CSCI 136
Data Structures &
Advanced Programming

Lecture 12

Fall 2017

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Administrative Details

- Lab 4 Today!
  - Try to answer questions before lab
- Mountain Day Madness!
  - If This Friday is Mountain Day
    - Lab 5 will go on-line this weekend
    - Problem Set 2---coming this Friday---will also go on-line this weekend (due next Friday at start of class)
    - And---OMG---we won't see you again until next Wednesday!!!

Last Time

- More about Mathematical Induction
  - For algorithm run-time and correctness
- More About Recursion
  - Recursion on arrays; helper methods
  - Recursion on Chains
- Strong Induction
- Linear and Binary Searching review

Today's Outline

- The Comparable Interface
- Basic Sorting
  - Bubble, Insertion, Selection Sorts
  - Including proofs of correctness

And, if time permits…

- Comparator interfaces for flexible sorting
- More Efficient Sorting Algorithms
  - MergeSort, QuickSort

Recall : Binary Search

```
public class BinSearch {

   public static int binarySearch(int a[], int value) {
        return recBinarySearch(a, value, 0, a.length-1); }

   protected static int recBinarySearch(int a[], int value, int
        low, int high) {

        if (low > high) return -1;
        else {
               int mid = (low + high) / 2;   //find midpoint
        if (a[mid] == value) return mid;     //first comparison
                                             //second comparison
        else if (a[mid] < value)             //search upper half
               return recBinarySearch(a, value, mid + 1, high);
        else                                 //search lower half
               return recBinarySearch(a, value, low, mid - 1);
   }
```

Recall: Binary Search

- Why does it work?
  - Because items can be ordered (they are *comparable*)
  - So they can be sorted then searched based on ordering
- Why is it fast?
  - Cut `search space in half with each comparison!
- Requires items to be *comparable*
- If items are not comparable, we typically need to do a *linear search*

Linear Search

- Complexity analysis of linear search:
  - Best case: O(1)
  - Worst case: O(n)
  - Average case: O(n)
  - Recall
    - Assume all locations equally likely
    - The average number of comparisons is

$$(1 + 2 + 3 + \ldots + n)/n = (n+1)/2, \text{ so } O(n)$$

- Here's a *generic* linear search method

Generic Linear Search Method

```
public class LinearSearchGeneric {
 // post: returns index of value in a, or -1 if not found
 // Note the <E> between static and int: a generic method!
    public static <E> int linearSearch(E a[], E value) {
        for (int i = 0; i < a.length; i++) {
            if (a[i].equals(value)) {
                return i;
            }
        }
        return -1;
    }
   public static void main(String args[]) {
        // search a String array
        System.out.println(linearSearch(args, "cow"));
        // search an Integer array
        Integer odds[] = new Integer[] { 1,3,5,7,9 };
        System.out.println(linearSearch(odds, 7));
    }
}
```

Linear vs. Binary Search

- Clearly binary is preferable
- But it requires ordered (i.e., sorted) data.
  - We need *comparable* items
  - Unlike with equality testing, the Object class doesn't define a "compare()" method 😟
  - We want a uniform way of saying objects can be compared, so we can write generic versions of methods like binary search
  - Use an interface! (We'll see two approaches)

Comparable Interface

- Java provides an interface for comparisons between objects
  - Provides a replacement for “<“ and “>” in recBinarySearch
- Java provides the *Comparable* interface, which specifies a method *compareTo()*
  - Any class that implements Comparable, provides compareTo()

```
public interface Comparable<T> {
    //post: return < 0 if this smaller than other
            return 0 if this equal to other
            return > 0 if this greater than other
      int compareTo(T other);
}
```

compareTo in Card Example

We could have written

```
public class CardRankSuit implements
      Comparable<CardRankSuit> {

   public int compareTo(CardRankSuit other) {
      if (this.getSuit() != other.getSuit())
         return getSuit().compareTo(other.Suit());
      else
         return getRank().compareTo(other.getRank());
   }
// rest of code for the class....
}
```

compareTo in Card Example

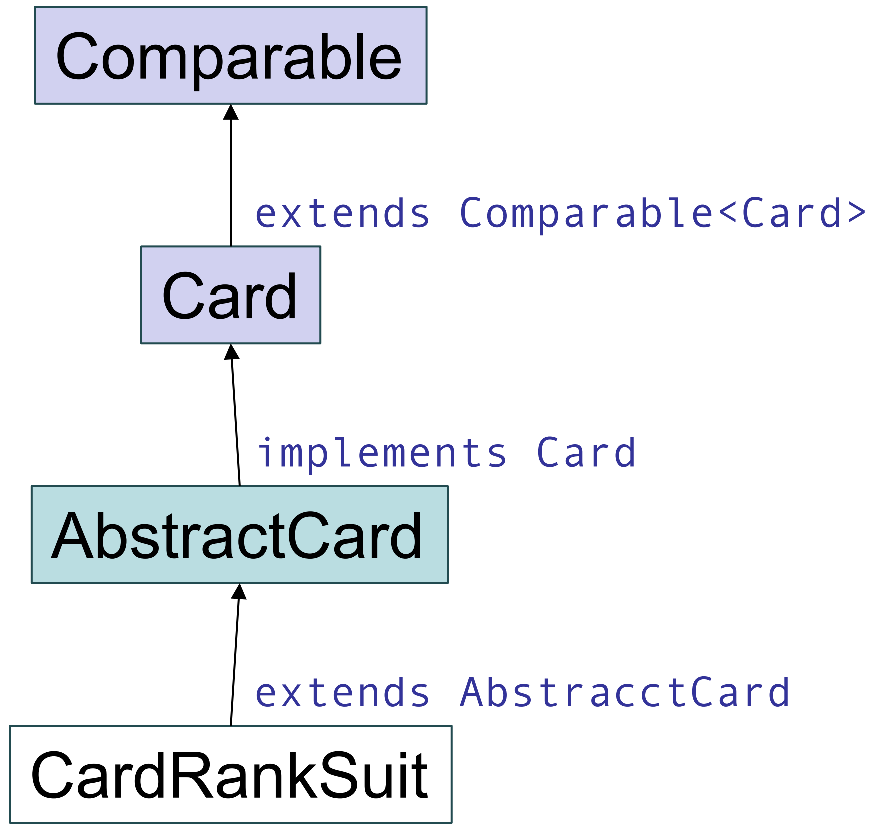
We actually wrote (in Card.java)

```
public interface Card extends Comparable<Card> {
   public int compareTo(Card other);
   // remainder of interface code
}
```

And in CardAbstract.java, we added

```
   public int compareTo(Card other) {
       if (this.getSuit() != other.getSuit())
          return getSuit().compareTo(other.Suit());
       else
          return getRank().compareTo(other.getRank());
   }
```

Class/Interface Hierarchy



- As a result, all of our implementations of the Card interface have comparable card types!

compareTo in Card Example

Notes

- Enum types implement Comparable and define compareTo
- The magnitude of the values returned by compareTo are not important. We only care if value is positive, negative, or 0!
- compareTo defines a *"natural ordering"* of Objects
  - There's nothing "*natural*" about it….
- We use the *BubbleSort* algorithm to sort the cards in CardDeck.java

Comparable & compareTo

- The Comparable interface (Comparable<T>) is part of the java.lang (not structure5) package.
- Other Java-provided structures can take advantage of objects that implement Comparable
  - See the Arrays class in java.util
  - Example JavaArraysBinSearch
- Users of Comparable are urged to ensure that *compareTo()* and *equals()* are *consistent*. That is,
  - x.compareTo(y) == 0 exactly when x.equals(y) == true
- Note that Comparable limits user to a *single ordering*
- The syntax can get kind of dense
  - See BinSearchComparable.java : a generic binary search method
  - And even more cumbersome….

ComparableAssociation

- Suppose we want an *ordered* Dictionary, so that we can use binary search instead of linear
- Structure5 provides a ComparableAssociation class that implements Comparable.
- The class declaration for ComparableAssociation is

…wait for it...

public class ComparableAssociation<K extends Comparable<K>, V>
    Extends Association<K,V> implements
    Comparable<ComparableAssociation<K,V>>

(Yikes!)

- Example: Since Integer implements Comparable, we can write
  - ComparableAssociation<Integer, String> myAssoc =
    new ComparableAssociation(new Integer(567), “Bob”);
- We could then use Arrays.sort on an array of these

Subset Sum

- Given an array a[] of integers and a target integer T, is there a subset of the integers in the array that sum to T?
- Example a[] = 10, 7, 12, 3, 5, 11, 8, 9, 1, 15:
  - T = 31? Yes: 10 + 7 + 5 + 9
  - T = 79? No. [Why?]
- How could we solve this problem?
  - Hint: Either we use a[0] or we don't….
  - Need: canMakeSumHelper(int set[], int target, int index)
- How could we prove our method was correct?

Complexity Analysis of Subset Sum

- The Subset Sum algorithm we wrote is slow.
- How slow?
- Let s_n be the *minimum* number of steps the algorithm might take on an array of size n.
  - $s_n \geq 1 + s_{n-1} + s_{n-1} > 2\, s_{n-1}$
  - $s_1 = 1$
- Claim: $s_n \geq 2^{n-1}$---an exponential *lower* bound
  - Proof: Induction. [Easy: try it for homework]
- Can also prove an *upper* bound of $O(2^n)$

Bubble Sort

- First Pass:
  - (**5** 1 3 2 9) → (1 **5** 3 2 9)
  - (1 **5** 3 2 9) → (1 3 **5** 2 9)
  - (1 3 **5** 2 9) → (1 3 2 **5** 9)
  - (1 3 2 **5** 9) → (1 3 2 5 **9**)
- Second Pass:
  - (**1** 3 2 5 9) → (**1** 3 2 5 9)
  - (1 **3** 2 5 9) → (1 2 **3** 5 9)
  - (1 2 **3** 5 9) → (1 2 3 **5** 9)
- Third Pass:
  - (**1** 2 3 5 9) -> (**1** 2 3 5 9)
  - (1 **2** 3 5 9) -> (1 **2** 3 5 9)
- Fourth Pass:
  - (**1** 2 3 5 9) -> (**1** 2 3 5 9)

http://www.youtube.com/watch?v=lyZQPjUT5B4

Sorting Preview: Bubble Sort

- CardDeck used BubbleSort to sort the deck
- Simple sorting algorithm that works by repeatedly stepping through the list to be sorted, comparing two items at a time and swapping them if they are in the wrong order
- Repeated until no swaps are needed
- Gets its name from the way larger elements "bubble" to the end of the list
- Time complexity?
  - $O(n^2)$
- Space complexity?
  - O(n) total (no additional space is required)

Sorting Preview: Insertion Sort

- 5 7 0 3 4 2 6 1
- 5 7 0 3 4 2 6 1
- 0 5 7 3 4 2 6 1
- 0 3 5 7 4 2 6 1
- 0 3 4 5 7 2 6 1
- 0 2 3 4 5 7 6 1
- 0 2 3 4 5 6 7 1
- 0 1 2 3 4 5 6 7

Sorting Preview: Insertion Sort

- Simple sorting algorithm that works by building a sorted list one entry at a time
- Less efficient on large lists than more advanced algorithms
- Advantages:
  - Simple to implement and efficient on small lists
  - Efficient on data sets which are already substantially sorted
- Time complexity
  - $O(n^2)$
- Space complexity
  - $O(n)$

Sorting Preview: Selection Sort

- 11 3 27 5 16
- 11 3 16 5 27
- 11 3 5 16 27
- 5 3 11 16 27
- 3 5 11 16 27

- Time Complexity:
  - $O(n^2)$
- Space Complexity:
  - $O(n)$

Sorting Preview: Selection Sort

- Similar to insertion sort
- Performs worse than insertion sort in general
- Noted for its simplicity and performance advantages when compared to complicated algorithms
- The algorithm works as follows:
  - Find the maximum value in the list
  - Swap it with the value in the last position
  - Repeat the steps above for remainder of the list (ending at the second to last position)

Some Skill Testing!

Selection sort uses two utility methods

Uses a swap method

```
private static void swap(int[]A, int i, int j) {
        int temp = a[i];
        A[i] = A[j];
        A[j] = temp;
}
```

And a max-finding method

```
// Find position of largest value in A[0 .. last]
public static int findPosOfMax(int[] A, int last) {
        int maxPos = 0;      // A wild guess
        for(int i = 1; i <= last; i++)
                if (A[maxPos] < A[i]) maxPos= i;
        return maxPos;
}
```

Some Skill Testing!

An Iterative Selection Sort

```
public static void selectionSort(int[] A) {
       for(int i = A.length - 1; i>0; i--)
           int big= findPosOfMax(A,i);
           swap(A, i, big);
       }
}
```

A Recursive Selection Sort (just the helper method)

```
public static void recSSHelper(int[] A, int last) {
       if(last == 0) return; // base case

       int big= findPosOfMax(A, last);
       swap(A,big,last);
       recSSHelper(A, last-1);
}
```

Some Skill Testing!

- Prove: recSSHelper (A, last) sorts elements A[0]…A[last].
  - Assume that maxLocation(A, last) is correct
- Proof:
  - Base case: last = 0.
  - Induction Hypothesis:
    - For k<last, recSSHelper sorts A[0]…A[k].
  - Prove for last:
    - Note: Using Second Principle of Induction (Strong)

Some Skill Testing!

- After call to findPosOfMax(A, last):
  - 'big' is location of largest A[0..last]
- That value is swapped with A[last]:
  - Rest of elements are A[0]..A[last-1].
- Since last - 1< last, then by induction
  - recSSHelper(A, last-1) sorts A[0]..A[last-1].
- Thus A[0]..A[last-1] are in increasing order
  - *and* A[last-1] ≤ A[last].
- So, A[0]…A[last] are sorted.

Comparators

- Limitations with Comparable interface
  - Only permits one order between objects
  - What if it isn't the desired ordering?
  - What if it isn't implemented?
- Solution: Comparators

Comparators (Ch 6.8)

- A comparator is an object that contains a method that is capable of comparing two objects
- Sorting methods can be written to apply a comparator to two objects when a comparison is to be performed
- Different comparators can be applied to the same data to sort in different orders or on different keys

```
public interface Comparator <E> {
    // pre: a and b are valid objects
    // post: returns a value <, =, or > than 0 determined by
    // whether a is less than, equal to, or greater than b
    public int compare(E a, E b);
}
```

Example

Note that Patient does not implement Comparable or Comparator!

```
class Patient {
    protected int age;
    protected String name;
    public Patient (String s, int a) {name = s; age = a;}
    public String getName() { return name; }
    public int getAge() {return age;}
}

class NameComparator implements Comparator <Patient>{
    public int compare(Patient a, Patient b) {
        return a.getName().compareTo(b.getName());
    }
} // Note: No constructor; a "do-nothing" constructor is added by Java
```

```
public void sort(T a[], Comparator<T> c) {
   …
   if (c.compare(a[i], a[max]) > 0) {…}
}
```

```
sort(patients, new NameComparator());
```

Comparable vs Comparator

- Comparable Interface for class X
  - Permits just one order between objects of class X
  - Class X must implement a compareTo method
  - Changing order requires rewriting compareTo
    - And recompiling class X
- Comparator Interface
  - Allows creation of “Compator classes” for class X
  - Class X isn’t changed or recompiled
  - Multiple Comparators for X can be developed
    - Sort Strings by length (alphabetically for equal-length)

Selection Sort with Comparator

```
public static <E> int findPosOfMax(E[] a, int last,
              Comparator<E> c) {
       int maxPos = 0       // A wild guess
       for(int i = 1; i <= last; i++)
              if (c.compare(a[maxPos], a[i]) < 0) maxPos = i;
       return maxPos;
}
public static <E> void selectionSort(E[] a, Comparator<E> c) {
       for(int i = a.length - 1; i>0; i--) {
            int big= findPosOfMin(a,i,c);
            swap(a, i, big);
       }
}
```

- The same array can be sorted in multiple ways by passing different Comparator<E> values to the sort method;

Merge Sort

- A *divide and conquer* algorithm
- Merge sort works as follows:
  - If the list is of length 0 or 1, then it is already sorted.
  - Divide the unsorted list into two sublists of about half the size of original list.
  - Sort each sublist recursively by re-applying merge sort.
  - Merge the two sublists back into one sorted list.
- Time Complexity?
  - Spoiler Alert! We'll see that it's O(n log n)
- Space Complexity?
  - O(n)

Merge Sort

- [8 14 29 1 17 39 16 9]
- [8 14 29 1] [17 39 16 9] split
- [8 14] [29 1] [17 39] [16 9] split
- [8] [14] [29] [1] [17] [39] [16] [9] split
- [8 14] [1 29] [17 39] [9 16] merge
- [1 8 14 29] [9 16 17 39] merge
- [1 8 9 14 16 17 29 39] merge

Merge Sort

- How would we implement it?
- First pass…

// recursively mergesorts A[from .. To] "in place"

```
void recMergeSortHelper(A[],  int from, int to)
    if ( from ≤ to )
        mid = (from + to)/2
        recMergeSortHelper(A, from, mid)
        recMergeSortHelper(A, mid+1, to)
        merge(A, from, to)
```

But *merge* hides a number of important details….

Merge Sort

- How would we implement it?
  - Review MergeSort.java
  - Note carefully how temp array is used to reduce copying
  - Make sure the data is in the correct array!
- Time Complexity?
  - Takes at most 2k comparisons to merge two lists of size k
  - Number of splits/merges for list of size n is log n
  - Claim: At most time O(n log n)…We'll see soon...
- Space Complexity?
  - O(n)?
  - Need an extra array, so really O(2n)! But O(2n) = O(n)

Merge Sort = O(n log n)

- [8 14 29 1 17 39 16 9]
- [8 14 29 1] [17 39 16 9] split
- [8 14] [29 1] [17 39] [16 9] split
- [8] [14] [29] [1] [17] [39] [16] [9] split
- [8 14] [1 29] [17 39] [9 16] merge
- [1 8 14 29] [9 16 17 39] merge
- [1 8 9 14 16 17 29 39] merge

log n

log n

merge takes at most n comparisons per line

Time Complexity Proof

- Prove for $n = 2^k$ (true for other n but harder)
- That is, MergeSort for performs at most
  - $n * \log(n) = 2^k * k$ comparisions of elements
- Base case: $k \le 1$: 0 comparisons: $0 < 1 * 2^1$ ✓
- Induction Step: Suppose true for all integers smaller than k. Let T(k) be # of comparisons for 2^k elements. Then
- $\underline{T(k) \le} 2^k+2*T(k-1) \le 2^k + 2(k-1)2^{k-1} \le \underline{k*2^k}$ ✓

Merge Sort

- Unlike Bubble, Insertion, and Selection sort, Merge sort is a divide and conquer algorithm
  - Bubble, Insertion, Selection sort complexity: $O(n^2)$
  - Merge sort complexity: $O(n \log n)$
- Are there any problems or limitations with Merge sort?
- Why would we ever use any other algorithm for sorting?

Problems with Merge Sort

- Need extra temporary array
  - If data set is large, this could be a problem
- Waste time copying values back and forth between original array and temporary array
- Can we avoid this?

Quick Sort

- Quick sort is designed to behave much like Merge sort, without requiring extra storage space

Merge Sort	Quick Sort
Divide list in half	Partition* list into 2 parts
Sort halves	Sort parts
Merge halves	Join* sorted parts

Recall Merge Sort

```
private static void mergeSortRecursive(Comparable data[],
                    Comparable temp[], int low, int high) {
   int n = high-low+1;
   int middle = low + n/2;
   int i;

   if (n < 2) return;
   // move lower half of data into temporary storage
   for (i = low; i < middle; i++) {
       temp[i] = data[i];
   }
   // sort lower half of array
   mergeSortRecursive(temp,data,low,middle-1);
   // sort upper half of array
   mergeSortRecursive(data,temp,middle,high);
   // merge halves together
   merge(data,temp,low,middle,high);
}
```

Quick Sort

```
public void quickSortRecursive(Comparable data[],
                    int low, int high) {
    // pre: low <= high
    // post: data[low..high] in ascending order
        int pivot;
        if (low >= high) return;

        /* 1 - place pivot */
        pivot = partition(data, low, high);
        /* 2 - sort small */
        quickSortRecursive(data, low, pivot-1);
        /* 3 - sort large */
        quickSortRecursive(data, pivot+1, high);
}
```

Partition

1. Put first element (pivot) into sorted position
2. All to the left of “pivot” are smaller and all to the right are larger
3. Return index of “pivot”

Partition

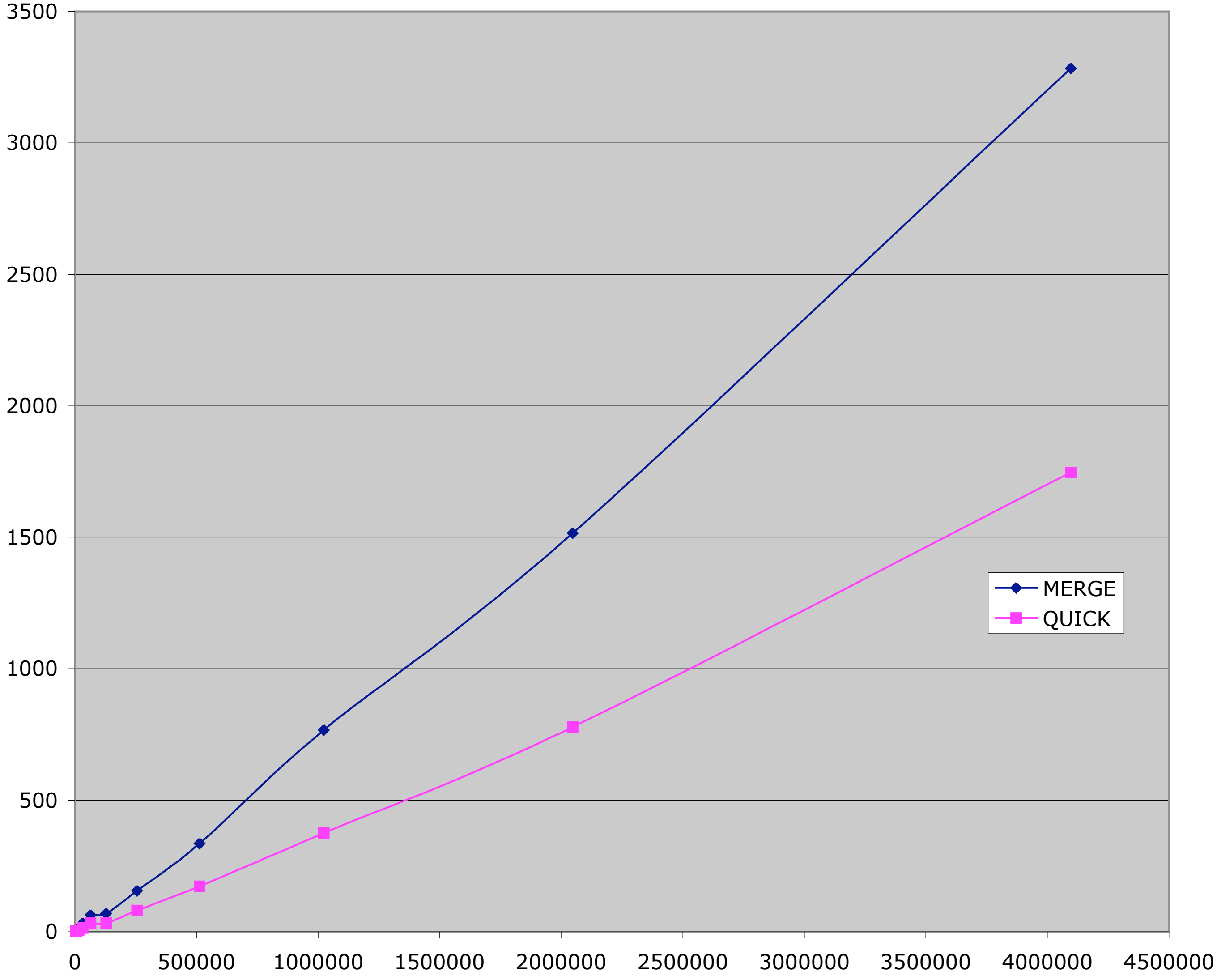
```
int partition(int data[], int left, int right) {
  while (true) {
    while (left < right && data[left] < data[right])
      right--;
    if (left < right) {
      swap(data,left++,right);
    } else {
      return left;
    }

    while (left < right && data[left] < data[right])
      left++;
    if (left < right) {
      swap(data,left,right--);
    } else {
      return right;
    }
  }
}
```

Complexity

- Time:
  - Partition is $O(n)$
  - If partition breaks list exactly in half, same as merge sort, so $O(n \log n)$
  - If data is already sorted, partition splits list into groups of 1 and n-1, so $O(n^2)$
- Space:
  - $O(n)$ (so is MergSort)
    - In fact, it's n + c compared to 2n + c for MergeSort

Merge vs. Quick



Food for Thought…

- How to avoid picking a bad pivot value?
  - Pick median of 3 elements for pivot (heuristic!)
- Combine selection sort with quick sort
  - For small n, selection sort is faster
  - Switch to selection sort when elements is <= 7
  - Switch to selection/insertion sort when the list is almost sorted (partitions are very unbalanced)
    - Heuristic!

Sorting Wrapup

	Time	Space
Bubble	Worst: $O(n^2)$ Best: O(n) - if "optimiazed"	O(n) : n + c
Insertion	Worst: $O(n^2)$ Best: O(n)	O(n) : n + c
Selection	Worst = Best: $O(n^2)$	O(n) : n + c
Merge	Worst = Best:: O(n log n)	O(n) : 2n + c
Quick	Average = Best: O(n log n) Worst: $O(n^2)$	O(n) : n + c

More Skill-Testing (Try these at home)

Given the following list of integers:

9 5 6 1 10 15 2 4

1) Sort the list using Bubble sort. Show your work!
2) Sort the list using Insertion sort. . Show your work!
3) Sort the list using Merge sort. . Show your work!
4) Verify the best and worst case time and space complexity for each of these sorting algorithms as well as for selection sort.