Lambda Calculus

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- Computability, Logic
- Lambda Calculus

- Why Lambda?

Only 3 "Interesting Parts" of Any PL

- Variables:
  - `x, y, z`
- Function Definitions:
  - `λx.x+3 == (lambda (x) (+ x 3))`
- Function Application:
  - `(λx.x+3) 6` → `6+3`
  - `(λa.(λb.a*b)) 3 4` → `(λb.3*b) 4` → `3*4`

Really, That's It?

```
int f(int x) { return x + 10; }
f(5);

let f = λx.x+10  in f(5)

(λf.f(5))(λx.x+10)
```


(λf.λx.f(f x))

```
(defun twice (f)
   (lambda (x) (f (f x))))
```

Paren Rules
- `A B C = (A B) C`
- `λx.xy = λx.(xy)`

```
  (λf.λx.f(f x))(λy.y+1)2
= ((λf.λx.f(f x))(λy.y+1)) 2
→ (λx.(λy.y+1)((λy.y+1)x)) 2
→ (λx.(λy.y+1)(x+1)) 2
→ (λx.(x+1)+1) 2
→ (2+1)+1
```

```
(λf.λx.f(f x))(λz.x+z)2 =
(λf.λa.f(f a))(λz.x+z)2 ->      (rename x)
([(λz.x+z)/f]λa.f(f a))2 =      (reduce f)
(λa.(λz.x+z)((λz.x+z)a))2 =     (substitution)
(λa.(λb.x+b)((λz.x+z)a))2 ->    (rename z)
(λa.(λb.x+b)([a/z](x+z))2 =     (reduce z)
(λa.(λb.x+b)(x+a))2 ->          (substitution)
(λa.[(x+a)/b](x+b))2 =          (reduce b)
(λa.x+(x+a))2 ->                (substitution)
[2/a](x+(x+a))=                 (reduce a)
x+x+2                           (substitution)
```



```
(λf.λx.f(f x))(λz.x+z)2 =
(λf.λa.f(f a))(λz.x+z)2 ->      (rename x)
([(λz.x+z)/f]λa.f(f a))2 =      (reduce f)
(λa.(λz.x+z)((λz.x+z)a))2 =     (substitution)
(λa.(λb.x+b)((λz.x+z)a))2 ->    (rename z)
```

Try a different step.
What is the final result?


```
(λf.λx.f(f x))(λz.x+z)2 =
(λf.λa.f(f a))(λz.x+z)2 ->      (rename x)
([(λz.x+z)/f]λa.f(f a))2 =      (reduce f)
(λa.(λz.x+z)((λz.x+z)a))2 =     (substitution)
(λa.(λb.x+b)((λz.x+z)a))2 ->
(λa.(λb.x+b)(x+a))2 ->
(λa.x+x+a)2 ->
 x+x+2
```

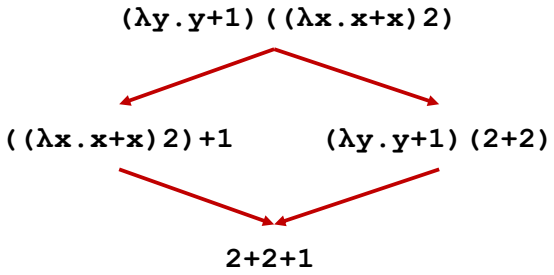


```
(λf.λx.f(f x))(λz.x+z)2 =
(λf.λa.f(f a))(λz.x+z)2 ->      (rename x)
([(λz.x+z)/f]λa.f(f a))2 =      (reduce f)
(λa.(λz.x+z)((λz.x+z)a))2 =     (substitution)
(λa.(λb.x+b)((λz.x+z)a))2 ->
(λa.x+((λz.x+z)a))2 ->
(λa.x+(x+a))2 ->
 x+x+2
```



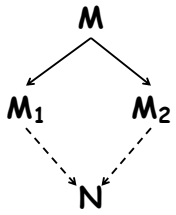
```
(λf.λx.f(f x))(λz.x+z)2 =
(λf.λa.f(f a))(λz.x+z)2 ->      (rename x)
([(λz.x+z)/f]λa.f(f a))2 =      (reduce f)
(λa.(λz.x+z)((λz.x+z)a))2 =     (substitution)
(λa.(λb.x+b)((λz.x+z)a))2 ->
(λb.x+b)((λz.x+z)2)->
(λb.x+b)(λz.x+2)->
 x+x+2
```


Confluence



Confluence

If $M \rightarrow M_1$ and $M \rightarrow M_2$ then $M_1 \rightarrow^* N$ and $M_2 \rightarrow^* N$ for some N



Is Functional Programming Better?

Pros?
Cons?