTree<E> root)
   {
        todo = new StackList<BinaryTree<E>>();
        this.root = root;
        reset();
   }

public void reset()
   {
        todo.clear(); // stack is empty; push on root
        if ((!root.isEmpty()) todo.push(root);
   }
```

Pre-Order Iterator

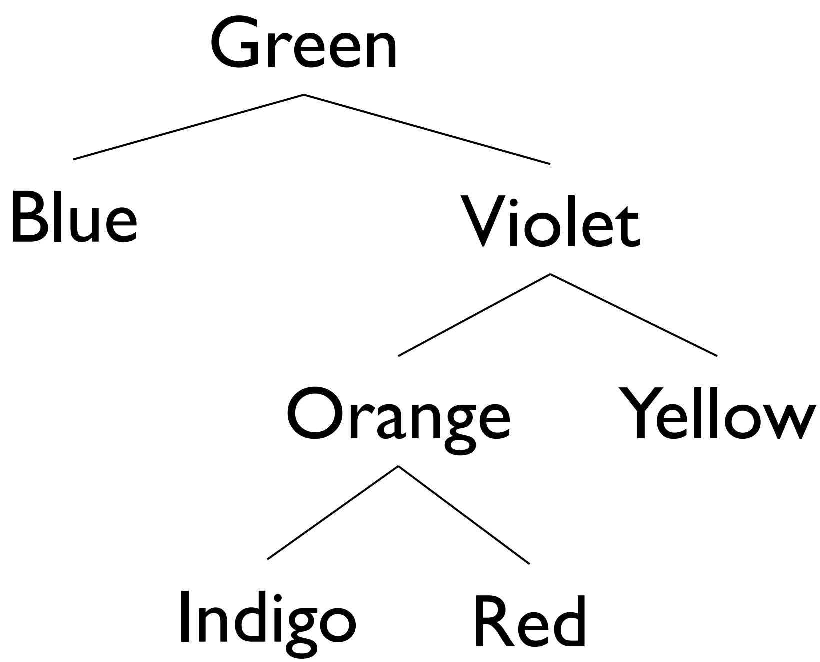
```
public boolean hasNext() {
       return !todo.isEmpty();
}

public E next() {
     BinaryTree<E> old = todo.pop();
     E result = old.value();

     if (!old.right().isEmpty())
           todo.push(old.right());
     if (!old.left().isEmpty())
           todo.push(old.left());
       return result;
   }
```

Tree Traversal Practice Problems

- Prove that levelOrder() is correct: that is, that it touches the nodes of the tree in the correct order (Hint: induction by level)
- Prove that levelOrder() takes O(n) time, where n is the size of the tree
- Prove that the PreOrder (LevelOrder) Iterator visits the nodes in the same order as the PreOrder (LevelOrder) traversal method

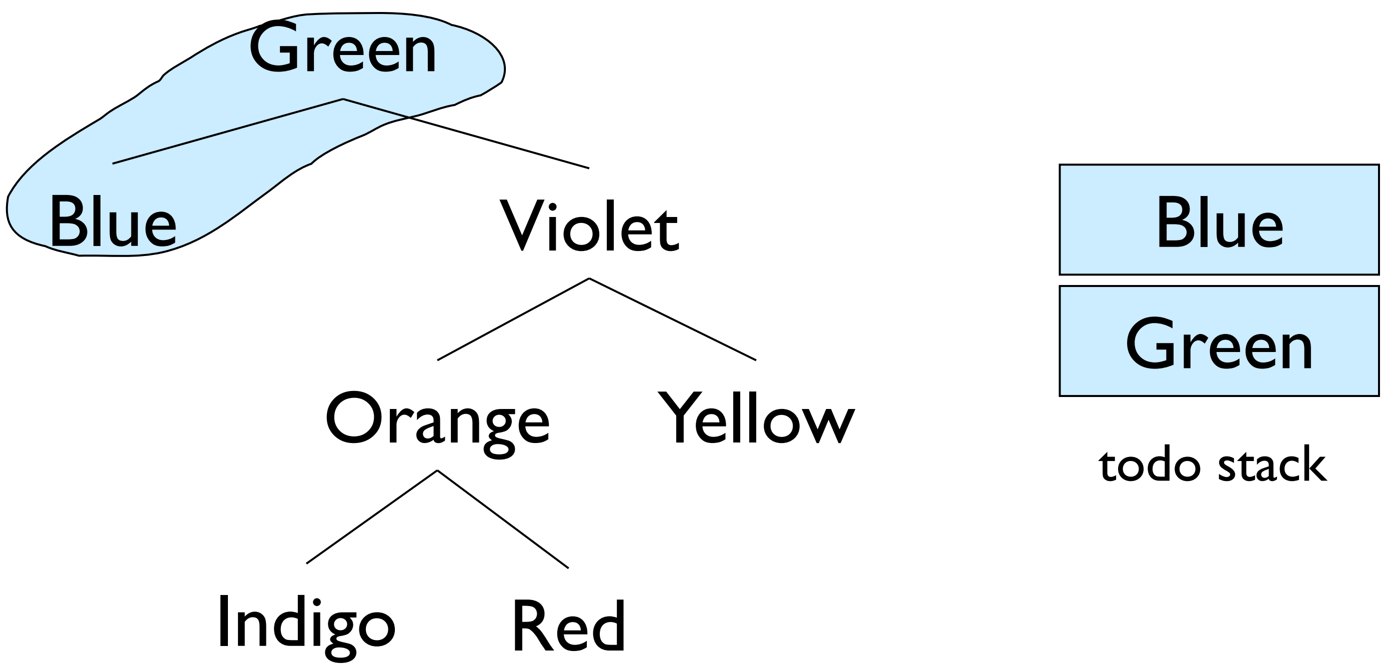
In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



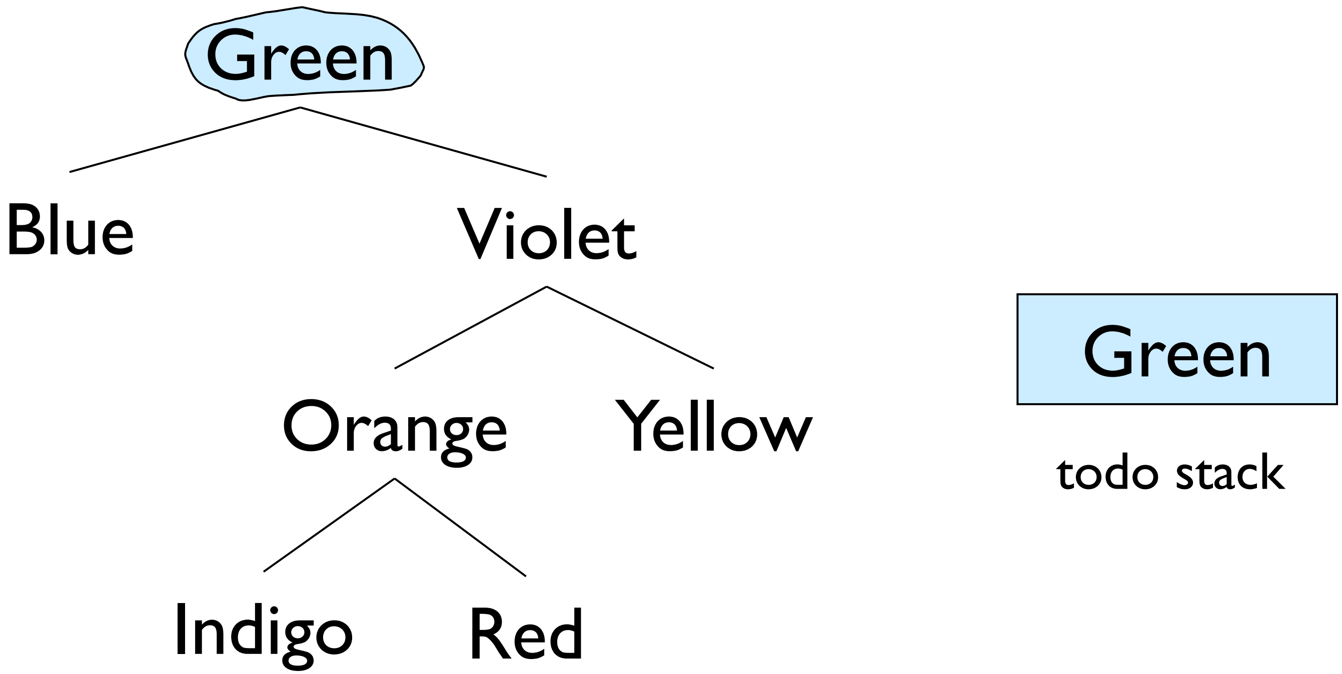
In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



In-Order Iterator

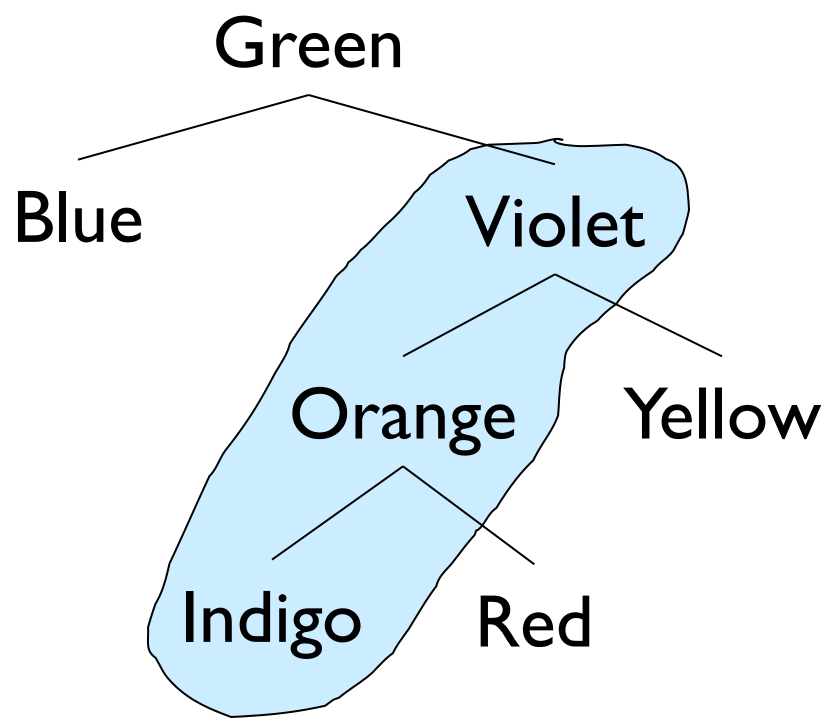
Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



B

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.

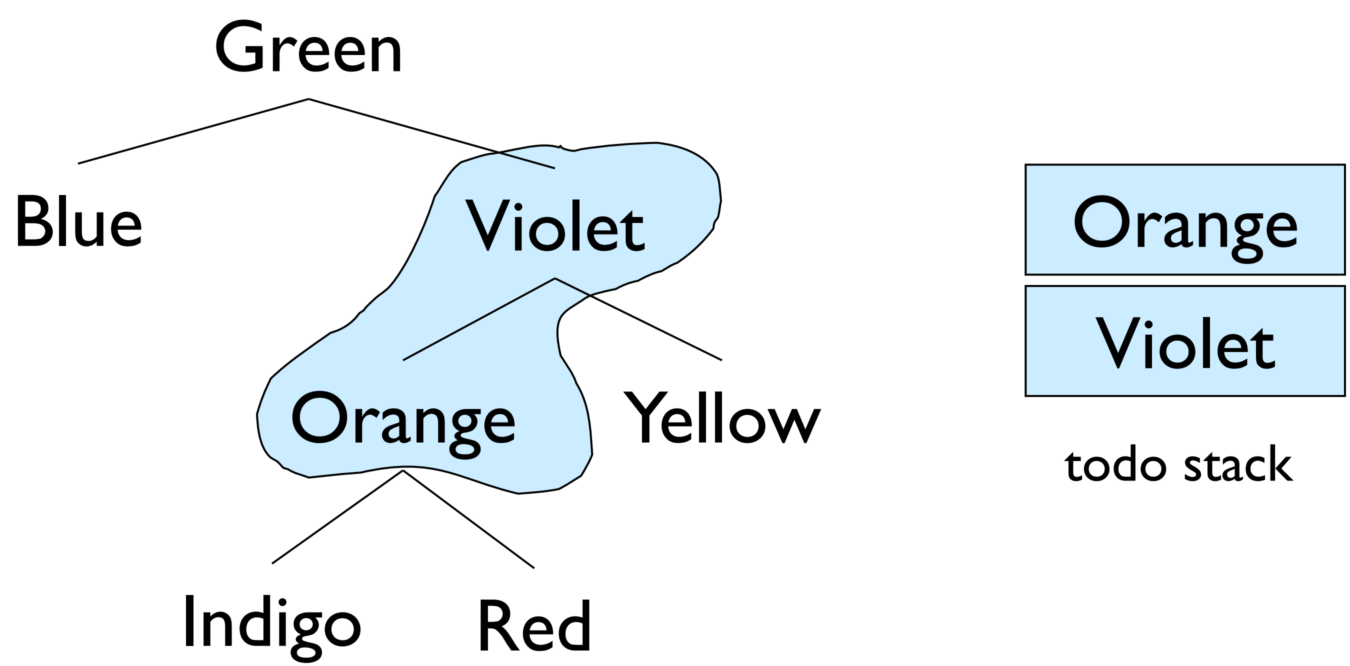


todo stack

B G

In-Order Iterator

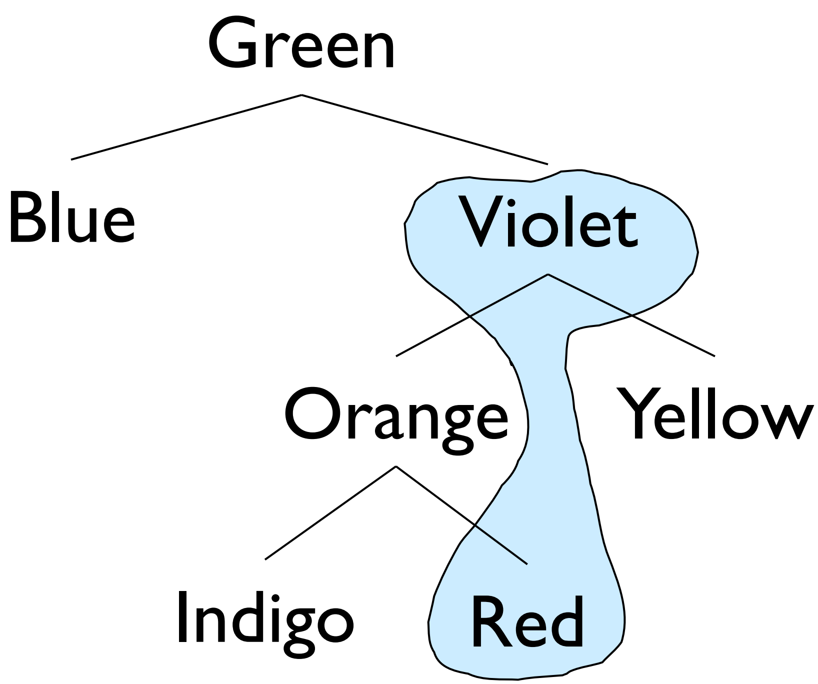
Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.

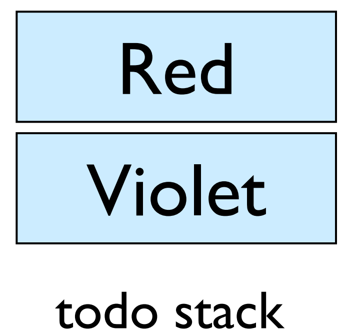


B G I

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.

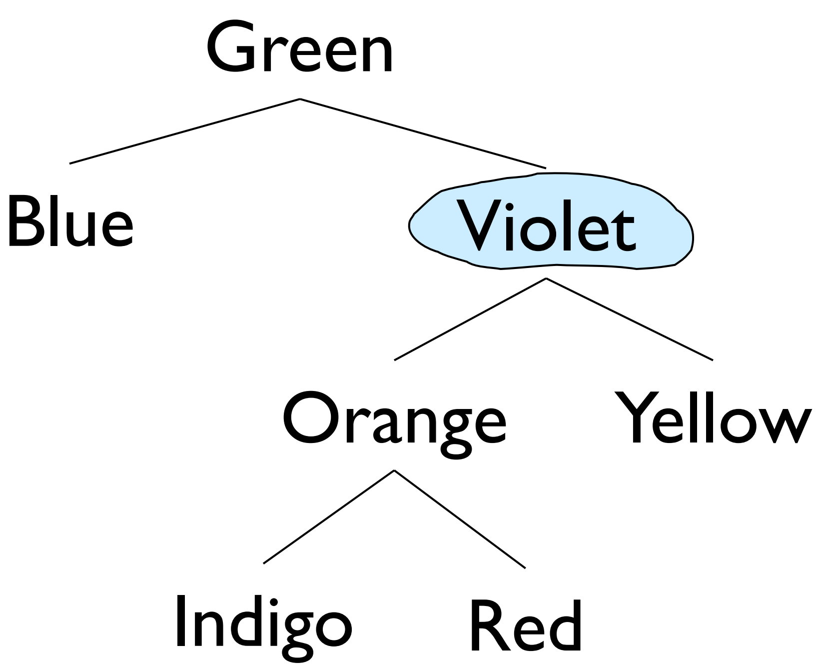


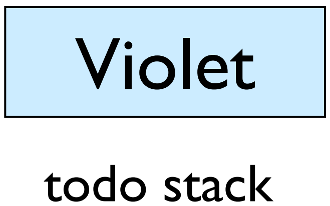


B G I O

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.

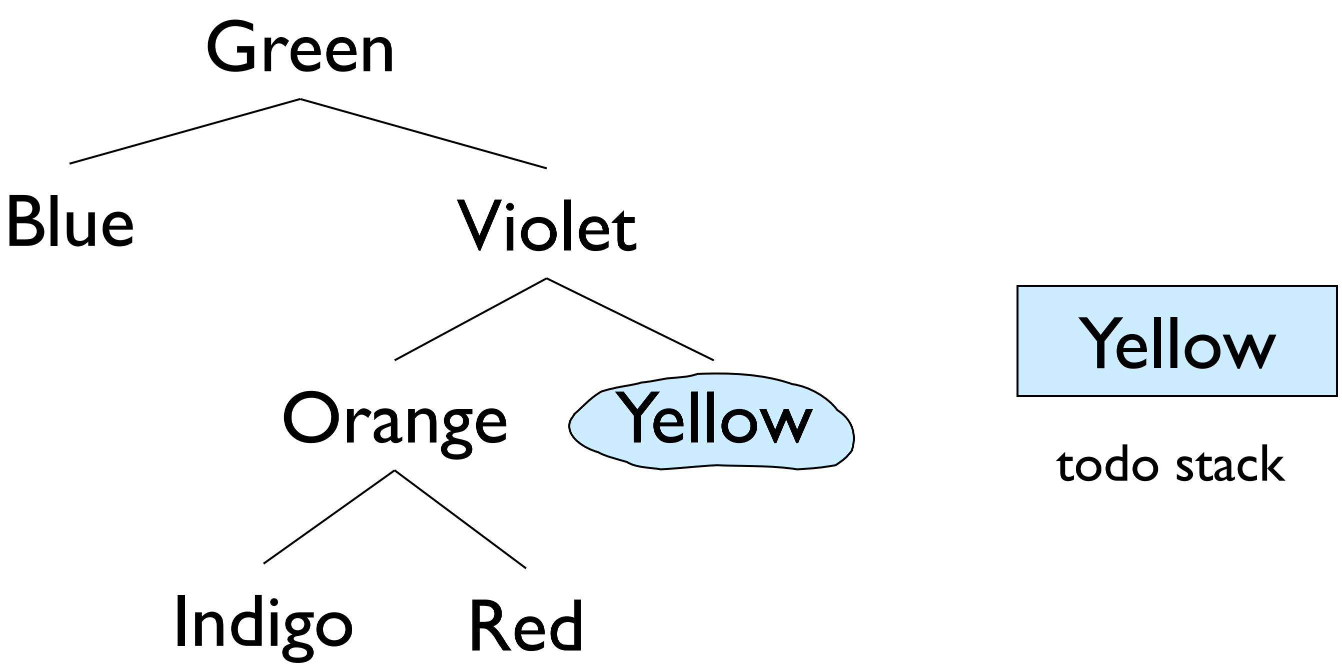




B G I O R

In-Order Iterator

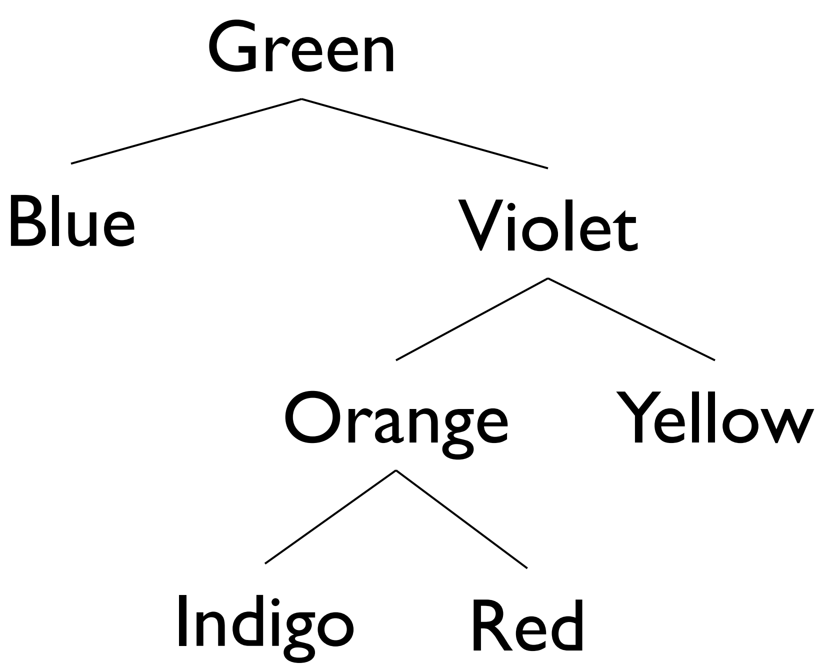
Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



B G I O R V

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



todo stack

B G I O R V Y

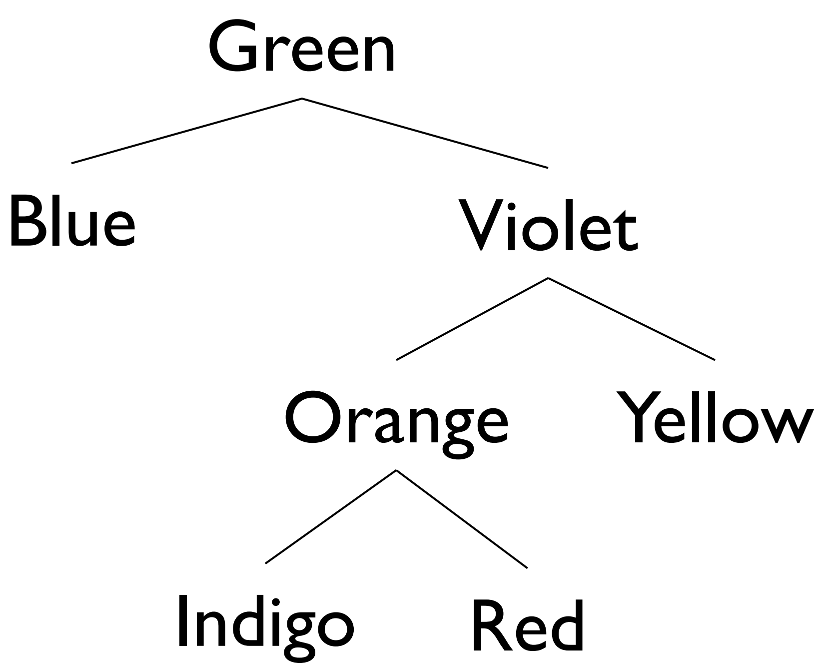
In-Order Iterator

- Outline: left - node - right
  1. Push left children (as far as possible) onto stack
  2. On call to next():
     - Pop node from stack
     - Push right child and follow left children as far as possible
     - Return node's value
  3. On call to hasNext():
     - return !stack.isEmpty()

Post-Order Iterator

- Left as an exercise…

Alternative Tree Representations



- Total # "slots" = 4n
  - Since each BinaryTree maintains a reference to left, right, parent, value
- 2-4x more overhead than vector, SLL, array, …
- But trees capture successor and predecessor relationships that other data structures don't…

Array-Based Binary Trees

- Encode structure of tree in array indexes
  - Put root at index 0
- Where are children of node i?
  - Children of node i are at 2i+1 and 2i+2
  - Look at example
- Where is parent of node j?
  - Parent of node j is at (j-1)/2

ArrayTree Tradeoffs

- Why are ArrayTrees good?
  - Save space for links
  - No need for additional memory allocated/garbage collected
  - Works well for full or complete trees
    - Complete: All levels except last are full and all gaps are at right
    - "A *complete* binary tree of height h is a full binary tree with 0 or more of the rightmost leaves of level h removed"
- Why bad?
  - Could waste a lot of space
  - Tree of height of n requires $2^{n+1}-1$ array slots even if only O(n) elements